PRACTICAL BUSINESS ARITHMETIC

KIGGEN

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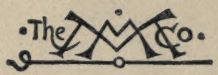

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PRACTICAL BUSINESS ARITHMETIC

BY

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PREFACE

THIS book has been prepared particularly for the use of teachers and pupils of continuation schools, pre-vocational classes, trade schools, and high schools offering courses in salesmanship, dressmaking, millinery, and domestic science. It presents the work as it has been practically conducted by the author during years of experience as teacher of continuation schools, and instructor in business methods in high schools.

It is designed to meet the educational needs of all those who wish to enter the business world and those who have entered it without a sufficient knowledge of the fundamental operations in arithmetic, or who lack the power to apply their knowledge to the solution of present-day business problems.

The chief consideration in the preparation of the book has been the needs of the pupil; and only such phases of work have been presented as are applicable to business. The problems selected are real, not imaginary, and are such as arise in the daily work or business surroundings of the pupil. The work is directed to the solution of problems by the shortest methods, thus minimizing the opportunity or likelihood of error and adding to the bread-winning power of the worker.

The author has not only endeavored to develop arithmetical skill, but, through the selection and solution of the problems given, has tried to instil into the minds of the pupils the character-making quality of individual responsibility in money matters, which is an outgrowth of the prac-

tice of thrift, and to show the loss resulting from waste, to the individual, to the community, and to the nation.

The author wishes to acknowledge her indebtedness to Mrs. Lucinda Wyman Prince, Director of the Prince School of Salesmanship, Boston, and Director of the Department of Education of the National Retail Dry Goods Association, New York, for valuable suggestions regarding the character and practicability of the work, and to the following educators and teachers, from whom she has received most helpful advice and assistance: Miss Mary Mellyn, Assistant Superintendent, Boston Public Schools; Miss Isabel Bacon, Federal Board of Vocational Education; Mr. Schuyler Herron, Director of Educational Institutions, First Federal Reserve, Boston; Mr. Herbert S. Weaver, Head Master, High School of Practical Arts, Boston; Mrs. Margaret Moore, Instructor in Dressmaking, and Miss Clara Gay, Instructor in Millinery, High School of Practical Arts; Miss Grace Blanchard, Head of Girls' Continuation School, Boston; Miss Alice Falvey, Instructor in Salesmanship, South Boston High School; and many other friends and co-workers who have given their assistance.

CONTENTS

PRACTICAL BUSINESS ARITHMETIC

CHAPTER I

ADDITION

When and how is addition used in business houses?

What are the advantages to the employer of being able to add with accuracy and rapidity?

To the customer?

To the employee?

To every citizen in any employment or capacity?

1. Fundamental processes. Rapid addition depends largely upon the ability instantly to combine two or more numbers into a single number.

1. The following combinations of numbers up to 10, selected from business problems, are those upon which employees most frequently fail in daily work.

ORAL EXERCISES

(1) Pronounce at sight the sum of each group:

a. from bottom to top;
b. from top to bottom;
c. from left to right;
d. from right to left.

(2) Continue drill until sums can be named at the rate of 125 a minute.

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*
3	7	8	6	3	9	7	4	2	8	9	7	6	5
8	3	7	3	2	3	4	5	9	4	6	7	4	7
—	—	—	—	—	—	—	—	—	—	—	—	—	—

o	p	q	r	s	t	u	v	w	x	y	z	a'	a''
9	8	6	4	9	8	6	3	7	5	9	8	2	3
8	6	5	3	4	9	6	5	9	5	5	8	7	9

(3) It is as easy to add 23 and 8 as it is to add 3 and 8. 3 and 8 are always equal to 1 ten and 1 unit; hence in adding 23 and 8 think of the tens as increased by 1, call the units 1, and the result is 31.

(4) Pronounce at sight the sum of each of the following groups, and continue series work until the sight of each combination causes an automatic response.

13	23	33	43	53	63	73	83	93	103	17	27	37	47	57
8	8	8	8	8	8	8	8	8	8	3	3	3	3	3
67	77	87	97	107	18	28	38	48	58	68	78	88	98	108
3	3	3	3	3	7	7	7	7	7	7	7	7	7	7
16	26	36	46	56	66	76	86	96	106	13	23	33	43	53
3	3	3	3	3	3	3	3	3	3	2	2	2	2	2
63	73	83	93	103	19	29	39	49	59	69	79	89	99	109
2	2	2	2	2	3	3	3	3	3	3	3	3	3	3
17	27	37	47	57	67	77	87	97	107	14	24	34	44	54
4	4	4	4	4	4	4	4	4	4	5	5	5	5	5
64	74	84	94	104	12	22	32	42	52	62	72	82	92	102
5	5	5	5	5	9	9	9	9	9	9	9	9	9	9
18	28	38	48	58	68	78	88	98	108	19	29	39	49	59
4	4	4	4	4	4	4	4	4	4	6	6	6	6	6

69	79	89	99	109	17	27	37	47	57	67	77	87	97	107
6	6	6	6	6	7	7	7	7	7	7	7	7	7	7

16	26	36	46	56	66	76	86	96	106	15	25	35	45	55
4	4	4	4	4	4	4	4	4	4	7	7	7	7	7

65	75	85	95	105	19	29	39	49	59	69	79	89	99	109
7	7	7	7	7	8	8	8	8	8	8	8	8	8	8

18	28	38	48	58	68	78	88	98	108	16	26	36	46	56
6	6	6	6	6	6	6	6	6	6	5	5	5	5	5

66	76	86	96	106	14	24	34	44	54	64	74	84	94	104
5	5	5	5	5	3	3	3	3	3	3	3	3	3	3

19	29	39	49	59	69	79	89	99	109	18	28	38	48	58
4	4	4	4	4	4	4	4	4	4	9	9	9	9	9

68	78	88	98	108	16	26	36	46	56	66	76	86	96	106
9	9	9	9	9	6	6	6	6	6	6	6	6	6	6

13	23	33	43	53	63	73	83	93	103	17	27	37	47	57
5	5	5	5	5	5	5	5	5	5	9	9	9	9	9

67	77	87	97	107	15	25	35	45	55	65	75	85	95	105
9	9	9	9	9	5	5	5	5	5	5	5	5	5	5

19	29	39	49	59	69	79	89	99	109	18	28	38	48	58
5	5	5	5	5	5	5	5	5	5	8	8	8	8	8

68	78	88	98	108	12	22	32	42	52	62	72	82	92	102
8	8	8	8	8	7	7	7	7	7	7	7	7	7	7

13	23	33	43	53	63	73	83	93	103
9	9	9	9	9	9	9	9	9	9

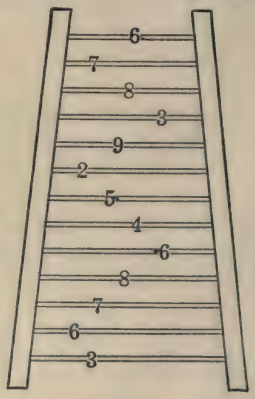
(5) The necessary repetitions may be made interesting by numerous devices for competitive activities:

Circle

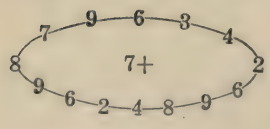
Go around circle adding central figure each time before adding figures on the edge of circle.

Ladder

Climb quickly to the top of the ladder without missing a round; if a failure is made, climb down and master the combination before climbing again to the top.



Race Track



Add middle figure to each figure on the edge in turn, then go swiftly around the track, adding middle figure again at the end.

(6) Name the sum in each of the following problems:

a. Add silently.
b. Add aloud.
c. Add up.
d. Add down.

4	5	3	5	6	8	9	5	1	2	6	9	4	5	3
6	7	0	8	6	2	8	5	2	0	5	9	0	4	7
7	3	6	3	5	7	6	9	3	4	0	3	3	4	2
3	2	8	6	0	0	7	6	4	1	9	4	9	3	1
1	9	2	7	9	6	3	3	0	0	1	0	6	9	7
2	0	9	2	0	3	8	0	5	6	7	8	8	6	9
9	6	7	9	8	9	5	8	6	8	8	7	7	8	0
1	8	1	5	1	5	2	7	7	3	0	0	5	7	6
8	7	4	6	7	4	7	9	0	1	3	9	0	8	8
3	3	0	7	5	0	9	7	8	9	5	6	1	0	5
7	5	8	4	4	8	0	0	9	7	2	3	2	2	9
5	2	7	8	0	6	6	1	9	2	7	5	9	6	7
6	9	6	3	8	7	8	2	0	4	0	2	6	7	3
3	8	3	1	6	3	7	9	8	1	9	8	7	1	2
2	0	2	0	7	0	4	8	7	8	8	7	4	8	9
0	7	9	9	3	9	0	0	6	6	6	5	3	7	0
9	6	0	8	0	8	9	3	1	8	0	6	8	3	6
8	9	8	6	9	1	8	6	5	7	5	1	7	4	8

2. Combinations of numbers from 11 to 19, inclusive.

The following combinations should be drilled upon and learned in the same way as those in (3) and (4) on p. 2. They are important for rapid addition; the instant calling of results should be required. In making these combinations the second number should be thought of as 1 ten and a given number of units, and not as a given number of units and 1 ten; thus in combining 15 and 17 we should think of 25 and 7; in combining 19 and 13, of 29 and 3.

ORAL EXERCISE

Name sums at sight:

12	13	11	18	11	11	13	12	12	17	11	16	16	13
13	15	17	13	11	12	13	14	12	14	16	16	15	16

19	18	17	12	14	15	19	16	15	19	17	18	15	18	14	14
19	17	17	15	19	15	13	19	17	18	19	18	18	12	18	14

11	18	14	12	11	16	19	14	16	13	16	12	17	18	11
19	11	13	17	14	12	15	15	17	11	14	19	13	16	15

2. Adding by groups.

9—53
7
4—37
6
8—27
4
9—15
6
53

In adding columns of figures one should think first of the sum of each group, then group the sums and name the sum of the groups. In the example given, think and name the sum of the first group as 15 without naming the figures of the group; the sum of the second group as 12; then the sum of 15 and 12 is thought and named. The sum of 27 and the next group 10, is thought of,— say 37; then 16, 53. In adding, name the results of the groups only as: 15, 27, 37, 53. The same problems may be added both up and down, affording double drill, and at the same time providing opportunity for detecting possible errors. If the same result is obtained from adding both ways, the sum is usually correct.

1. Adding two columns.

Two columns of figures may be added at once by practice. The tens should be combined first and the units added to

their sums, page 2, (3). The following problems may be used for practice in single and double column adding.

ORAL EXERCISE

Drill until the group sums can be named quickly, adding both up and down:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*
17	75	67	38	45	67	70	36	83	19
29	14	95	84	24	13	56	67	54	56
13	85	76	10	98	28	17	22	30	75
31	38	24	42	76	83	98	71	28	42
19	17	53	17	67	98	30	23	34	13
40	24	92	50	19	14	33	47	31	88
37	95	13	63	14	76	42	85	15	71
84	71	77	28	67	8	12	56	10	48
22	36	38	15	32	25	19	42	42	50
45	22	24	78	78	32	30	98	70	23
—	—	—	—	—	—	—	—	—	—

k	*l*	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*
76	33	68	76	32	98	57	71	89	78
5	22	19	58	11	21	10	95	4	23
21	76	47	40	94	15	34	11	31	42
19	89	94	32	26	4	19	24	17	12
30	71	32	39	18	9	56	32	67	13
72	48	25	36	45	27	81	16	54	25
25	50	10	44	31	32	9	48	43	67
42	23	74	9	6	99	30	23	91	32
8	9	99	32	9	45	76	9	10	9
14	35	8	17	56	52	21	75	36	28
—	—	—	—	—	—	—	—	—	—

2. Practice adding by groups. Practice in grouping will lead to great proficiency in adding accurately and rapidly.

WRITTEN EXERCISE

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*
180	614	768	234	297	1097	25	854	321	6798
385	718	347	723	178	485	1678	9	2753	5
276	259	905	348	781	234	428	9876	1046	813
324	753	179	150	345	76	96	3714	6978	1644
943	187	897	347	543	1865	1865	2943	287	876
432	981	317	501	971	2843	402	108	56	2454
738	728	182	753	508	804	45	96	4523	43
597	578	859	408	845	68	2825	4196	874	19
365	401	854	587	578	6751	98	3068	19	8607
276	374	548	423	748	234	654	5862	2698	841

k	*l*	*m*	*n*	*o*	*p*	*q*
46	6731	92,483	12,345	40,020	836,427	155,208
818	432	65,789	8,027	95,160	359,208	143,346
8	3542	92,475	36	84	674,128	72,109
2	2089	68,941	13,652	3,070	342,108	46,343
67	763	56,634	7,209	16,898	85,096	272,654
138	48	78,924	18,164	9	27	180,348
849	96	82,116	3,456	23,274	8,560	9,674
9324	75	28,354	20,004	85,615	902,209	780,965
876	120	19,175	948	24,934	65,498	65,498
23	1896	14,628	18,272	648	876	165,349
62	45	32,158	14,138	15,001	161,511	461,020
491	1921	13,462	2,156	3	41,000	13,092
4632	1462	11,631	98	4,271	16,781	244,444
91	3	4,567	24	914	1	3,323

r	*s*	*t*
811,202	186,195	777,777
96,315	365,754	234,568
509,732	467,989	101,393
81,694	356,878	426,751
576,352	987,566	808,909
46,758	106,662	50,672
25,196	811,309	85,109
808,174	828,352	76
78,968	808,099	8
1,678	99	303,968

3. Horizontal addition. Horizontal addition is used in some kinds of invoicing, time sheets, sales sheets, statistical reports, and in extending the items of an account. Time is saved by adding the numbers as they stand. Care must be taken to combine units of the same order only.

ILLUSTRATIVE EXAMPLE:

Check

156 + 324 + 289 = 769
342 + 165 + 824 = 1331
565 + 278 + 324 = 1167
396 + 152 + 297 = 845

Check 1459 + 919 + 1734 = 4112

ORAL EXERCISE

1. Add from left to right: verify by adding from right to left.

2. Add by columns; check the result by adding the horizontal and vertical totals. If these totals are not alike, the

student should find and correct the error without help from the teacher:

a

385+416+212+279=?
672+289+783+578=?
524+753+829+987=?
346+273+654+528=?
548+653+936+364=?

\+ + + =

b

911+788+882+739+531=?
314+408+516+408+504=?
647+583+357+136+897=?
763+345+263+413+187=?
581+317+231+517+486=?

\+ + + + =

c

562+614+212+119+432=?
284+482+738+817+288=?
672+473+928+265+471=?
518+891+657+627+335=?
525+827+762+417+178=?

\+ + + + =

3. Copy and add the following numbers horizontally, beginning at the left:

a. 9, 3, 4, 2, 8, 7, 6, 5, 9, 8, 4, 7, 6, 8
b. 63, 28, 93, 45, 64, 73, 82, 97, 25, 81
c. 456, 328, 297, 694, 865, 937, 489
d. 7082, 3695, 8716, 3998, 7959, 6825
e. 9980, 3576, 2190, 8998, 7684

4. Add horizontally from left to right, then from right to left, the columns of numbers in Ex. (6) page 5.

4. Methods of attaining accuracy and speed. The surest way to secure accuracy and speed is through frequent repetition, quickly done.

1. It is always an advantage to find groups of figures aggregating 10, hence whenever these combinations appear in

addition problems, make use of them in the following way and thus simplify the work:

6	10	3		9		Look through problems first to find combinations making 10 or 20; add column up, "jumping" the 10's,—5, 15, 25, 35, 45; add column down, "jumping" the 10's,—10, 20, 30, 40, etc.
4		4	10	2	20	
3	10	3		9		
7		2		5		
9	10	7	10	1		
1		1		4	20	
8	10	6		7		
2		2	10	3		
5		2		9		
45		8		5	20	
		38		6		
				60		

EXERCISES

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*
3	8	6	4	6	8	4	6	6	6
2	4	4	3	1	4	8	7	3	5
1	6	1	1	8	9	7	8	4	1
4	2	9	7	2	7	5	7	9	8
2	9	2	7	5	7	3	6	3	2
8		2	2	2	7	2	6	3	2
3	1	5	1	5	7	2	4	5	6
7	7	8	8	8	9	4	6	1	3
4	3	8	6	8	8	4	8	8	5
2	6	5	4	4	3	7	7	2	4
3	2	7	1	9	6	3	8	7	1
5	9	3	2	1	1	3	5	2	2
4	3	1	9	0	5	8	6	1	2
2	4	9	9	3	6	6	9	9	3
1	8	3	4	8	9	0	5	6	5
2	8	4	6	0	2	6	0	5	6

k	*l*	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*
7	8	4	5	4	5	2	9	2	8
7	6	3	7	3	2	3	9	2	8
4	4	8	1	9	2	6	2	6	4
3	1	2	2	6	8	7	8	8	2
9	3	7	4	5	2	2	7	7	9
9	6	7	2	4	5	2	5	1	8
7	6	3	9	5	2	3	4	2	1
8	9	5	2	8	7	8	3	9	7
2	7	4	9	8	9	7	9	1	6
7	3	1	8	3	1	3	2	8	4
3	5	6	7	9	9	2	9	4	3
6	2	6	5	5	0	5	4	8	8
1	2	8	4	4	1	6	7	6	6
7	9	2	3	9	4	9	8	7	6
3	1	7	6	2	9	3	3	7	2
6	2	3	7	6	6	4	9	3	3

2. When five figures are in consecutive order, the sum may be found by multiplying the middle figure by 5; the sum of any number of consecutive figures is found by taking one-half the sum of the first and last figures and multiplying by the number of figures in the column to be added.

ILLUSTRATIVE EXAMPLE:

6 ½ of (6+8) =7; 7 ×3 =21; or multiply middle figure
7 by (3)
8
21

By inspection find the sum of:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*
4	2	7	11	9	13	22	19	41	56	67	72	87	90
5	3	8	12	10	14	23	20	42	57	68	73	88	91
6	4	9	13	11	15	24	21	43	58	69	74	89	92

ORAL EXERCISES

Find by inspection the sum of each of the following:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*
10	21	43	67	29	49	24	80	92	69	18	1	39	84
11	22	44	68	30	50	25	81	93	70	19	2	40	85
12	23	45	69	31	51	26	82	94	71	20	3	41	86
13	24	46	70	32	52	27	83	95	72	21	4	42	87
14	25	47	71	33	53	28	84	96	73	22	5	43	88
15	26	48	72	34	54	29	85	97	74	23	6	44	89
16	27	49	73	35	55	30	86	98	75	24	7	45	90
17	28	50	74	36	56	31	87	99	76	25	8	46	91
18	29	51	75	37	57	32	88		77	26	9	47	92
		52	76	38	58	33	89		78	27	10	48	93
		53	77	39	59	34	90			28	11	49	94
							91				12	50	

3. The sum may be found by multiplication when the figure is repeated several times. State at sight the sum of the following:

ORAL EXERCISE

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*	*o*
8	9	6	18	23	98	74	13	76	84	47	19	86	49	69
8	9	6	18	23	98	74	13	76	84	47	19	86	49	69
8	9	6	18	23	98	74	13	76	84	47	19	86	49	69
8	9	6	18	23	98	74	13	76	84	47	19	86	49	69
4	9	6	18	23	98	74	13	76	84	47	19	86	49	69
4	3	7	18	36	89	47	13	9	84	47	19	74	49	69
4	3	7	18	36	89	47	52	9	12	54	19	74	49	69
	3	7	18	36	89	47	52	9	12	54	98	74	8	99
		7		36		47	52	9	12	54	98	74	8	99
								9		54	98	74	8	99
										54	98	74	8	99
											98		8	99

4. Many of the errors made by the average employee during business hours result either from irregularity in the placing of figures, or from poorly made figures. It is important that figures be neat, legible, and of uniform size. Skill in writing figures from dictation should be cultivated, and exercises given until the pupil can write numbers rapidly, placing units of the same order in the same vertical column. Speed should not be secured at the expense of legibility.

1 2 3 4 5 6 7 8 9 0

(1) See how many times the following numbers can be written in one minute:

a. 663498	*d.* $8065.28	*g.* $301875.56
b. 725634	*e.* $4080.79	*h.* $593701.89
c. 817906	*f.* $2985.44	*i.* $246805.47

(2) Increase the speed gradually until from 150 to 200 figures can be written in a minute, keeping the spacing between the lines and between the columns uniform.

5. In order to insure accuracy in reading United States money, the word "dollars" should be called with each amount, the word "cents" being omitted except where there are no dollars. Thus in reading, "$300.52," say "three hundred dollars, fifty-two."

WRITTEN EXERCISE

Write from dictation and find the sum of:

$2.35, $1.95, $132.50, $196.84, $372.85, $64.78, $56.10, $241.89, $75.10, $615.20, $408.89, $5608.42, $3267.91, $24.86, $516.59, $780.46, $90.57, $3400.50, $500.40, $800.90, $520.70, $100.95, $410.70, $90.80, $699.70, $422.80.

6. Dollars and cents are sometimes expressed without the dollar sign and the decimal point, thus: \$28.16 may be written 28^{16}; \$5.06 may be written 5^{06}.

WRITTEN EXERCISE

Add horizontally:

a. 25^{36}, 50^{48}, 61^{24}, 19^{98}, 23^{75}, 10^{33}, 64^{85}, 40^{52}, 99^{75}

b. 86^{75}, 48^{87}, 95^{46}, 52^{50}, 42^{28}, 60^{60}, 85^{23}, 90^{56}, 87^{42}, 16^{10}

c. 49^{52}, 37^{15}, 67^{43}, 84^{96}, 28^{32}, 47^{66}, 85^{17}, 49^{64}, 115^{25}

7. In a recent analysis of 1000 errors on sales slips in a large department store, 416 were due to mistakes in carrying the wrong figure when adding the next column. This can be avoided by recording the total sum of each column separately. In adding long columns of figures (more than ten numbers) this practice should be followed. This is sometimes known as the "Civil Service Method," because it is so much used in government offices.

ILLUSTRATIVE EXAMPLE:

	\$15.98	
	4.50	
	27.35	
Add: (1)	84.96	*Check:* (2)
	52.67	
49	18.75	32
64	3.98	46
46	27.74	64
32	62.81	49
	74.15	
\$372.89		\$372.89
	49	
	64	
	46	
	32	
	\$372.89	

5. Checking results. The simplest way to check addition is to add the columns in reverse order; if the sums obtained by both processes agree, the work is usually correct. In the illustrative example, check the result.

1. Beginning at the bottom of the right hand column, add each column in regular order and write the entire total, as shown in (1), Ex. 7, p. 15. Beginning at the top of the left hand column, add each column and write the entire totals as shown in (2), Ex. 7, p. 15. Then add totals obtained by the first and second additions, and compare results. The results being equal, the work is assumed to be correct.

WRITTEN EXERCISE

Copy, find the sum, and check:

a	*b*	*c*	*d*
$ 3.39	$ 2,468.25	$12,564.32	$ 45,832.19
53.24	356.72	87,905.16	98.75
27.50	20,508.97	218.74	6,075.18
40.08	1,234.18	8.96	8.95
50.63	92.54	4,007.29	30,419.98
24.75	675.89	80,906.32	2.36
30.10	30,907.84	28.75	98.75
36.52	7.68	3,472.99	7,894.23
116.70	95.27	87.54	10,287.05
50.70	10,348.96	50,679.25	9,687,438.45
16.22	116.74	4.87	37,296.24
153.86	32.28	24.98	81.16
30.70	808.64	67,348.95	303.49
8.63	1.98	2,758.65	80,709.58
26.72	74.36	10.74	468,372.25
56.46			90.98
145.60			

e	*f*	*g*
$ 156,518.85	$ 323,542.46	$2,620,178.35
89,653.27	88,575.57	2,582,614.92
9,845.00	24,567.89	1,361,426.42
578,716.15	13,579.86	1,029,011.60
384.68	19,668.42	991,834.48
74,264.36	427,687.59	721,216.74
947,862.87	2,070,965.74	32,994.36
36,045.80	849,327.68	18,917.45
165,891.73	236,478.54	416,918.35
5,364.50	754,268.86	16,019.40
724,683.42	733,261.42	7,896.75
92,785.25	321,918.96	1,707,834.62
459,482.30	73,293.60	1,875.98
7,850.00	13,050.00	13,291.75
97,524.64	8,254.68	1,518,253.46
457,364.80	37,074.84	1,211,949.57
9,013,682.25	795,125.25	39,874.26
62,943.12	821,203.11	7,241,837.75
5,438.80	612,746.08	156,432.78
5,278,610.04	842,628.02	216,224.64
364,528.74	721,354.98	84,916.59
124,347.61	78,129.60	312,692.48
28,752.19	2,023,015.58	888,850.69
17,632.28	983,107.29	82,162.37
367,849.19	131,300.60	3,596,038.69
251,963.17	152,537.56	859,063.96
86,532.78	384,569.46	15,271.58
374,289.61	78,398.58	2,828.54
45,032.06		
1,364,463.58		
37,850.64		

2. The most convenient, reliable, and practical method of testing the accuracy of the work in addition, has been given; checking results, by casting out the 9's or 11's, may also be used.

3. The excess of 9's in any number is equal to the excess of 9's in the sum of its digits. Thus, the sum of the digits in the number 5432 (5+4+3+2) is 14. Divide the sum, 14, by 9, and the remainder, 5, is the excess.

ILLUSTRATIVE EXAMPLE 1: Suppose we wished to check the result, 2074, in the illustrative example given, by casting out the 9's. Add the digits in each of the addends and divide the sum by 9. The remainder is called "the excess." (It is the remainder which would be found if the entire number were divided by 9.) After the excess of nines in each addend has been found, add these excesses, and divide the result by 9. The remainder (or excess) should equal the excess in the sum in the illustrative example.

Excesses	
796——	4
384——	6
516——	3
378——	0
2074——	13

Excess in the sum, 2074, equals 4. This is also the excess in the sum of the excesses, 13.

The excess in the sum must equal the excess in the sum of excesses.

What is the excess of 9's in 248? 560? 2348? 4132? 9364?

ILLUSTRATIVE EXAMPLE 2: The addition of digits to find the excess of 9's may be simplified by ignoring 9's, combinations which equal 9, and multiples of nine; thus, in finding the excess of 9's in 9364, begin at the left, reject the first digit 9, the sum of the next two digits (3+6) 9, and the single 4 will be the excess of 9's in the entire number.

Excesses	
4156——	7
3297——	3
1036——	1
2473——	7
6124——	4
17086——	22 = sum of the excesses.

Excess in the sum (17086) equals 4. This is also the excess in the sum of the excesses.

4. Casting out nines is not an absolute proof of the accuracy of any process. It will not disclose a transposition of figures (such as 698 for 968), nor can the omission or addition of 9's or ciphers be detected.

5. Casting out elevens. The excess of 11's in any number is equal to the sum of the digits in the odd places (beginning with units) minus the sum of the digits in the even places. If the sum of the digits in the even places is greater than the sum of the digits in the odd places, add 11, or a multiple of 11, to the sum of the digits in the odd places. Thus: the excess of 11's in 479 is 6, (9+4—7); the excess of 11's in 217 is 8, (7+2—1=8); the excess of 11's in 4286 is 7, (6+2+11)—(8+4).

Illustrative Example 3: By casting out the 11's, show that 355+657+324+436+249=2021.

	Excess		*Excess*
355	=5+3—5=3		
657	=7+6—5=8	7+7+5+8+3=30	
324	=4+3—2=5		
436	=6+4—3=7	30=2 elevens and 8 over.	
249	=9+2—4=7		
2021	30		
(1+0+11)—(2+2)=8			

6. Errors not located by casting out nines can be found by casting out elevens.

Check examples *a*, *b*, *c*, *d*, on p. 16, by casting out 9's.

Check examples *e*, *f*, *g*, on p. 17, by casting out 11's.

Write from dictation, find the sum of, and check results by casting out nines.

(1) $28.36, $125.78, $142.17, $50.65, $175.38, $196.45, $98.39, $156.42, $38.07, $24.80, $342.24, $196.08.

6. Drills. One of the best possible drills in addition may be found in the following exercise. The results should be named at the rate of 125 a minute.

ORAL EXERCISE

Count or add by

a. 2's from 19 to 69	*f.* 7's from 1 to 78
b. 3's from 1 " 77	*g.* 8's from 32 " 192
c. 4's from 3 " 75	*h.* 9's from 12 " 102
d. 5's from 1 " 61	*i.* 11's from 14 " 113
e. 6's from 3 " 81	*j.* 12's from 16 " 160

Beginning at 3 count by 6's to 39; going on from 39 count by 7's to 81; from 81 count by 8's to 129; from 12 count by 9's to 192.

1. Complete the following sales sheet; add by columns and by lines; check the work by adding the vertical and horizontal totals.

SUMMARY OF DEPARTMENTAL SALES FROM JUNE 1, 1919, TO JUNE 13, 1919

Days	Suit Dept.		Millinery		Yard Goods		Under-wear		Hosiery		Totals	
Mon....	698	75	165	20	516	45	825	96	615	25		
Tues....	901	50	298	50	349	28	712	49	479	36		
Wed....	857	37	568	00	654	49	954	32	752	12		
Thurs...	763	24	764	15	878	65	876	67	369	18		
Fri.....	504	12	652	25	496	96	479	87	475	25		
Sat.....	1008	65	857	16	985	49	758	56	684	19		
Mon....	972	20	401	50	649	17	636	50	721	35		
Tues....	659	45	912	45	156	48	519	27	846	19		
Wed....	768	37	356	60	489	98	487	65	796	69		
Thurs....	812	96	825	12	374	16	652	45	958	27		
Fri.....	479	89	965	78	512	29	918	19	438	59		
Sat.....	276	65	826	65	674	42	548	72	652	27		
Total												

DEPARTMENTAL SALES FOR WEEK ENDING NOVEMBER 31, 1921

Day	Furs	Rugs	Carpet	Furniture	Upholstery	Total
Mon.....	936.45	1102.17	978.56	932.24	876.75	?
Tues.....	1026.45	1078.32	972.28	868.27	767.15	?
Wed.....	1140.15	1276.42	1096.24	912.45	524.96	?
Thurs.....	1076.84	1395.45	975.42	1192.18	675.38	?
Fri.....	1061.35	1428.17	828.45	1036.75	542.16	?
Sat.....	1248.78	1528.32	1065.28	1978.25	862.12	?
Total.....	?	?	?	?	?	?

Busy Section of a Large City

Find the total sales for each day in the five departments indicated above.

Find the total sales for the week in each department.

Find the total sales made during the week.

2. Find the footing of each of the following inventory extensions, allowing two minutes for the adding of each column:

a	*b*	*c*	*d*
$2,861.54	$1,856.17	$1,765.48	$ 12.96
297.36	28.98	3,056.23	1,364.87
568.62	2,011.54	154.30	198.45
1,204.98	625.70	38.99	388.49
36.57	201.52	516.68	54.57
158.96	84.96	305.41	9.98
123.42	312.53	1,079.45	1,260.03
568.75	363.90	54.60	75.63
121.42	272.83	387.58	285.79
8.96	50.61	196.61	346.30
94.75	127.27	124.54	162.75
56.84	68.18	84.56	84.65
160.32	1.75	133.27	219.63
156.74	864.53	26.31	305.48
1,298.02	98.57	450.43	54.45
365.24	508.19	8.25	165.26
695.38	402.18	29.98	1,265.12
109.18	1,254.83	872.71	375.08
78.45	663.81	1,509.98	211.06
9.64	401.35	672.36	38.08
158.73	74.36	115.50	409.47
196.50	9.78	46.50	116.78
60.75	89.65	78.80	84.89
724.18	196.53	244.58	956.78
150.08	89.75	78.50	75.68
75.38	560.16	430.98	297.24
852.63	153.20	568.03	1,675.25
324.10	846.19	297.01	487.02
65.65	1,805.67	308.09	609.15
234.18	125.45	1,845.32	349.50
509.06	478.08	560.18	98.75
87.24	59.61	67.50	416.30
280.12	143.56	365.12	178.06
2,938.98	68.50	724.38	1,924.85

3. Write answers to the following questions for each of the accompanying tally-sheets:

Net C. O. D. sales made by No. A; net chg. sales; total credits received by No. A; total no. of sales made by No. A.

Answer these same questions applied to No. B, No. C, No. 28, No. 29, and No. 30.

CHG.			C. O. D.			CHG.			C. O. D.		
No. A			*Date*			*No. B*			*Date*		
	Dollars	Cents		Dollars	Cents		Dollars	Cents		Dollars	Cents
1	35	00	1	15	00	1	2	25	1	3	75
2	22	50	2	22	00	2	1	50	2	9	75
3	39	50	3	19	50	3	6	00	3	6	48
4	25	00	4	35	00	4	4	00	4	7	95
5	29	50	5	12	50	5	3	65	5	10	50
6	45	00	6	65	00	6	8	50	6	11	75
7	14	50	7	15	00	7	4	50	7	18	50
8	12	50	8	35	00	8	10	00	8	16	75
9	15	00	9	55	98	9	4	50	9	19	75
10	29	50	10	48	00	10	12	50	10		
11	16	50				11	11	00			
12	15	00				12	3	75			
13	25	00		CREDITS		13	25	00		CREDITS	
14	19	50		16	50	14	5	00		8	50
15	17	50		17	50	15	30	50		4	50
16	78	00		18	50	16	3	50		11	00
17	65	00		12	50	17	22	50		29	75
18	48	00		5	95	18	29	75		6	00
19	14	50		65	00	19	4	50		18	75
20	18	50				20	7	50			
21	9	50				21	6	00			
22	12	50				22	24	98			
23	11	50				23	18	75			
24	12	50				24	15	25			
25	5	95				25	16	75			

CHG.			C. O. D.			CHG.			C. O. D.		
No. C			*Date*			*No. 28*			*Date*		
	Dollars	Cents		Dollars	Cents		Dollars	Cents		Dollars	Cents
1	4	25	1	10	50	1	10	50	1	14	25
2	2	79	2	8	65	2	9	25	2	18	75
3	9	54	3	7	09	3	13	50	3	14	75
4	11	25	4	8	25	4	8	00	4	28	75
5	8	65	5	3	98	5	10	00	5	13	50
6	19	75	6	6	50	6	12	50	6	9	78
7	10	50	7	2	78	7	11	75	7	16	75
8	8	75	8	1	65	8	14	25	8	12	87
9	19	75	9	9	78	9	14	75	9	23	75
10	3	69	10			10	18	00	10		
11	7	50				11	29	65			
12	8	32				12	17	50			
13	7	15		CREDITS		13	9	25		CREDITS	
14	10	12		8	65	14	13	50		8	00
15	9	98		7	15	15	3	50		11	75
16	10	25		10	25	16	11	75		9	25
17	9	15		7	25	17	14	00		3	50
18	11	10		9	98	18	18	00		22	50
19	8	25				19	22	50		18	25
20	16	98				20	31	75			
21	7	25				21	18	25			
22	18	12				22	15	98			
23	9	98				23	12	75			
24	10	60				24	9	58			
25						25	10	75			

Discuss the advantages and disadvantages of the exchange system of merchandise: (1) to the store; (2) to the customer.

How may the efficiency of a sales person be determined by her number of daily credits?

CHG.		C. O. D.		CHG.		C. O. D	
No. 29		*Date*		*No.* 30		*Date*	
Dollars	Cents	Dollars	Cents	Dollars	Cents	Dollars	Cents
1 12	50	1 18	75	1 50	75	1 19	50
2 25	00	2 38	00	2 69	50	2 5	00
3 45	00	3 45	00	3 42	85	3 25	00
4 55	00	4 28	75	4 75	00	4 22	50
5 100	00	5 39	75	5 84	50	5 7	50
6 38	00	6 42	50	6 76	75	6 48	00
7 65	00	7 37	50	7 81	50	7 28	50
8 32	50	8 19	75	8 9	75	8 15	75
9 125	00	9 24	50	9 10	98	9 14	50
10 45	50	10 18	75	10 9	98	10 16	98
11 85	00			11 11	75		
12 110	00			12 62	75		
13 55	00	CREDITS		13 9	98	CREDITS	
14 135	50	45	00	14 4	75	75	00
15 50	00	85	00	15 45	00	9	98
16 165	00	78	50	16 28	50	11	75
17 78	50	18	50	17 22	50	28	50
18 98	75	67	50	18 16	50	16	50
19 18	75	38	50	19 13	50	58	75
20 78	50			20 58	75	30	00
21 67	50			21 65	00	19	50
22 89	00			22 28	50		
23 67	50			23 30	00		
24 38	75			24 19	50		
25 24	50			25 15	75		

4. The United States Government made the following purchases of Liberty Bonds during the years 1918–1920.

First loan, $25,115,000; second, $371,215,000; third, $278,685,500; and fourth, $368,115,000. What was the total amount of these bond purchases?

5. The cost of purchases made for various departments in a

store during a month is shown by the following table. The total amount of the invoice is found by horizontal addition; the total amount of the purchases for each department, by vertical addition.

(1) Find the total of each invoice.

(2) Find the total purchases for each department during the month.

(3) Find the total purchases for all departments during the month.

(4) Check all additions by casting out nines.

Invoice No.	Date	Dept. 8	Dept. 9	Dept. 10	Dept. 11	Total
1	Oct. 1	165 32	258 65	356 42	365 19	
2	Oct. 2	298 25	650 42	458 25		
3	Oct. 3	324 17		567 42	548 25	
4	Oct. 3		564 25	398 65	312 25	
5	Oct. 5		817 32	254 32	498 32	
6	Oct. 12	652 75	256 34	412 74		
7	Oct. 15	538 14	169 17		476 52	
8	Oct. 20	487 32	425 32	358 29	100 00	
9	Oct. 26	506 18	126 18		565 32	
10	Oct. 28	612 25		854 19	327 49	
11	Oct. 29	432 19	564 28	212 52	375 45	
12		498 32	257 28		465 32	
Total						

6. The sales report made by a number of sales people during a certain week is shown by the following table.

(1) Find the total sales made by each sales person during the week and enter these totals at the bottom of the table, in their proper places.

(2) Find the total sales for each day and enter these totals in the column at the right.

(3) Find the total sales made by all during the week.

	F. Ward	J. Cox	S. New	B. Ford	Total
Monday	$428.65	$396.87	$746.84	$512.16	
Tuesday	512.42	254.59	819.29	498.03	
Wednesday	365.98	658.17	725.13	397.64	
Thursday	427.16	548.32	698.98	428.56	
Friday	519.24	495.20	554.49	568.76	
Saturday	476.84	617.35	728.64	602.19	
Totals					

Why is the information obtained from the above tabulation a help to the sales manager of the store?

In what way do these comparisons of different sales people determine their efficiency?

Why are such records helpful to the sales force?

7. Add the following, without placing the amounts in columns.

a. $9.56, $80.72, $725.89, $698.76, $2.86, $14.50

b. $2.03, $6.90, $0.25, $500.00, $24.17, $196.24, $0.18

c. $95.20, $11.09, $0.89, $35.52, $8.36, $11.20

d. $152.25, $36.47, $5.75, $19.48, $103.03

e. $1.59, $0.18, $200.64, $5.16, $206.60, $17.70

8. Complete each clerk's weekly sales in the following tables.

9. Add horizontally and find the store's total daily sales.

WEEKLY SALES

	Miss Dean	Miss Fisk	Miss Long	Miss Moore	Daily Sales for the Store
Monday	$28.25	$15.65	$36.72	$26.65	?
Tuesday	42.19	22.98	28.72	38.74	?
Wednesday	37.56	19.75	35.19	29.56	?
Thursday	54.19	24.24	27.98	32.74	?
Friday	42.68	18.98	27.56	28.68	?
Saturday	37.15	19.50	19.98	37.19	?
Totals	?	?	?	?	?

WEEKLY SALES

	Miss Sears	Miss Ross	Miss Riley	Miss Reed	Daily Sales for the Store
Monday	$126.48	$97.28	$132.19	$179.36	?
Tuesday	98.75	98.15	165.37	137.42	?
Wednesday	79.68	87.75	158.84	156.78	?
Thursday	95.52	89.68	126.15	164.19	?
Friday	118.75	85.54	149.32	132.25	?
Totals	?	?	?	?	?

CHAPTER II

SUBTRACTION

When is subtraction used in business houses?
Name employees who make use of it in their daily work.
In what way would you make use of it as a customer?
Why is a knowledge of subtraction dependent upon a knowledge of addition?

7. Fundamental processes. The ability to subtract accurately and quickly is acquired only by continued practice; the process of finding the difference between numbers is made easier if the pupil is able to see at a glance what number added to the smaller of two numbers will produce the larger. Thus if 19 is to be subtracted from 28, the pupil should think of 9, the number which added to 19 produces 28. This method of subtraction makes use of the addition combinations previously taught, and so combines the two processes.

ORAL EXERCISE

Name the number which, added to the smaller number, makes the larger one, in each of the following:

a	3	5	4	6	7	6	8	8	7	9	9	8	9	8	9
	1	3	2	3	4	2	5	4	3	3	7	1	4	2	6
b	12	11	12	11	13	16	12	13	11	14	15	16	19	12	14
	9	3	8	2	6	5	7	4	9	6	8	7	7	4	8
c	17	18	16	14	12	15	19	18	19	18	19	17	16	17	19
	8	9	5	9	4	6	11	13	13	12	14	12	11	13	15

d	23	24	22	21	26	28	29	24	32	37	36	39	33	27	38
	19	17	19	13	17	16	13	16	19	18	15	14	18	19	17
e	35	44	63	53	56	74	63	52	61	75	97	53	62	96	59
	29	25	24	27	29	35	26	29	27	37	29	28	27	37	27
f	95	82	71	76	75	69	67	90	81	62	94	98	72	61	83
	36	24	37	58	37	36	28	86	27	55	55	39	45	58	94

g	\$2.00	\$5.00	\$10.00	\$10.00	\$5.00	\$5.00	\$10.00
	1.25	1.50	4.25	8.75	2.75	3.15	7.75
h	\$10.00	\$10.00	\$10.00	\$20.00	\$20.00	\$20.00	\$20.00
	6.38	8.20	6.42	12.75	11.50	16.25	18.35

i	\$20.00	\$50.00	\$50.00	\$50.00	\$50.00
	14.95	25.50	35.45	31.20	27.75

8. Change and the cash register. Why is it an advantage to the clerk making change to give the largest coins and bills that can be used?

To the employer?

To the customer?

1. In the ordinary business transactions of a store, the simplest and most accurate method of making change is to add to the price of the goods purchased, enough change to make the sum equal the amount of money offered in payment. Thus, if a person buys goods amounting to 87¢, and gives in payment \$1.00, the cashier in making change will return to the customer 3 cents, and a dime, saying \$.87, .90, \$1.00, which means \$0.87+.03 equals .90; .90+.10=\$1.00.

2. In making change, always add to the price of the purchase, beginning with the smallest coins.

ORAL EXERCISE

a. What change should be given for a one dollar bill if the following purchases were made? Name the coins given as change. Use the largest denominations possible.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1.	28¢	11¢	49¢	38¢	29¢	57¢	76¢	26¢	13¢	45¢	52¢	23¢	32¢	17¢
2.	61¢	81¢	33¢	64¢	63¢	75¢	56¢	46¢	74¢	19¢	48¢	42¢	78¢	73¢
3.	14¢	51¢	88¢	24¢	55¢	37¢	96¢	31¢	62¢	87¢	71¢	79¢	36¢	39¢

b. What change should be given for a two dollar bill if the following purchases were made? Name the coins given as change. Use the largest denominations possible.

\$1.16, \$1.24, \$1.17, \$0.39, \$1.98, \$0.67,
\$1.32, \$1.74, \$1.48, \$1.40, \$1.69, \$1.76,
\$1.33, \$1.27, \$1.55, \$1.88, \$1.92, \$1.79.

c. Name the bills or coins and the amount of change to be given from \$5.00 for each of the following purchases; use as few pieces of money as possible.

\$2.50, \$2.87, \$3.65, \$0.87, \$3.86, \$2.65,
\$4.89, \$3.18, \$2.98, \$0.96, \$4.12, \$1.76,
\$2.77, \$4.49, \$2.91, \$3.72, \$4.50, \$1.37,
\$2.69, \$3.98.

3. The picture below shows one of the cash registers now so much used in the large markets and stores. When a purchase is made amounting to \$3.45, the clerk pushes in the amount keys (\$3, \$.40, \$.05) and the operating buttons, which opens the cash drawer. This registers the purchase on a sales slip within,

Cash Register

pushes up cards that show the amount of purchase, and opens the cash drawer. If the clerk receives \$5.00, he opens the drawer and takes out the change, thus: a nickel, a half dollar and a dollar. He then puts the 5-dollar bill into the drawer and hands change and slip to purchaser saying as he does so, "Three fifty, four dollars, five dollars."

Following are the amounts of some purchases and the sum of money paid the clerk in each case:

(1) Tell which keys of the cash register should be pushed

in, which pieces of money should be taken from the drawer, and in what order.

If you were the clerk in the following transactions what should you say to the purchaser in each case, as you hand out the change?

Purchase	*Money Received*	*Purchase*	*Money Received*
a. $1.69	$ 2.00	*k.* $ 2.16	$ 3.00
b. .73	1.00	*l.* 1.19	1.25
c. 2.19	5.00	*m.* 10.17	12.00
d. 2.17	2.50	*n.* 8.19	10.00
e. 3.69	10.00	*o.* 6.59	10.00
f. 1.48	2.00	*p.* 3.19	4.00
g. 1.17	2.00	*q.* 6.28	7.00
h. 11.85	20.00	*r.* 3.13	4.00
i. 12.73	15.00	*s.* 7.07	7.50
j. 5.67	10.00	*t.* 8.09	10.00

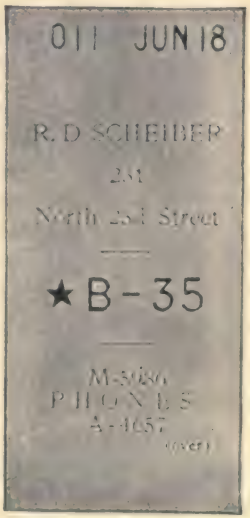
4. Cash registers are adapted to the special needs of the various businesses in which they are used. The following records are kept by the cash register used at a soda fountain:

(1) The sales check shows date, consecutive number, amount of sale, and initial of clerk making sale. Two checks are printed, one of which may be given to the customer as a receipt.

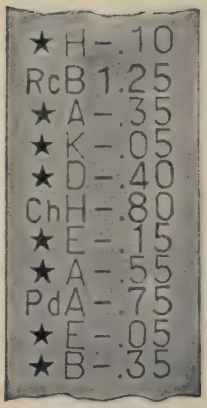
Add upward; subtract across:

(a) $528.13 − $349.28
210.80 − 96.92
602.05 − 125.08
413.23 − 356.98

(b) $720.00 − $298.07
600.02 − 58.79
315.46 − 175.84
1,008.18 − 328.42



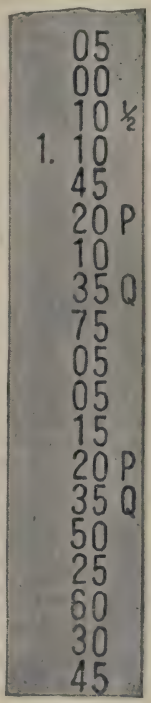
Sales Check



Sales Slip

(2) The sales slip shows a record of each transaction, the kind of sale, and clerk making sale. This is one means of discovering which clerks are paying the most attention to business.

(a) Make a collection of sales checks or parts of checks from as many different stores as possible. Compare the slips and find out the differences. State the advantages of one kind of check over another and give reasons for your opinion.



Sales Slip

(3) The sales slip keeps a record of ice-cream sold in bulk; it also tells how many quarts, pints, and half-pints have been sold, and the amount of money received for each.

(a) Do you know of any stores today where sales checks are not given when a purchase is made? If so, state the circumstances. Can you give any personal experience where a sales check has been of special value to you? Relate the incident.

(b) Give all the reasons you can think of why the use of a cash register is preferable to the making out of a sales slip by a sales person in a busy department.

ORAL EXERCISE

a. Give all the reasons you can think of as to why such a register would be of use to the business man. To the employer. To the customer. Why is the sales slip important? How much information does it give the employer?

Ice Cream Record

CASH SALES

Sales No. 10 — *Date 4–8–19*

1	12	13	21	7	89	41	7	07
2	9	50	22	9	36	42	8	24
3	7	75	23	4	50	43	6	93
4	9	25	24	2	75	44	2	46
5	8	50	25	8	24	45	8	24
6	8	75	26	3	71	46	6	80
7	8	19	27	2	89	47	7	55
8	7	23	28	9	50	48	5	31
9	7	50	29	10	37	49	1	19
10	6	98	30	4	25	50	1	25
11	7	13	31	6	89			
12	8	25	32	9	54			
13	9	09	33	7	17	CREDITS		
14	10	00	34	8	12		10	00
15	9	56	35	1	23		6	75
16	8	08	36	4	56		2	89
17	13	13	37	7	89		7	17
18	9	24	38	9	87		6	54
19	6	75	39	6	54		5	31
20	6	50	40	3	21			

CASH SALES

Sales No. 49 — *Date 12–19–20*

1	3	58	21	10	02	41	4	13
2	8	25	22	8	12	42	3	28
3	1	17	23	6	75	43	2	79
4	2	49	24	4	13	44	6	85
5	6	84	25	11	12	45	7	97
6	7	15	26	9	03	46	5	61
7	11	10	27	2	75	47	4	85
8	12	19	28	1	64	48	3	71
9	8	24	29	1	89	49	1	49
10	9	13	30	2	75	50		98
11	6	75	31	8	71			
12	4	98	32	3	79			
13	3	79	33	4	97	CREDITS		
14	2	87	34	5	65		11	10
15	6	75	35	4	89		6	75
16	1	18	36	6	25		2	87
17		98	37	3	87			98
18	2	04	38	2	98		1	49
19		13	39	6	75		4	13
20	1	95	40	1	24			

b. Name the coins, bills, and the amount of change to be given from $10.00 for each of the following purchases. Use the largest denominations possible.

$7.98, $6.65, $8.98, $7.50, $2.49, $3.24, $0.69, $6.98, $1.56, $2.05, $4.24, $5.92, $7.65, $5.17, $3.28, $7.35, $7.48, $9.43, $3.64, $1.98, $2.87, $3.45, $4.69, $5.66, $5.18, $4.98, $8.75, $3.19, $6.73

c. Name the coins, bills, and the amount of change to be given from $20.00 for each of the following purchases. Use as few pieces of money as possible.

$15.75, $16.75, $ 8.93, $12.24, $13.62, $ 0.76, $ 1.65, $19.09, $18.75, $12.19, $13.64, $11.98, $11.37, $13.13, $16.27, $17.11, $10.98, $19.19, $17.05, $17.14, $18.46

d. What is the total amount of the cash sales of salesperson No. 10, in the Ice Cream record, page 35? What is the total amount of the credits? What are the total returns minus the credits? Answer these same questions to the tally of salesperson No. 49.

ILLUSTRATIVE EXAMPLE: Find the difference between 916 and 278.

Solution:
916 Think "8+8=16" and write 8
278 8 (1 carried plus 7) plus 3 are 11; write 3.
638 3 (1 carried plus 2) plus 6 are 9; write 6.

Check: 638+278=916.

5. For all practical computation the "making change" or Austrian method of subtraction is the best. Use it below:

	Cost of Mdse. Pur.	Money Paid		Cost of Mdse. Pur.	Money Paid
a	19¢, 17¢, 13¢	$ 1.00	*f*	$4.50, $2.75, $3.09	$ 20.00
b	15¢, 28¢, 39¢	1.00	*g*	8.15, 7.65, 1.02	20.00
c	29¢, 17¢, 99¢	3.00	*h*	4.49, 6.15, 7.23	20.00
d	77¢, 29¢, 56¢	4.00	*i*	3.21, 2.79, 1.98	10.00
e	98¢, 79¢, 67¢	5.00	*j*	6.75, 10.10, 3.27	50.00

WRITTEN PROBLEMS

Find the differences in the following amounts, doing the individual problems by the addition method; check answers. Time: ten minutes.

a	*b*	*c*	*d*
$3,150.25	$5,178.75	$6,000.15	$10,000.52
618.32	1,345.87	329.61	7,856.27
e	*f*	*g*	*h*
$6,800.79	$5,254.26	$6,000.15	$10,000.52
3,976.10	3,000.69	897.70	748.89
i	*j*	*k*	*l*
$1,529.45	$1,352.28	$8,109.17	$ 2,030.50
967.36	849.94	756.28	1,396.87
m	*n*	*o*	*p*
$3,178.01	$4,453.17	$4,001.02	$ 9,125.35
1,969.28	1,500.28	3,739.39	849.67
q	*r*	*s*	*t*
$7,381.80	$1,865.29	$7,021.12	$ 1,516.17
1,695.90	909.78	3,090.84	789.08

9. Checking results. The best way to check the work in subtraction is to add the remainder to the subtrahend. If the sum is equal to the minuend, the work is correct.

ILLUSTRATIVE EXAMPLE:

$$\begin{array}{r} 27{,}936 \\ -8{,}495 \\ \hline 19{,}441 \end{array} \qquad \textit{Check: } 8{,}495 + 19{,}441 = 27{,}936$$

1. Subtraction may also be proved by casting out either the 9's or 11's, in practically the same way as in addition.

ILLUSTRATIVE EXAMPLE: Prove by casting out the 9's that 27,935—7368 = 20567.

Solution.

	Excesses
27935	—8
—7368	—6
20567	—2

Explanation: Casting out the 9's from the minuend leaves 8 for a remainder; casting out the 9's from the subtrahend leaves a remainder of 6. 6 subtracted from 8 leaves a remainder of 2.

Casting out the 9's from the remainder leaves 2. Hence the work is correct.

2. To prove subtraction by casting out 11's (Sec. **5**, page 19).

ILLUSTRATIVE EXAMPLE:

27,936 = 7	Explanation: Casting out 11's from the minuend leaves a remainder of 7.
—8,495 = 3	
19,441 = 4	Casting out the 11's from the subtrahend leaves a remainder of 3.

Casting out the 11's from the remainder leaves 4. Hence the work is correct.

This method of checking subtraction is seldom used.

10. The complement method. The complement of a number is the difference between that number and a unit of the next higher order: thus, the complement of 8 is 2; the complement of 9 is 1; the complement of 56 is 44.

1. Instead of subtracting a number less than 10 from a given number, add its complement; the result will be too large by 10. Thus, 15—9 = 6, or 15 + 1 = 16 (too large by 10).

2. Instead of subtracting two numbers less than 10 from a given number, add their complements; the result will be too large by 20, because two complements were added. Thus, 52—8—7 = 37 or 52 + 2 + 3 = 57 (too large by 20; therefore subtract 20; result, 37).

3. If the complements of three members less than 10 are

added, the result will be too large by 30. 87—9—6—8=64, or 87+1+4+2=94 (too large by 30).

4. 52—9+6—8+3=44, or 52+1+6+2+3=64 (20 must be subtracted from the result, because two complements were added).

5. 896+184—236—58—169—76+287=?

896
+184
—236
— 58
—169
— 76
+287
828

Solution: Begin at the top of the units' column and add downward, thus:

6+4+4+2+1+4+7=28

Write the 8. Four complements were added, therefore the result is 40 too large. Drop the 2 in 28 and also 2 from the 9 at the top of tens' column. Begin at the top of the tens' column and add downward, thus:

7+8+7+5+4+3+8=42

Four complements were added, therefore 40 must be subtracted from the result, leaving 2, which is written in the tens' column.

Begin at the top of the hundreds' column and add downward, thus:

8+1+8+9+2=28

Two complements were added, therefore 20 must be subtracted from the result, leaving 8, which is written in the hundreds' column. Result: 828.

Find the results in the following examples, by the addition of complements:

a 156+287—198—24+352+657—176—99.

b 1876+1942—1324—876—52+8756—198.

c 6572—3214—298+7165+2148—657—289—672.

d 9278+1675—876—794—87+6359—872—524.

e 4369—2751—38+6429+1768—365—297—875.

6. The "making change" method of subtraction is the simplest one in finding the difference between a minuend and several subtrahends.

Illustrative Example: The total of a day's sales in the suit department of a large store was $742.50; credits, $75.50, $58.75, and $125.25 were deducted. How much money did this department bring to the store?

Solution: Arrange the numbers as shown in the margin; (5+5+0) = 10. 10 and 0 = 10; write 0. Three (1 carried in addition to the 1 borrowed) +7+5 = 15; fifteen and 0 are 15; write 0, decimal point. Six (1 carried) +8+5 and 3 are twenty-two; write 3. Four (2 carried) +5+7 and 8 are 24; write 8. Three (2 carried) and 4 are 7; write 4.

$$\begin{array}{r} \$742.50 \\ 75.50 \\ 58.75 \\ +125.25 \\ \hline =\$483.00 \end{array}$$

Check: \$483.00+\$125.25+\$58.75+\$75.50 = \$742.50

EXERCISE

Find the amount or balance each of the following persons has remaining on deposit, and check the work:

	Deposit		Checks		Balance
a W. E. Barnes,	\$1,025	\$150.	\$ 75.	\$256	?
b J. A. Smith,	968	49.	112.	218	?
c F. L. Young,	1,150	198.	165.	215	?
d A. M. Brown,	756	25.50	125.	119	?
e M. E. Cobb,	975	165.	212.	198	?
f A. L. Swift,	1,200	208.	150.65	72	?

Making Sales

11. Problems in buying and selling.

(1) A dealer bought 5 pieces of lawn containing 50 yards each. He sold 39 yards from the first piece; 47 yards from the second piece; 46 yards from the third piece; 41 yards from the fourth piece; and 49 yards from the fifth piece. He sold what was left from each piece as a separate remnant. How many yards did he buy? How many yards did he sell? How many yards did he sell as remnants?

(2) Goods on hand at the beginning of a week: $156.75; goods purchased during the week: $78.50; goods sold during the week: $197.75; credits for the week, $40.70. What was the value of the stock on hand, at the end of the week?

(3) Goods on hand January 1, 1915: $7,258.88; purchases during the year amounted to $5,684.90; sales during the year amounted to $11,975.75. What was the value of the stock on hand at the close of the year?

(4) A merchant bought 700 yards silk for $650.00; 4,500 yards muslin for $215.00; 600 yards linen for $498.00. He sold the whole for $1,500.00. Gain, how much?

(5) A bank had $3,192.00 on hand Monday morning. $1,724.00 was deposited during the day and $2,987.00 was drawn out; on Tuesday $3,030.00 was deposited and $1,876.00 was drawn out. How many dollars were on hand Wednesday morning?

(6) A merchant commenced business with $9,500.00. The first year he gained $1,650.00; the second year he lost $2,987.00; the third year he gained $1,876.00; and the fourth year he lost $987.00. How much had he left at the end of the fourth year?

(7) The distance from Chicago to Buffalo is 523 miles; from Chicago to New York, 980 miles; from New York to Boston, 200 miles. How far is it from Buffalo to New York? From Boston to Chicago via New York?

(8) A merchant bought goods to the amount of $10,108.00. He paid $6,850.00 for help and $385.00 for rent. How much must he sell his goods for in order to make $1,500.00?

(9) How many years since the discovery of America in 1492?

(10) A bank account amounting to $875.00, had the following checks drawn against it; $75.16, $37.29, $54.79, $67.98. Find the balance.

(11) Find the total loss to the store from the following mark-down sale:

THE ONLY "MARK DOWN" OF THE YEAR

1 Pink brocaded chiffon wrap with marabou trimming. Was $50.00. Now.....$ 12.50
1 Rose broadcloth wrap, lined with Liberty satin. Was $70.00. Now.....$ 25.00
1 Black Liberty silk velvet wrap, lined with yellow Liberty satin. Was $85.00. Now.....$ 45.00
1 Black net wrap, embroidered in gold and colors, finished with bands of rose asphodel silk. Was $110.00. Now..$ 55.00
1 Maise Liberty orion satin coat, handsomely embroidered. Was $165.00. Now.....$ 65.00
1 Black chiffon wrap, brocaded with gold, lined with Liberty asphodel silk. Was $135.00. Now.....$ 50.00
1 Black net wrap, embroidered in gold and colors, lined with King's blue Liberty satin. Was $135.00. Now.....$100.00
1 Black net wrap with allover gold and silver embroidery. Was $110.00. Now.....$ 55.00
1 Black net wrap with ecru lace embroidery. Was $55.00. Now.....$ 25.00
1 Black caracul wrap, lined with Liberty satin. Was $85.00. Now.....$ 25.00
1 Brown chiffon wrap, brocaded with velvet. Was $45.00. Now.....$ 12.50
1 Cerulean blue Liberty silk velvet wrap. Was $125.00. Now.....$ 25.00
1 Rose broadcloth Eileen cape. Was $25.00. Now.....$ 15.00
1 Cream broadcloth cape. Was $32.50. Now.....$ 15.00
1 Rose Atrion damask tea gown, lined throughout with white silk, rose satin and cream lace trimming. Was $60.00. Now.....$ 38.00
1 Gray Thracian crêpe tea gown, hand-embroidered trimming and muslin collar and cuffs. Was $65.00. Now...$ 38.00

(12) COMPARATIVE SALES RECORD

Department No.	Dec. 1919	Dec. 1920	Increase or decrease
1	$ 8,654.35	$ 8,925.72	?
2	12,350.28	11,045.50	?
3	9,276.29	10,748.32	?
4	15,024.00	14,952.98	?
Total.....			

(13) Find the balance of cash of the following account:

	Receipts				Expenditures		
1921				1914			
Feb. 1	Mdse.	$6,320	00	Feb. 1	Mdse.	$275	00
2	"	425	75		"	1,840	25
3	"				"	94	00
11	"				"	315	00
12	"	506	30		"		

14. The following items were charged to the account of James R. Fleming, during the month of April, 1921. What was the total amount due April 30, 1921?

April 1, balance due $39.21; April 2, 15 yd. serge at $2.50 a yd., a dozen towels at $1.25 apiece, 2 dozen napkins at $4.50 a dozen; April 16, one dozen sheets at $.98 each, 9 pillow cases at $6.00 a dozen; April 29, 5 books at $1.75 each, 2 table cloths at $3.75 each. The following sums were paid: April 3, $25.75; April 20, $32.00.

(15) A convenient way of keeping a record of cash received and paid is shown by the following blank. Rule a blank similar to the one below, and complete the work.

Date	Cash Received		Cash Paid			Daily Cash
	Cash Sales	On Account	Purchases	On Account	Expenses	
Aug. 10						$155.75
Aug. 10	$210.50	$115.65	$ 62.78	$ 56.25	$ 5.25	?
11	175.80	49.49	103.45	98.17	2.78	?
12	356.25	165.78	78.96	225.36	24.12	?
13	210.49	49.49	24.89	16.49	6.50	?
14	376.08	108.50	198.50	67.75	32.08	?
15	289.10	97.69	47.88	312.96	5.50	?
	?	?	?	?	?	?

a. Find the daily cash balances. Add the receipts from cash sales and receipts on account, and subtract the amounts listed under "Cash Paid." Thus,

$155.75 + $210.50 + $115.65 — $62.78 — $56.25 — $5.25 = ?

b. Find the totals for each column.

(16) The first year the Exchange Trust Company established the Christmas Savings Club (1913) the total deposits were $200,000. In 1919, the total deposits were $1,350,000. What was the increase in 6 years?

(17) Do you belong to a Christmas Savings Club connected with any bank? What are the advantages to yourself of becoming a member of such a club? To your family? To your co-workers?

(18) Have pupils bring to class their daily business problems.

(19) John Smith borrowed of a bank at one time $968, at another $659, and at another, $756. How much does he owe if he has paid $1,575?

(20) A grocer bought goods to the amount of $9,000 in 1920. He paid out $4,365 in salaries and $850 for rent. For how much must he sell his goods in order to clear $2,500?

(21) Mr. Lee is worth $25.850. He has invested $9650 in property, $6850 in mortgages, and the rest in stocks. How much has he invested in stocks?

(22) A merchant bought 1200 yards of silk for $1500, 1800 yards of gingham for $675, and 500 yards of cretonne for $312.50: he sold the whole for $5000. How much did he gain?

(23) What was the amount of money gained by a merchant who sold for $25,000 goods that he bought for $6,020.50?

(24) Find the gain in each of the following sales, and the total gain on all sales:

Selling Price	*Cost*	*Gain*
10,052.00	6,759.65	?
25,000.00	13,185.75	?
7,050.65	4,312.00	?
18,365.25	14,978.67	?
8,090.50	3,796.68	?

CHAPTER III

MULTIPLICATION

1. Do you use multiplication in your daily work? How?

2. Do other employees use it? How much? Where?

3. How does accurate and rapid work in multiplication add to the efficiency of the employee? How is it a help to the employer? To the customer? To every individual in the business world?

4. What is the process called when unequal numbers (such as $7+8+6+9=30$) are combined?

5. $8+8+8+8=?$ How do the numbers in problem 5 compare with those in problem 4? Could the sum of the numbers in problem 5 be found by any shorter process? $4\times8=?$ What is the first process called? The second? Which is shorter?

6. Could the sum of the numbers in problem 4 be found by any shorter process? Why?

7. How does addition differ from multiplication?

8. $8\times56=?$ $9\times85=?$

9. If 1 yard of cloth costs 75¢, what will 13 yards cost? $75\times13=?$

10. In problem 8, do the numbers refer to any particular kind of object or measure?

11. In problem 9, to what do the numbers refer?

12. Fundamental processes. There are 72 primary facts of multiplication contained in the tables of 2, 3, 4, 5, 6, 7, 8, 9, 10, that must be perfectly memorized before the pupil can become skilled in the process.

1. Study these until the recognition of the combination causes instant response.

DRILL WORK

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1.	2	13	7	11	12	5	8	7	11	6	5	9	7	12	11
2.	5	4	3	7	9	6	12	9	7	4	8	13	10	8	7
3.	7	10	8	4	7	8			6	10	12	4	8	6	5
4.	4	9	4	8	6	7		9X	5	7	6	11	5	4	8
5.	9	12	10	5	10	9	13	11	12	9	4	7	9	10	6
6.	6	11	5	6	8	3	7	4	2	8	9	3	6	7	9
7.	8	3	7	9	4	2	5	6	8	3	7	8	11	9	12

(1) Using central figure as multiplier, multiply in columns from top to bottom, bottom to top, left to right, right to left.

(2) Make quick changes in centre figure and repeat drill, working for accuracy and speed until the response in the different combinations is instantaneous.

(3) Add by 9's to 90. Write results as follows:

9
18
27
36
45
54
63
72
81
90

Note interesting facts in the table of 9's.

a. Look at units figures in table and note the decrease in regular order from the top downward.

b. Look at the tens figures in the table and note increase from top downward.

c. Note that the sum of the digits in each product is 9.

How will the knowledge of these facts be helpful to you?

(4) Beginning at zero, count by 6's to 90; by 9's to 135;

by 12's to 180; by 10's to 200; by 7's to 105; by 8's to 120; by 15's to 150; by 13's to 117; by 11's to 165; by 14's to 126; by 17's to 153; by 18's to 162; by 19's to 171; by 20's to 200.

ILLUSTRATIVE EXAMPLE: Multiply 568 by 27.

Solution: There is no practical advantage in beginning to multiply by the lowest order of units in the multiplier; it is simpler to begin with the highest order, placing the result under the same denomination as the multiplier. The essential thing to remember in either process is that (1) units times units give units and the result must be placed under the units; (2) that units times hundreds are hundreds and the result must be placed under the hundreds, etc.; the adding of the partial products showing the necessity of placing units under units, etc.

568	568
×27	×27
1136	3976
3976	1136
15336	15336

13. Concrete and abstract numbers. Numbers that refer to objects of any kind or measure are *concrete*, as: 5 yd.; 9 books; 12 schools. Numbers used without reference to any particular kind of object or measure, are *abstract*, as: 8, 12, 16.

1. The multiplier is always an abstract number; the multiplicand and product are *like numbers* (units of the same name and kind).

ILLUSTRATIVE EXAMPLE: $46 × 25 =

$46	multiplicand
25	multiplier
$1,150	product

(1) Which of the following numbers are concrete? 17; 25; 32; 12 apples; 6 tops; 13 chains; 15; 39; 87 skirts; 11 berries; 19 hammocks; 36; 42; 17; 8 melons. Why?

(2) In what respect are 15 yards and 27 yards alike? Can you think of any respect in which 4 books and 3 pencils are alike? Units of the same kind and name are like numbers. Mention three like numbers; five that are unlike. Why are all abstract numbers like numbers?

(3) 9 books$\times$6=54 books. Which of these numbers are concrete? Which is abstract? Of what use is the abstract number? Which of these are like numbers? Why?

(4) Which is more: $3\times5\times2$ yards or $3\times2\times5$ yards or $2\times5\times3$ yards? Does the order in which the factors (*makers* of the product) are used affect the product? Give several examples to prove this.

2. Only one factor can be concrete; the product and the concrete factor must be like numbers. Why?

3. The multiplicand and the multiplier together make the product; hence they are factors (makers).

4. When both multiplicand and multiplier are abstract, the product is a multiple of either of the factors.

14. Checking results. The work may be checked by multiplying first by one method and then by the other; or by remultiplying, interchanging the multiplier and the multiplicand.

WRITTEN EXERCISE

Find the product in the following problems, beginning to multiply by the highest order of units in the multiplier; check the work:

a. \$ 785$\times$654	*f*. \$2,978$\times$659
b. \$ 321$\times$872	*g*. \$3,256$\times$197
c. \$ 978$\times$789	*h*. \$4,872$\times$826
d. \$ 659$\times$187	*i*. \$3,752$\times$658
e. \$1,876$\times$672	*j*. \$4,875$\times$354

ILLUSTRATIVE EXAMPLE: Multiply 8,006 by 8,274.

Solution: (1)

8,274	
8,006	
66192	
49,644	
66,241,644	

(2)

8,006	
8,274	
64048	
16012	
56042	
32024	
66,241,644	

In the above problems which method is shorter? Why? When one of two numbers to be multiplied contains one or more zeros or ones, it is always easier to take that number as the multiplier; since the product of any number multiplied by 0 is 0, the product of 8274 multiplied by the tens and hundreds of the multiplier need not be written. In practice, select the number for multiplier that will make the least work. To do this we must think of it as an abstract number. The problem may be checked as stated above, § **14.**

1. Long multiplications may be checked by casting out the nines, as in the following example:

Proof of Multiplication.

ILLUSTRATIVE EXAMPLE: Prove that $856 \times 38 = 32528$.

Solution: Casting out the 9's from the multiplicand leaves a remainder of 1; casting out the 9's from the multiplier leaves a remainder of 2. The product of these remainders is 2, and it is the same as the remainder after the 9's have been cast out from the product. Hence, the work is supposed to be correct.

$856 = 1$
$38 = 2$
$32528 = 2$

EXERCISE

(1) Find the cost of:

1280 rugs at $105	170 boxes soap at $4.08
350 suits at $196	1650 pounds chocolate at 59¢
125 pair shoes at $5.50	1376 yards muslin at 39¢
450 books at $1.98	152 pounds coffee at 59¢
650 yards silk at $3.75	5260 bushels wheat at $1.25

(2) Find the product in the following examples, and check the work by casting out the nines:

a. 964×7896	*f.* 8726×9070
b. 7788×4387	*g.* 3009×2784
c. 1056×3097	*h.* 8159×6732
d. 8999×2879	*i.* 4706×8572
e. 4006×3275	*j.* 4664×9325

15. Short methods. 1. If one of the two numbers to be multiplied has two or more digits alike, it is easier to take that number as the multiplier. Thus, to multiply 8596 and 4444, it is easier to take 4444 as the multiplier.

2 To multiply by 10, 100, 1000, 10,000, etc. 30 is how many times 10? 80 is how many times 8? How does 3 compare in value with 30? Compare 8 with 80. How many ciphers are in the number 10? Annexing one cipher to a number does what to the number? State a short way of multiplying by 10. $7\times10=?$ $9\times10=?$ $13\times10=?$ $26\times10=?$ $58\times10=?$

(1) 300 is how many times 3? 800 is how many times 8? How many ciphers in the number 100? Annexing two ciphers to a number does what to the number? State a short way of multiplying a number by 100.

(2) 3000 is how many times 3? 8000 is how many times 8? How many ciphers in the number 1000? State a short way of multiplying by 1000; by 10,000.

(3) An integer may be multiplied by 10, 100, 1000, etc., by annexing to it as many ciphers as there are ciphers in the multiplier. Every zero annexed to an integer multiplies it by 10.

(4) What use can you make of this knowledge in your work?

(5) Read aloud the following numbers multiplied by 10, 100, 1000: 6; 9; 13; 250; 22; 83; 600; 170; 135; 108; 1004; 3026; 2083; 908; 50; 810; 92; 301; 45.

(6) \$3 is how many times \$0.30? \$8 is how many times \$0.80? \$400 is how many times \$0.40?

(7) Removing the decimal point one place to the right in United States money, multiplies the number by —? Removing the decimal point two places to the right, multiplies the number by —?

(8) Read aloud the following numbers multiplied by 10; 100; 1000:

$0.87; $0.09; $0.01; $0.10; $0.08; $0.15; $0.12; $0.25; $0.50; $0.75; $0.33; $0.98; $0.79; $0.19; $3.50; $7.75; $8.24; $3.12; $2.72; $8.98; $10.10; $25.36; $15.19; $32.68; $150.06; $205.20; $315.30; $4451.02; $38.00; $7249.10; $0.99; $6285.03.

3. Removing the decimal point one place to the right, multiplies the number by 10; two places to the right, by 100; three places to the right, by 1000, etc.

(1) To multiply by 5, 50, 500; by $0.05, $0.50.

a. What part of 10 dozen is 5 dozen? What part of 10×5 is 5×5? Compare 5×5 with 10×5; 50×5 with 100×5. How is 10×96 found? How may 5×96 be found from 10×96? $5\times96=?$

$$2)960$$
$$480$$

Tell an easy way to multiply any integer by 5.

b. How is 100×36 found? How may 50×36 be found from 100×36? $50\times36=?$

$$2)3600$$
$$1800$$

How may any integer be multiplied by 50?

c. What part of 1000 is 500? How is any integer multiplied by 1000? How may any integer be multiplied by 500? $500\times64=?$ $500\times28=?$ $500\times96=?$ $500\times38=?$ $500\times72=?$

d. State an easy way to multiply by 5; by 50; by 500; by 5000.

4. Any integer may be multiplied by 5 by annexing one cipher and dividing by 2.

5. Any integer may be multiplied by 50 by annexing two ciphers and dividing by 2.

6. Any integer may be multiplied by 500 by annexing three ciphers and dividing by 2. Why?

(1) Read each of the following numbers multiplied by 5; by 50; by 500: 8, 12, 18, 24, 36, 50, 90, 128, 164, 250, 296, 1020, 1242, 2468, 3296, 5672.

(2) What is the price of 10 books @10¢? @5¢? @$1.00? @50¢? @$5.00?

(3) Find the cost of each of the following quantities of goods at 10¢, 5¢, $1.00, 50¢, $5.00 per yard: 18 yd., 36 yd., 98 yd., 52 yd., 112 yd., 150 yd., 300 yd., 46 yd., 252 yd., 198 yd., 650 yd., 456 yd., 1000 yd.

7. To multiply by 25; by 75.

(1) 25 is what part of 100? How is any integer multiplied by 100? Removing the decimal point two places to the right, multiplies the decimal by 100. Since 25 is ¼ of 100, show that to multiply by 25 is the same as to multiply by 100 and divide by 4.

(2) What relation is 75 to 25? Annexing two ciphers to a number, dividing that product by 4, and then multiplying by 3, multiplies a number by 75.

(3) Read each of the following numbers multiplied by 25; by 75: 12, 36, 28, 32, 14, 84, 50, 62, 144, 250, 342, 156, 228, 306, 198, 242, 1050, 1260.

(4) Find the cost of:

18 yd. cloth @ 25¢; @ 50¢; @ 75¢
42 yd. cloth @ 25¢; @ 50¢; @ 75¢
64 yd. cloth @ 25¢; @ 50¢; @ 75¢
24 yd. cloth @ 25¢; @ 50¢; @ 75¢
12 yd. cloth @ 25¢; @ 50¢; @ 75¢
32 yd. cloth @ 25¢; @ 50¢; @ 75¢
84 yd. cloth @ 25¢; @ 50¢; @ 75¢
96 yd. cloth @ 25¢; @ 50¢; @ 75¢
128 yd. cloth @ 25¢; @ 50¢; @ 75¢
132 yd. cloth @ 50¢; @ 75¢
400 yd. cloth @ 50¢; @ 75¢
560 yd. cloth @ 50¢; @ 75¢
1280 yd. cloth @ 50¢; @ 75¢
960 yd. cloth @ 50¢; @ 75¢
1872 yd. cloth @ 50¢; @ 75¢

(5) Name some goods that are sold at these popular prices: 5¢; 10¢; $1.00; 50¢; 25¢; 75¢.

(6) How will your knowledge of these "short cuts" in multiplication help you?

8. Bargain prices in large stores are quoted at 99¢, 98¢, 19¢, 49¢, 29¢, 69¢, 11¢. Following are some "short cuts" in multiplying "bargain prices."

(1) To multiply by 9. 9 = 10 minus what number? How many times a number must be taken from 10 times the number to leave 9 times the number? State a short way of multiplying by 9.

Thus $9 \times 156 = 1560$ (10×156)
-156 (1×156)
1404

(2) 99 = 100 diminished by 1; hence 156 multiplied by 99 = 15600
-156
15444

Check by using the addition method of subtraction.

(3) To multiply by 9: multiply the number by 10, and then by 1, and subtract the results; to multiply by 99: multiply the number by 100, and then by 1 and subtract the results; to multiply by 98: multiply by 100, and then by 2, and subtract the results; thus: $156 \times 98 = 15600$
-312
15288

Multiply the following numbers by 9, by 99, by 98, the short way:

24, 56, 45, 19, 32, 123, 208, 365, 150, 425

(4) To multiply by 19, 49, 29, 69.

$19 = 20 - 1$

(a) $156 \times 19 = \begin{cases} 156 \times 20 = 3120 \\ 156 \times 1 = -156 \end{cases}$ State an easy way to multiply by 19.

2964

$49 = 50 - 1$

(b) $156 \times 49 = \begin{cases} 156 \times 50 = 7800 \\ 156 \times 1 = -156 \end{cases}$ State an easy way to multiply by 49.

7644

$29 = 30 - 1$

(c) $156 \times 29 = \begin{cases} 156 \times 30 = 4680 \\ 156 \times 1 = -156 \end{cases}$ State an easy way to multiply by 29.

4524

$69 = 70 - 1$

(d) $156 \times 69 = \begin{cases} 156 \times 70 = 10920 \\ 156 \times 1 = -156 \end{cases}$ State an easy way to multiply by 69.

10764

(5) To multiply by 11 or any multiple of 11.

Illustrative Example (1): $356 \times 11 = ?$ Bring down the units' figure of the multiplicand. The second figure of the product is the sum of the units and tens; $(5+6)$. The third figure of the product is the sum of the tens and the hundreds, with the carrying figure $(5+1+3)$. The fourth figure is the hundreds figure, including the carrying figure if there is one. $356 \times 11 = 3916$.

568×33.

Illustrative Example (2): 33 is 3 times 11. Multiply by 11 as explained in Illustrative Example (1) (mentally) and multiply the result by 3 at the same time, thus: (1) In multiplying by 11 the first figure of the product would be 8, which multiplied by 3 is 24. Write the units' figure in the product which is 4, and carry 2. (2) $6+8=14$: $3 \times 14 = 42$, which with 2 to carry, makes 44. (3) $5+6=11$: $3 \times 11 = 33$, which with 4 to carry makes 37. Write

7 as the third figure in the product, and carry 3. (4) $3 \times 5 = 15$, which with 3 to carry $= 18$; write 18, making 18,744.

ORAL EXERCISE

Find the cost of the following and give answers at sight:

36 yd. silk @ 25¢	156 yd. @ 49¢	125 yd. @ 44¢
25 doz. h'dk'ch'fs @ 50¢	275 yd. @ 11¢	320 yd. @ 22¢
84 yd. silk @ 75¢	350 yd. @ 50¢	465 yd. @ 99¢
29 yd. gingham @ 19¢	1260 yd. @ 75¢	256 yd. @ 88¢
150 yd. flannel @ 98¢	360 yd. @ 50¢	852 yd. @ 66¢
42 yd. cloth @ 49¢	298 yd. @ 10¢	
160 yd. lining @ 69¢	420 yd. @ 5¢	
172 yd. muslin @ 39¢	476 yd. @ 50¢	
256 yd. braid @ 11¢	387 yd. @ 11¢	
342 yd. tape @ 5¢	1620 yd. @ 75¢	

9. To multiply any two numbers in the teens, as 16 by 17:

(1) Annex a zero to the sum of either of the numbers and the units' figure of the other number.

(2) Add the product of the units' figures to this result.

Solution: $16+7=23$, annexing a zero $= 230$

$7 \times 6 =$ 42

272

16. Multiplication by the complement and supplement method.

1. Complement Method: ILLUSTRATIVE EXAMPLE: Multiply 89×96.

Solution: (1) The complement of 89 is 11; the complement of 96 is 4.

(2) The product of the complements is (11×4) 44. Write 44 as the units and tens figures in the product.

(3) Subtract the complement of either number from the other number (89—4 = 85) and write the result, 85, as the remaining part of the product, making 8544.

2. Supplement Method:

The supplement of a number is the difference between the number and the next lower power of 10. The supplement of 18 is 8; of 165 is 65; of 1075 is 75.

ILLUSTRATIVE EXAMPLE: Multiply 105×109.

105
×109
945
105
11445

Solution: (1) The supplement of 105 is 5; of 109 is 9.

(2) Take the product of the supplements for the units and tens figures of the product. ($9 \times 5 = 45$.)

(3) Add the supplement of either number to the other number, ($105 + 9 = 114$) and write the sum as the remaining part of the product, making 11,445.

Multiply by supplement or complement method:

a. 108×102	*d.* 103×109
b. 98×89	*e.* 89×97
c. 96×88	*f.* 111×109

EXERCISES

(1) Find the cost of 16 aprons at 17¢ apiece; at 19¢ apiece; at 14¢ apiece.

(2) Find the cost of 13 bags at 15¢ each; at 18¢ each; at 17¢ each.

(3) Find the cost of 18 baskets at 19¢ each; at 17¢ each; at 18¢ each.

(4) It takes the fleece from 20 sheep to supply the uniform for 1 United States soldier. The United States had two million soldiers in Europe in 1918. How many sheep were needed to supply the necessary uniforms?

(5) A sales slip contained the following items: What was the total value of the goods purchased?

6 yd. carpet @	$1.98	13 yd. binding @	.19
12 yd. fringe @	.75	50 yd. matting @	.69
48 yd. carpet @	2.00	68 yd. tape @	.11

(6) Find the value of:

a. 215 tons coal @ $5.98
b. 3658 bu. rye at @ 98¢
c. 3578 bu. wheat @ $1.06
d. 4296 bu. barley @ $1.02
e. 2658 bbl. pears @ $2.20
f. 7856 bbl. onions @ $1.09
g. 2561 bbl. squash @ $2.98
h. 1030 bu. beets @ 79¢
i. 2964 bu. turnips @ $1.50
j. 500 bu. corn @ 69¢
k. 4681 bu. peaches @ $3.98
l. 864 bu. tomatoes @ $4.99
m. 6128 bu. potatoes @ $2.98
n. 7864 bu. apples @ $3.96
o. 2458 bu. peas @ $3.05

(7) State the product of:

a. 96×25
b. 38×50
c. 18×75
d. 108×25
e. 136×50
f. 164×75
g. 72×50
h. 108×5
i. $62.50×100
j. $3.75×1000
k. 480×50
l. 150×98
m. 196×10
n. 648×75
o. 288×75
p. 1152×75
q. 675×25
r. 1260×75

(8) Find the cost of:

260 doz. buttons @ $3.00
400 doz. stockings @ $6.00
850 yd. silk @ $1.50
317 yd. tape @ 11¢
60 yd. silk @ $1.98
216 yd. velvet @ $3.08
145 yd. lace @ $2.98
178 watches @ $58.99
365 hats @ $12.50
285 books @ 99¢
176 lamps @ $4.98

(9) Find the cost of:

11 yd. goods @ 28¢; 67¢; 59¢; 36¢; 76¢; $1.24; $2.36; $5.60;

$8.10; $5.13; $8.75; $9.98; $16.25; $25.30; $19.06; $4.49; $18.60; $24.28; $15.75.

17. Problems.

(1) An upholsterer bought 169 yards cretonne at $0.98 a yard; 19 packages tacks at $0.05 a package; 225 yd. braid at $0.11 a yd.; 25 yd. plush at $3.98 a yd.; 32 yd. fringe at $.75 a yd. He gave in payment a check for $400. How much change should he receive?

(2) A dealer bought 568 pieces of cloth, 98 yd. in each piece, at 11¢ a yd. He sold it at 19¢ a yd. What did he receive for it? What was his gain?

(3) What is the freight charge on 36,500 lbs. cotton goods at 99¢ per 100 lbs.?

(4) A customer bought 11 yd. silk at $2.98 a yd.; 29 yd. ribbon at 99¢ a yd.; 19 yd. muslin at 75¢ a yd. She gave the clerk a 100-dollar bill in payment. How much change should she receive?

(5) A cloth merchant sold three lots of serges. The first contained 19 pieces averaging 29 yd. each at $1.98 per yard; the second contained 11 pieces averaging 49 yd. each at $2.75 per yard; the third contained 69 pieces averaging 25 yd., at $1.69 per yard. What was the value of the whole?

(6) Bought of Jordan Marsh Co., 69 yd. muslin at 25¢ a yd.; 98 yd. table linen at $1.50 a yd.; 69 yd. silk at $1.75 a yd.; 49 yd. carpet at $1.98 a yd.; what was the whole amount?

(7) If a commission merchant buys 150 pieces of velvet containing 49 yd. each at 98¢ a yard and sells the velvet at $1.19 a yard, how much does he make?

(8) What is the cost of a telegram containing 28 words at 40¢ for the first 10 words, and 3¢ for each additional word?

(9) The number of cotton bales produced in the United States in the year 1800 was 155, 556. In the year 1910, the crop was 12,132,000 bales. How many times as great as the 1910 crop was the 1800 crop?

(10) It costs $12.25 per acre to cultivate, fertilize, pick, gin, bale, and market cotton. What is the cost of 215 acres?

CHAPTER IV

DIVISION

A piece of goods containing 12 yd. sold for 96¢. What was the selling price of 1 yard?

How many dimes should you give in exchange for a dollar bill?

If 6 days' sales amount to $654.00, what is the average amount of sales per day?

How is division used in the department store? In other business houses? How is skill in division largely dependent upon multiplication?

18. Concrete and abstract numbers.

4 times 8 yd. = 32 yd.; 32 yd. ÷ 8 = 4 yd.; 32 yd. ÷ 4 yd. = 8. From this is seen that:

1. If the dividend (the number divided) and divisor (the number by which the dividend is divided), are concrete numbers, the quotient (the result obtained by dividing), is an abstract number and;

2. If the dividend is concrete and the divisor abstract, the quotient is a concrete number like the dividend.

19. Measuring and partition. There are two kinds of division:

1. 32 yd. ÷ 8 yd. = 4, is sometimes called measuring, because 32 yd. is measured by 8 yd. and;

2. 32 yd. ÷ 4 = 8 yd. is sometimes called partition, because 32 yd. is divided into 4 equal parts.

EXERCISE

(1) In the following table use each number in the left hand column as a divisor and the other numbers in the

same line as dividends. Give quotients and remainders quickly.

5	60	24	32	25	78	49	51	65	28	36	19	15
4	24	32	17	23	42	13	4	48	20	33	50	36
9	45	27	19	34	13	54	48	56	62	71	80	108
8	32	72	19	13	27	35	48	63	71	56	49	64
6	72	54	32	28	17	37	49	41	65	25	78	84
7	49	35	41	36	22	56	64	79	29	50	59	77
10	40	35	27	38	96	54	63	50	81	19	32	25
12	60	108	25	37	96	75	62	49	76	28	84	95
11	77	81	23	17	35	46	50	99	27	42	87	100
3	45	36	28	17	19	25	32	41	54	49	63	72

(2) Solve:

18 bu. ÷ 9 = ?	18 bu. ÷ 9 bu. = ?	$136.43 ÷ 7 = ?
$48 ÷ 5 = ?	$32.75 ÷ 5 = ?	$16.60 ÷ 4 = ?
128 ft. ÷ 4 ft. = ?	$150 ÷ 15 = ?	$5.25 ÷ $1.75 = ?

(3) If twelve books cost $47.40, what is the cost per book?

Illustrative Example: Solution $47.40 ÷ 12 = $3, with a remainder; subtracting 12 times $3 the result is $11.40 in dividend; $11.40 ÷ 12 = $0.95,

$3.95 Quotient
making divisor 12)47.40 Dividend
36. = 12 × 3
11.4 undivided
10.8 = 12 × .9
.60 undivided
.60 = 12 × .05

the quotient $3.95.

(4) At $1.25 a book, how many books can be bought for $45.00?

Solution: The divisor contains cents and it is better to change both dividend and divisor to cents before dividing, thus:

36
125)4500

$45.00 will buy 36 times as many books as $1.25 or 36 books.

(5) At $1.50 per yard, how many yards can be bought for $450.00?

(6) If 6 rugs sell for $30.54, what is the selling price of 1 rug?

(7) At 16¢ a yard, how many yards of cloth can be bought with $5.20?

(8) If you earn $1.25 a day, what is your earning power per hour? Per week? Per year?

(9) If 19 books cost $1.52, what will 28 books cost at the same rate? 36 books? 49 books? 150 books?

(10) If 32 yards of cloth cost $96, for how much per yard must it be sold to gain $18? To gain $24? To gain $28? To gain $52? To gain $100?

(11) At $1.98 per yard, how many yards of cloth can be bought for $15.84? For $5.94? For $23.76? For $13.86? For $21.78?

(12) A merchant bought a dozen shirts for $15.60. He sold them at a gain of $6.00 on the lot. What was the selling price of each shirt?

(13) If a clerk receives $2500 a year and pays $36 a month for board and room, $2.60 a month for laundry, $396 a year for clothing, $16.82 to a dentist, $11.50 to a doctor, $256.00 for incidentals, how much money has he left at the end of the year to invest in business? What are his average monthly savings?

(14) A grocer has 64 salesmen. His total weekly payment for their services is $1024.00. What is the average weekly salary of each salesman?

(15) How long will it take a man to earn $483 if he earns $3.50 a day?

(16) If 58 books cost $43.50, what will be the cost of 2 dozen books?

Long division.

ILLUSTRATIVE EXAMPLE: (1) Divide 5683 by 147.

(complete operation)

$$\begin{array}{r} 38 \\ 147\overline{)5683} \\ 441 = 3 \times 147 \\ \hline 1273 \text{ undivided} \\ 1176 = 8 \times 147 \\ \hline 97 \text{ undivided} \end{array}$$

Check: $(38 \times 147) + 97 = 5683$.

(required work)

$$\begin{array}{r} 38\ \dfrac{97}{147} = \text{quotient} \\ \text{divisor } 147\overline{)5683} = \text{dividend} \\ 441 \\ \hline 1273 \\ 1176 \\ \hline 97 \end{array}$$

The remainder cannot always be written as a part of the quotient, for example: At \$15 each, how many suits can be bought for \$56? We cannot say "$3\frac{11}{15}$ suits," but 3 suits and \$11 remaining.

20. Short methods. The process of long division may be much shortened by omitting the partial products and writing only the partial dividends. Drill on this will not only be found valuable as a mental training, but of real value in lessening work.

ILLUSTRATIVE EXAMPLE: (1) Divide 78,024 by 56.

Solution: Omit writing the products; subtract mentally and write the remainder only; ($1 \times 56 = 56$; 56 subtracted from $78 = 22$; omit the writing of 56). Work it out as follows:

$$\begin{array}{r} 1393 \\ 56\overline{)78024} \\ 220 \\ 522 \\ 184 \\ 16 \text{ remainder} \end{array}$$

$1 \times 6 + 2 = 8$; $1 \times 5 + 2 = 7$: Bring down 0
$3 \times 6 + 2 = 20$; $3 \times 5 + 2 + 5 = 22$: Bring down 2
$9 \times 6 + 8 = 62$; $9 \times 5 + 6 + 1 = 52$: Bring down 4
$3 \times 6 + 6 = 24$; $3 \times 5 + 2 + 1 = 18$: Remainder = 16

ILLUSTRATIVE EXAMPLE (2): Divide 78024 by 56.

$$\begin{array}{r} 8\overline{)78024} \\ 7\overline{)9753} \\ 1393 - 2 \end{array}$$

The factors of the divisor (56) are 8 and 7. Divide by the factors, 8 and 7, using short division. To find the remainder, multiply the first divisor, 8, by the remainder.

$2(8\times2=16)$. Remainder 16. Therefore, $78024\div56=1393$, with a remainder of 16.

ILLUSTRATIVE EXAMPLE (3): Divide 342789 by 18000.

Problems in which the divisor contains ciphers to the right may be shortened by cutting off as many figures to the right of the dividend as there are ciphers to the right of the divisor.

$$19\frac{789}{18000}$$
$$18000)342789$$

The figures so cut off (as 789 in illustrative example) are always the whole or part of the entire divisor.

Problems

(1) How many automobiles at $1,800 each can be purchased for $6,354,000?

(2) At $2.50 a day, how long will it take you to earn $1000?

(3) The Union Pacific Railroad is 1819 miles long and cost $147,093,435. What was the average cost per mile?

(4) A salesman earning $3.50 a day, earned $343.00. How many days did he work?

(5) If a clerk saves $38 a month, how long will it take him to save $2,128?

(6) A merchant bought 1260 rugs for $989,100; 28 were damaged. He sold the rest for $1,005,312. What was the selling price of each rug? What did he gain?

1. Dividing by 10, 100, 1000, etc.

(1) What is the short way of multiplying by 10? $8\times10=?$ $80\div10=?$

(2) How many times is 10 contained in 50? 70? 20? 30? 150? 600? 840? 960? 320?

(3) Cutting off a cipher in 40 divides it by what number?

(4) $100\times8=?$ $800\div100=?$ How many times is 100 contained in 800? 900? 1000? 300? 200? Cutting off two ciphers

divides 500 by what? Cutting off three ciphers divides 15,000 by what?

2. Cutting off the units' figure in any number divides the number by 10; cutting off the tens' and units' figures divides the number by 100; cutting off the hundreds', tens', and units' figures divides the number by 1000.

How many times is \$0.10 contained in \$1? \$0.01 in \$1? \$0.001 in \$1? What is $\frac{1}{10}$ of 60? of 7600? $\frac{1}{100}$ of \$60? of \$600? What is $\frac{1}{1000}$ of \$1? of \$10? of \$100?

3. Moving the decimal point one place to the left in United States money divides the amount by 10; two places to the left divides the amount by 100; three places to the left divides the amount by 1000, etc.

ORAL EXERCISE

(1) State the quotient when each of the following numbers is divided by 10: 30; 40; 70; 90; 1900; 1500; 6200; 16; \$28; \$34; \$156; \$321; \$7.80; \$78; \$36.50; \$3.65; \$183.05; \$18.30; \$28.50.

(2) State the quotient when each of the following numbers is divided by 100: 300; 500; 800; 1000; 15000; 24000; \$160; \$456; \$2872; \$3527.

(3) Divide by 1000: \$2856; \$38000; \$4567; \$3287.

(4) To divide by 5: $20 \div 5 = ?$ Cutting off the right hand figure and multiplying by $2 = ?$ State a short way of dividing by 5. Divide at sight by 5: 120; 240; 600; 380; 750.

(5) To divide by 50: $8400 \div 50 = ?$ $(8400 \div 100) \times 2 = ?$ Divide by 50 at sight: 750; 1200; 300; 1000; 450.

(6) To divide by 25: $625 \div 25 = ?$ $(625 \div 100) \times 4 = ?$ Why does casting off two figures from the right of the divisor, and multiplying by 4, divide the number by 25? Divide by 25 at sight: 500; 675; 450; 750; 300; 1000; 27500.

(7) To divide by 75: $1200 \div 75 = ?$ $(1200 \div 100) \times 4) \div 3 = ?$ Why does cutting off the two right hand figures of the dividend, multiplying the result by 4, and dividing by 3, divide the number by 75? Divide at sight by 75: 800; 900; 1200; 1500.

(8) To divide by 20: Divide by 10 and divide result by 2. $480 \div 20 = ?$ $(480 \div 10) \div 2 = ?$ Divide at sight by 20: 640; 320; 1260; 480; 500.

(9) To divide by 30: Cut off right hand figure and divide by 3. Why does cutting off right hand figure and dividing by 3, divide by 30? Divide at sight by 30: 540; 720; 960; 840; 1260.

(10) Try to show short methods of dividing by 200; by 600; by any multiple of 10.

(11) State the quotient of:

a. $834 \div 10$	*h.* $\$3150 \div 1000$	*o.* $45000 \div 90$
b. $368 \div 100$	*i.* 3850 yd. $\div 1000$	*p.* $560 \div 10$
c. $32756 \div 1000$	*j.* 3245 yd. $\div 100$	*q.* $560 \div 80$
d. $310208 \div 10000$	*k.* 3456 yd. $\div 1000$	*r.* $245600 \div 400$
e. $\$156 \div 100$	*l.* 2872 yd. $\div 100$	*s.* $56000 \div 8000$
f. $\$819.24 \div 10$	*m.* $8400 \div 1200$	*t.* $7560 \div 8$
g. $\$56.70 \div 100$	*n.* $5600 \div 800$	*u.* $80{,}000 \div 2456$

(12) State the quotient of:

a. $12560 \div 40$	*f.* $8620400 \div 400$
b. $28440 \div 30$	*g.* $9621600 \div 600$
c. $70800 \div 50$	*h.* $6412800 \div 800$
d. $42630 \div 70$	*i.* $2821200 \div 400$
e. $84060 \div 90$	*j.* $18600 \div 300$

(13) How would these "short cuts" help you in work in a store? In business life?

21. Checking results. Proof of division by casting out 9's or 11's.

1. Division is proved by casting out either the 9's or 11's in the same manner as in multiplication. The excess of 9's or 11's in the divisor should equal the excess of 9's or 11's in the dividend, minus the excess of 9's or 11's in the remainder, if any.

ILLUSTRATIVE EXAMPLE: Prove that $8326 \div 23 = 362$.

5 1 2

Solution: 23)8326(362. Casting the 9's out of the divisor and quotient, the remainders are 5 and 2 respectively. Their product is 10 and the excess is 1; this number corresponds to the remainder in the dividend after the 9's have been cast out of it. The work is therefore presumably correct.

(1) Divide and check:

8560 yards by 68	\$3,648.15 by 72
7300 hours by 52	\$40,508.02 by 648
\$96,584 by 124	\$30,410.15 by 726
\$37,800 by 350	\$2,096.87 by 1956
\$349.75 by 85	\$320,889 by 67

WRITTEN EXERCISE

(1) Determine without dividing whether \$1.98 is the quotient of $\$720.72 \div 364$.

(2) Determine without adding whether $487+95+385+486+793+43+861+459$ is 3609.

(3) Determine without multiplying whether \$5914.58 is the product of 83 times \$71.26.

(4) Prove by casting out the 9's that $16456—7239 = 9217$.

(5) Prove by casting out 9's and 11's the correctness of the following sales:

a. 28 waists @ \$7.50 each, cost \$210.00
b. 342 yards muslin @ \$0.49 each, cost 167.58

c. 1 dozen blankets @ $2.25 each, cost . . . . . $ 27.00
d. 36 yards carpet @ $3.75 per yard, cost 135.00
e. 15 yards velvet @ $6.50 per yard, cost 97.50

22. Division by continued subtraction.

ILLUSTRATIVE EXAMPLE: Divide 968 by 256, and prove the work. Method: Subtract the divisor from the dividend, and from the successive remainders, until a remainder smaller than the divisor is found. The number of subtractions is the quotient.

968
−256 = 1st subtraction
712
−256 = 2d subtraction
456
−256 = 3d subtraction
200 = Remainder

Proof: $3 \times 256 + 200 = 968$, therefore, $968 \div 256 = 3$, with a remainder of 200.

EXERCISE

Divide by continued subtraction, and prove the work.

(1.) $67214 \div 1942$
(2.) $8654 \div 2356$
(3.) $98742 \div 41352$
(4.) $24165 \div 18122$
(5.) $9674 \div 2875$

23. Review. Miscellaneous problems.

(1) What is the total amount on a tally sheet showing the following amounts received for sales?

$1.98 $0.49 $0.75 $0.25 $10.00 $4.05 $0.98 $0.89
$6.50 $0.39 $0.58 $9.16 $0.87 $12.50 $6.79 $8.19
$5.21 $ .49 $ .19.

(2) A sales girl had the following amounts to add on her tally sheet:

$4.60 $2.98 $3.41 $12.56 $4.48 $1.48 $3.45 $2.69 $4.98
$1.87 $3.46 $2.98 $1.59 $1.79 $3.98 $4.48 $5.59 $0.89
$9.97 $0.98

What was the total amount?

(3) What change should be returned if a $5 bill were received for goods purchased for: 98¢, 39¢, 42¢, 69¢, 87¢, 19¢, 39¢, 49¢?

(4) The sales in several different departments of a dry goods store for one day were as follows: $625; $75; $1220; $865; $3800; $1750; $1250; $875; $560; $978; $375. What was the total amount of the day's sales?

(5) The total value of the exports of the United States in 1913 was $2,615,261,082. The value of the imports in the same year was $1,923,470,775. How much did the value of the exports exceed that of the imports?

(6) A 100-dollar bill was given in payment for the following purchases. What amount of change was returned?

hat, $7.98; purse, $0.97; coat, $12.98; stockings, $4.75; suit, $25.98; waist, $5.98; veil, $1.75; underwear, $7.50; gloves, $1.25; shoes, $8.50; collar, $3.49; skirt, $4.49.

(7) What was the total amount of sales from the dressgoods department if the receipts were as follows:

$84.24	$8.08	$7.98	$0.15	$7.75	$8.95	$37.00	$3.24
$12.50	$0.87	$18.02	$62.00	$1.00	$4.50	$0.08	$5.90
$8.15	$0.07	$10.50	$1.25				

(8) In a certain mill 140,000 yards of dress goods are made in a day. How many yards are made in a week of 6 days?

(9) There are 560 fibres of warp and 408 of filling in one yard of cloth. Each fibre of warp contains 28 strands; each fibre of filling contains 56 strands. How many strands are there in the yard?

(10) The yearly cotton crop of the United States in a certain year was 16,000,000 bales. What was its value at $48 per bale?

(11) What is the average production of each of the 240 button factories of the United States, if the yearly value of their combined product is $7,500,000?

(12) There are 30,000,000 pins made in the United States in one day. How many pins are made in a year of 300 working days?

(13) The average value of a Waltham watch is $19; what is the value of Waltham's yearly production of 600,000 watches?

(14) The cotton gin was invented in 1792. How many years ago was it?

(15) What is the total cost of a telegram of 35 words, if you pay 25¢ for the first 10 words and 2¢ for each additional word?

(16) What is the total yearly amount earned by the following large manufacturing industries of Boston:

Boots and shoes,	$30,102,317
Printing and publishing,	25,065,956
Confectionery,	15,161,583
Foundry and machine works,	12,971,641
Miscellaneous,	8,647,997
Tobacco,	6,948,214

(17) What will you pay for trimming for a waist if it takes 9 inches for one side of the front, 6 inches for the opposite side, 12 inches for each sleeve, 3 inches for the belt, and 13 inches for the neck, and the trimming is 50¢ a yard?

(18) A buyer bought merchandise for three departments. He paid $200 for the merchandise of the first department; $95 less for the second department; and for the third $59 less than for the first and second. What did he pay for the merchandise for the three departments?

(19) If your wages the first year you work are $5 a week; the second year $6.50 a week; the third $8 a week; and the fourth $10 a week; what is your average weekly wage for the four years? What have you earned in the four years, if you work 50 weeks each year?

(20) Forty hats were bought for $148. At what price should they be sold to make 50¢ on each hat?

(21) What is the weight of 8 bales of cotton weighing respectively 565 lbs., 498 lbs., 615 lbs., 507 lbs., 487 lbs., 603 lbs., 818 lbs., 575 lbs.?

(22) A cotton dealer bought a quantity of cotton in New Orleans for $6000. It cost him $550 for freight to New York; $75 for cartage; $300 for storage; $75 for insurance. For how much must he sell it to gain $5000?

(23) A sheep owner bought 600 sheep at $5 a head. He paid $180 for hay; $36 for grain; 15¢ a head for shearing; $30 for barn rent. He sold the wool at 25¢ a lb., the weight of each fleece being 5 lbs. He sold his sheep at $6 a head. Did he gain or lose, and how much?

(24) I paid $8.75 for a serge dress; $25 for an evening dress; $7 for alterations; $1 for a sateen petticoat; $5 for a silk petticoat; $3 for a negligee; $3 for shoes; $10 for boots and slippers; $5 for a sweater; $1 for rubbers; $7 for corsets; $1 for brassieres; $4 for stockings; $4 for gloves; $8.50 for a hat; $9.38 for underwear; $3.69 for a night dress; $2.98 for neckwear; and $5.17 for other articles. What was the cost of my winter outfit?

(25) I paid $34.50 for a suit; $7.50 for a linen dress; $2 for a petticoat; $10 for boots and shoes; $3.75 for stockings; $3.15 for gloves; $5.49 for a hat; $5.79 for underwear; $11.50 for waists; $4.50 for neckwear. What was the total amount spent for my summer outfit?

(26) What have I left out of $250 after paying for both my summer and winter outfits?

CHAPTER V

UNITED STATES MONEY AND DECIMALS

1. How many dimes does it take to make a dollar? What part of a dollar is a dime? In the number \$8.59 what figure stands for the dimes?

2. How many one-cent pieces does it take to make one dollar? What part of one dollar is one cent? In the number \$8.59 what figure stands for the cents?

3. What part of one cent is one mill? What part of one dollar is one mill? In the number \$8.59 what figure stands for the dollars? tenths of a dollar? hundredths of a dollar? What name is given to the point which separates the whole number of dollars from the parts of a dollar?

24. United States money. United States currency has a decimal system of notation. The decimal point separates the dollars from the parts of a dollar, or cents and mills. The first figure at the right of the decimal point denotes dimes or tenths of a dollar; the second figure at the right of the decimal point represents cents or hundredths of a dollar; the third figure at the right of the decimal point represents mills or thousandths of a dollar. (Mill means thousandths.)

1. The dollars are written as whole numbers at the left of the decimal point, with the dollar sign (\$) written before them. Where the number of cents is less than ten, a cipher must be written in the first place at the right of the decimal point (representing dimes); thus six dollars and five cents is written: \$6.05.

(1) Read: 4.5 dollars; 8.002 dollars; 5.01 dollars; .1 of a dollar; .10 of a dollar; .020 of a dollar; 36.5 dollars; 45.003 dol-

lars; 80.08 dollars; .09 of a dollar. What coin is worth a tenth of a dollar?

(2) Write with the dollar sign, decimal point and figures: 4 cents, 25 cents, 3 dollars and five mills, 287 dollars, 63 cents, 40 cents.

(3) Four dimes equals how many cents? Would you say that the price of a ball was 3 dimes or thirty cents? Why? What do we commonly say instead of 6 dimes and 7 cents? Instead of 8 dimes and 9 cents? 456 cents equals how many dollars and how many cents?

(4) Write with dollar sign, decimal point and figures: five thousand eight hundred six cents; nine tenths dollars; 2 tenths dollars; 7 and 5 tenths dollars; 12 and 1 tenth dollars; 15 and 5 hundredths dollars; 24 and 50 hundredths dollars; 6 and 5 thousandths dollars; 9 and 25 thousandths dollars; 48 and 356 thousandths dollars.

(5) Read as dollars, dimes, cents, and mills, then as dollars, tenths, hundredths, thousandths of dollars:
$26.285; $5.091; $42.010; $68.011; $50.505; $20.020; $85.06; $69.278; $5.001; $195.026; $300.300; $5.009.

(6) Write in words:
$565.418; $18.028; $500.005; $60.060; $1.011.

(7) How many mills in 5½¢; in a dime; in 23¢? How many cents in 560 mills? in 50 mills? in 24 mills? in 10 mills? in 428 mills?

25. Units of value. The unit of value for the United States is the dollar; from this unit are derived other units, using decimal divisions and multiples. Thus:

1. The gold coins are:

(1) Double eagle — value $20
(2) The eagle " $10
(3) The half eagle " $ 5
(4) The quarter eagle " $ 2.50

2. The silver coins are:

(1) The dollar — value \$1.00 or 100¢
(2) The half dollar " 50¢
(3) The quarter dollar " 25¢
(4) The dime " 10¢

3. The nickel coin is:

(1) The five cent piece.

4. The bronze or copper coin is:

(1) The one cent piece.

5. The paper money is:

(1) United States notes or greenbacks.
(2) Gold certificates.
(3) Silver certificates.
(4) Treasury notes.
(5) National bank notes.

Paper money is issued in the following denominations: \$1, \$2, \$5, \$10, \$20, \$50, \$100, \$500, \$1000.

26. Decimals. 1. What does each 5 in 55 represent? How do they compare in value? Compare the values of each figure in the number 111; in \$8.88. What does each figure represent as dollars, dimes, cents? As dollars, tenths of a dollar, hundredths of a dollar?

2. In what other way may 1/10 be written? 1/100? 1/1000?

3. Units expressed by figures at the left of the decimal point are integers and represent whole things; units expressed by figures at the right of the decimal point represent tenths, hundredths, thousandths, etc., or parts of whole things, and are called decimal fractions or decimals. (The word decimal is derived from the Latin word decem, ten.) In

reading decimals the word "and" is used only where the decimal point comes. Thus, in reading 0.906 or .906, say nine hundred six thousandths.

4. Read: 0.8; 0.05; 0.38. How many places must be used to express completely any number of hundredths? Read: 0.008; 0.090; 0.036; 0.825. How many places must be used to express completely any number of thousandths? Read: 0.0008; 0.00080; 0.000800; 0.080405; 0026; 0.004321. How many places must be used to express completely any number of ten-thousandths? Any number of hundred-thousandths? Any number of millionths?

27. Numeration table. The following table shows the relation of integers and decimals with their decreasing and increasing orders to the right and to the left of the decimal point.

NUMERATION TABLE

Orders	Hundred-Millions	Ten-Millions	Millions	Hundred Thousands	Ten-Thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths	Millionths
	4	0	3	8	7	9	2	3	5	.	2	0	6	7	3	1
Periods	Millions			Thousands			Units				Thousandths			Millionths		

1. Read:

(1) 8.7, 42.002, 81.081, 6.6205, 0.087, 300.1014

(2) 50.050, 800.800, 10.100, 5.006, 40.0040, 31.0084

(3) 78.50, 300.0402, 2000.002, 5.50011, 84.84

(4) 9.217, 100.001, .000001, 7.0057, 160.0160

(5) 20.1, 49.0152, 12.36876, .0007, .076850

2. Write decimally:

Five and fifty thousandths; eight and eighty hundredths; one and ten thousandths; thirty and thirty ten thousandths; two and two hundred six millionths; one thousand and one thousandth; twenty-five and twenty-five hundred thousandths; one hundred four millionths; thirty and thirty thousandths; one hundred two millionths; eight hundred fifty-six millionths; three hundred thousandths.

28. Decimal ciphers. In the number 2.8 there are 2 integral units and 8 tenths of a unit; in the number 2.80 there are 2 integral units, 8 tenths of a unit and 0 hundredths of a unit; in the number 2.800 there are 2 units, and 8 tenths of a unit, 0 hundredths of a unit and 0 thousandths of a unit; 2.800 is therefore equal to 2.8. Decimal ciphers annexed to, or omitted from the right of any number, do not affect its value.

1. Read the following (1) as printed and (2) in their simplest forms:

0.30; 28.020; 0.620; 0.8000; 3.6480; 18.350; 0.00270; 27.3010; 2.2000; 8000.8000

29. Addition of decimals.

1. What is the sum of 0.9, 0.08, 0.0058?
2. What is the sum of 0.1, 0.39, 0.010?
3. Find the sum of three-tenths; twenty-eight hundredths; three and three hundredths.

ILLUSTRATIVE EXAMPLE: Add 16.16, 8.075, 146.008, 95.7, 3.056.

Solution: Write the numbers so that their decimal points stand in the same vertical column; units will then come in one column, the tenths in another, etc. Add as in integers, placing the decimal point in the sum, directly under the decimal point in the several numbers added.

16.16
8.075
146.008
95.7
3.056
268.999

WRITTEN EXERCISE

Find the sum of:

(1) 8.6, 126.45, 4.328, 5.0028.

(2) 238.29, 42.864, 206.339, 9.002008, 4801.040801.

(3) 0.168, 3.0028, .00029, 26.0301, 854.090801.

(4) 28.008, 15.0002, 3000.003, 70.00070, 200.2008.

(5) $47.258, $80.102, $658.175, $834.005, $2000.005.

(6) Find the sum of thirty-two hundred fifty-six and one hundred two thousandths; twenty-four and three hundred two millionths; eight and eight millionths; one and one thousandth; forty millionths.

30. Subtraction of decimals.

1. From .9 take .3; from .28 take .16; from .165 take .108.

ILLUSTRATIVE EXAMPLE: From 100.001 take 56.987.

Solution: Write the numbers so that the decimal points stand in the same vertical column. Subtract as in integers, placing the decimal point in the remainder directly under the decimal points in the numbers subtracted.

$$\begin{array}{r} 100.001 \\ 56.987 \\ \hline 43.014 \end{array}$$

WRITTEN EXERCISE

(1) Find the difference between:

a. 6.3148 and 2.7348

b. 28.2034 and 2.0008

c. 1.0050 and .7826

d. $1365.284 and $197.3068

e. 8 and 3.2139

f. 14 and 6.00398

g. .008 and .00009

h. 468.256 and 48.0707

(2) From the sum of forty-eight ten thousandths, two hundred twelve and six thousandths, and three hundred five hundred thousandths, take the sum of one hundred six millionths and one hundred twenty-eight thousandths.

(3) $125.795—$49.875 = ?

(4) What were my earnings for five weeks if I earned $7.75 the first week, $8.29 the second week, $9.36 the third week, $10 the fourth week and $11.50 the fifth week? What did I have left if I spent on an average $3.25 each week?

(5) Goods on hand at the beginning of the week, $37.625; goods purchased during the week, $42.875; goods sold during the week, $78.25; returned goods for the week, $15.75. What was the value of the goods on hand at the end of the week?

(6) A customer bought buttons for $1.08, ribbons for 62½¢, muslin for $1.875, needles for 19¢, a dress for $17.50. What change should she receive from a 50-dollar bill?

(7) Mr. Smith had on hand January 1, 1915, goods worth $6500. During the year he bought goods to the amount of $7865.85, and sold goods to the amount of $8325.52. What was the value of the goods on hand at the end of the year?

(8) If you spend $39.87 for cloth, how much change will you receive from a 50-dollar bill?

(9) A merchant sold cloth which cost him $275.75 at a gain of $89.45. What was the selling price?

(10) A merchant had on hand January 1, goods amounting to $4702.80. During the year his purchases amounted to $3948.87. He lost by fire, goods valued at $601.98, for which he received from an insurance company $525.00. December 31 his inventory showed his goods to be worth $6275.08. How much did he gain or lose on his goods during the year?

31. Multiplication of decimals. Skilful use of decimals in every day life depends chiefly upon the ability to multiply and divide instantly by 10 or some power of 10, by moving the decimal point one place to the right when multiplying, and one place to the left when dividing. This skill can be acquired only by practice.

1. Multiply by 10: 8.00, .05, 1.65, .048, 9.1.

2. How many times $.01 is $1.00? $1.00 is how many times $.01? How many times $.08 is $8.00? How many times $1.75 is $175? Moving the decimal point two places to the right multiplies the number by—?

3. How many times $.001 is $1.00? How many mills does

it take to make a dollar? Moving the decimal point three places to the right multiplies the number by—?

4. Moving the decimal point one place to the right multiplies the value of the decimal by 10; two places to the right multiplies the value by 100; three places to the right multiplies the value by 1000, etc.

5. Multiply the following numbers by 10; by 100; by 1000:

9.3, 28.75, 38.796, .0157, 275.8, 327.51, $67.18, $325.505, 202.08, $95.178, $16.20, $1.375, $81.206, $5.005.

6. What is a short way of multiplying by .1? By .01? By .001?

7. Multiply the following numbers by .1; by .01; by .001: 208.25, 15.85, 2.875, 25.608, 9.175.

ILLUSTRATIVE EXAMPLE (1): Multiply 36.28 by 4200.

Solution: 4200 is 42 times 100; multiply by 100 by removing the decimal point two places to the right. The result is 3628; 42 times 3628 equals 152376, the required product. In multiplying begin with either the lowest or the highest digit in the multiplier as shown in Example 1, and Example 2.

(1)
$$\begin{array}{r} 3628 \\ 42 \\ \hline 7256 \\ 14512 \\ \hline 152376 \end{array}$$

(2)
$$\begin{array}{r} 3628 \\ 42 \\ \hline 14512 \\ 7256 \\ \hline 152376 \end{array}$$

8. State a short way of multiplying a decimal by any number of 10's, 100's, 1000's, etc.

9. Find the cost of:

(1) 800 yd. cloth @ 56¢; 900 yd. @ 79¢

(2) 320 yd. cloth @ 42¢; 600 yd. @ 57¢

(3) 200 yd. cloth @ 39¢; 1600 yd. @ 9¢

ILLUSTRATIVE EXAMPLE (2): Multiply 26.358 by 9.

Solution: 26.358 equals 26,358 thousandths; multiplied by 9 equals 237,222 thousandths or 237.222. Thousandths multiplied by a whole number must equal thousandths.

$$\begin{array}{r} 26.358 \\ 9 \\ \hline 237.222 \end{array}$$

ILLUSTRATIVE EXAMPLE (3): Multiply 26.358 by .09.

Solution: The multiplier, $.09 = 9 \times .01$; therefore multiply by 9 and .01. Multiplying by 9 as in Example 1, the product is 237.222. Multiply by .01 by moving the decimal point in the product two places to the left. The result is 2.37222; from this it will be seen that the number of decimal places in the product is equal to the sum of the decimal places in the multiplicand and multiplier.

$$\begin{array}{r} 26.358 \\ .09 \\ \hline 2.37222 \end{array}$$

10. In multiplying 32.06 by 7.08, can you tell before multiplying how many integral places there will be in the product? How many decimal places? Explain.

11. How many integral places will there be in each of the following products? How many decimal places?
13.6×3.028; 36.05×4.298; 354.1×306.52; 3.428×28.167; 132.8×02; 1030×8.08; 305×8; 3.42×10.01.

12. Find the product of:

(1) $1000 \times .001$; $100 \times .001$; $10 \times .1$

(2) $1000 \times .01$; $100 \times .1$; $10 \times .1$

(3) $565 \times .1$; $565 \times .001$; $565 \times .01$; 565×1.1

(4) 100×25; 100×2.5; $100 \times .25$; 100×250; $100 \times .5$; 100×5.6

(5) $1000 \times .428$; $1000 \times .036$; $1000 \times .02$; 1000×1.04

(6) 1000×2.354; 1000×5.02; 1000×1.1

(7) Compare the product of $3600 \times \$0.08$ with $100 \times 36 \times \$0.08$, or with $36 \times \$8$.

(8) Compare the product of 4000×324.642 with $1000 \times 4 \times 324.642$, or with 4×324642.

(9) A merchant buys 3000 yd. cotton @ $\$0.06\frac{1}{2}$ and gives in payment two 100-dollar bills. How much change should he receive?

32. Division of decimals. Any number may be divided by 10, 100, 1000, by moving the decimal point as many places to the left as there are ciphers in the divisor.

1. What part of \$1 is \$.10? What part of \$8 is \$.08? What

part of \$5 is \$.50? Removing the decimal point one place to the left divides the decimal by 10. To divide a decimal by 10 is to find one-tenth (.1) of it or to multiply it by .1.

2. What part of \$36 is \$.36? What part of \$125 is \$1.25? What part of \$1350 is \$13.50? What part of .1 is .001? What part of 28 is .0028? Each removal of the decimal point one place to the left divides the decimal by 10.

3. What is $\frac{1}{10}$ of \$20? Compare \$20.00 and \$2.00. What change in the place of the decimal point will give $\frac{1}{10}$ of \$20? What is $\frac{1}{10}$ of \$80? Compare \$80 and \$8.00. How many places is the decimal point moved to the left in dividing by 10?

4. Moving the decimal point one place to the left in \$1000 gives what? \$100 is what part of \$1000? Moving the decimal point two places to the left in \$1000 gives what? Compare \$1000 and \$10. Moving the decimal point two places to the left divides a decimal by —?

5. Moving the decimal point three places to the left in \$1000 gives what? Compare \$1 and \$1000. Moving the decimal point three places to the left divides the number by—?

6. State a short way of dividing by 10; by 100; by 1000.

7. Read the answers of the following problems:

(1) $5643 \div 10$; $5643 \div 100$; $5643 \div 1000$

(2) $591.4 \div 100$; $67.4 \div 10$; $8.4 \div 100$; $7.3 \div 1000$; $1.02 \div 1000$

(3) $8.2 \div 10$; $8.2 \div 1000$; $8.2 \div 100$; $658 \div 1000$

(4) $.5010 \div 1000$; $.5010 \div 10$; $.5010 \div 100$; $4268 \div 100$

(5) $.0325 \div 10$; $.0325 \div 100$; $.0325 \div 1000$; $40.2 \div 10$; $4.2 \div 100$; $4.2 \div 1000$

8. 20 apples divided by $5 = 4$ apples. $.20 \div 5 = .04$. Note that the denomination of the quotient and the dividend is the same.

9. Show that $.6 \div 2 = .3$; that $.06 \div 2 = .03$; that $.006 \div 2 = .003$; that $.036 \div 2 = .018$; that $.018 \div 2 = .009$.

(1) Note that in each of the above problems the divisor is a whole number; compare the number of decimal places in the dividend and in the quotient of each of the above examples.

10. From the illustration we see that when the divisor is an integer there are as many decimal places in the quotient as there are in the dividend.

ILLUSTRATIVE EXAMPLE (1): $8.6502 \div 26$.

Note that each quotient figure is of the same order of units as the right hand figure of the partial dividend used to obtain it. Divide as in integers, placing the decimal point in the quotient directly above the decimal point in the dividend.

Check: $26 \times 0.3327 = 8.6502$. Hence the work is correct.

$$
\begin{array}{r}
0.3327 \\
\hline
26)\ 8.6502 \\
7.8 = 26 \times .3 \\
\hline
.85 \text{ undivided} \\
.78 = 26 \times .03 \\
\hline
.070 \text{ undivided} \\
.052 = 26 \times .002 \\
\hline
.0182 \text{ undivided} \\
.0182 = 26 \times .0007 \\
\hline
\end{array}
$$

11. Find the value of the following:

(1) $.1236 \div 6$; $6.44 \div 7$; $.924 \div 12$.

(2) What is the value of 1 franc if a 5 franc piece is worth \$0.965?

(3) What is the value of 1 yd. of cloth if 56 yd. sell for \$70.00?

(4) How many times is \$.05 contained in \$0.25?

(5) How many times is \$0.30 contained in \$0.60?

(6) What is the denominator of the dividend in example (4)? Of the divisor? What kind of a number is the quotient?

12. Tenths divided by tenths give what kind of a number for the quotient? Hundredths divided by hundredths give what? Thousandths divided by thousandths?

13. From these illustrations we observe that when the dividend and the divisor are of the same denomination, the quotient obtained, to the limit of the dividend given, is a whole number.

(1) Divide: 3.6 by 1.2; 6.5 by 1.3; 7.5 by 1.5; 62.5 by 5; 8.4 by 1.2.

14. Multiplying both dividend and divisor by the same number does not change the value of quotient; therefore, when the divisor is not an integer, multiply both dividend and divisor by that power of 10 which will make the divisor an integer, and divide as in United States money.

ILLUSTRATIVE EXAMPLE (2): Divide 0.7488 by .96.

Note that the divisor may always be made an integer by moving the decimal point in the dividend as many places to the right as there are decimal places in the divisor. Hence the number of decimal places in the quotient will equal the excess of the number of decimal places in the dividend over those of the divisor.

```
    .78
96) 74.88
    67.2
    7.68
    7.68
```

Should there be a remainder after all the decimal places in the dividend are used, annex decimal ciphers and continue the division as far as indicated.

WRITTEN EXERCISE

Divide:

a. 6375 by .024
b. 2.71348 by .308
c. 20.340 by 6780
d. .00268 by .067
e. 3.2376 by .0019
f. .5568 by .96
g. 435.24 by 279
h. 840 by .1200
i. 72.056 by .0008
j. 126.702 by .0006
k. 796.784 by .0076
l. 175.0 by 3.50
m. 1.05 by 3.50
n. 175 by .3500
o. 1750 by .35
p. 175 by 35000
q. 1750 by .0035

33. Problems.

(1) What is your yearly income if you work 48 weeks and receive $9.75 a week?

(2) How many yards of ribbon can you buy with $16.32, at $.24 a yard?

(3) How many tickets at $0.32 each, can be bought with $16.00?

(4) How many pieces of cloth 5.625 yards long can be cut from a bolt of cloth containing 675 yards?

(5) How many bows of ribbon each containing 1.5 yards, can be made from 5 pieces containing 12 yards each?

(6) How many rosettes can be made from 9 yards of ribbon if each rosette contains 1.8 yd.?

(7) Find the cost of 58 books at $1.125; 575 yards muslin at $0.19; 625 yards lace at $0.875; 60 pairs of shoes at $4.75.

(8) If one pound of English money is worth $4.88, how many pounds of English money would you receive for $100 United States money?

(9) A cotton plantation contains 750 acres. If 250 lbs. are raised on one acre, and 500 lbs. make a bale, how many bales are raised on this plantation? What is this cotton worth at 12¢ a pound?

(10) A house and lot cost $3900. The lot cost .625 as much as the house. What was the cost of each?

(11) A retail dealer bought 398 yd. cloth for $696.50. He sold it for $2.25 a yard. What was his gain per yard?

(12) A merchant bought 325 yd. of cloth at $2.875 per yard, and 165 yd. at $3.50. At what average price per yard should he sell the lot to make an average profit of $1.50 a yard?

(13) What is the annual income of a man who receives $288 a month?

(14) What is his yearly rent bill at the rate of $42.50 per month?

(15) He pays $9.50 a month for eight months of the year for fuel. What is the cost for heat?

(16) If his grocery bill averages $7.50 per week what is his annual bill for groceries?

(17) His gas bill averages $3.50 and his electric light bill $3.75 per month. What is his year's bill for gas and electricity?

(18) He spends $375 for clothing for the year, and allows $2.75 per week for other expenses. What does he save in a year?

(19) Find the price of the following pieces of merchandise:

a. 338 pairs silk hose @ $1.29=
b. 373 " " " @ $1.15=
c. 564 " " " @ $ .85=
d. 38 ladies' suits @ $19.75=
e. 29 " " @ $35.50=
f. 64 " " @ $22.50=
g. 83 " " @ $16.25=

(20) Find the cost of the following and prove by casting out 9's and 11's:

a.	17 table cloths	@ $6.50	*m.*	29 " "	@ $6.75
b.	23 " "	@ $8.25	*n.*	38 " "	@ $7.25
c.	49 " "	@ $3.75	*o.*	14 bed spreads	@ $1.45
d.	34 " "	@ $5.85	*p.*	78 " "	@ $2.65
e.	38 doz. napkins	@ $4.25	*q.*	93 " "	@ $2.95
f.	87 " "	@ $6.75	*r.*	47 " "	@ $1.35
g.	65 " "	@ $5.95	*s.*	14 silk sweaters	@ $18.50
h.	143 towels	@ $ .18	*t.*	28 " "	@ $22.50
i.	192 "	@ $ .38	*u.*	19 " "	@ $28.50
j.	29 "	@ $ .75	*v.*	87 " "	@ $ 6.95
k.	13 pr. blankets	@ $8.75	*w.*	43 wool sweaters	@ $7.95
l.	18 pr. blankets	@ $4.25	*x.*	87 white skirts	@ $2.95

(21) Determine the correctness of the following:

a. 129 pr. of gloves were purchased for $49.02—price per pair 38¢.

b. 373 pr. of gloves were purchased for $55.95—price per pair 15¢.

c. 654 pr. of gloves were purchased for $189.66—price per pair 29¢.

CHAPTER VI

THRIFT

34. Thrift problems.

1. A boy receives 40 cents an hour for chopping wood. He works 2½ hours a day 6 days every week for 50 weeks. How many 50-dollar Liberty Bonds can he buy with his year's earnings?

2. John's father left him a life insurance which he invested so as to receive $45 a year. How much is this a month? His mother pays $9 a month for rent. What part of the rent will this pay?

3. John delivers papers before and after school. He receives $1.50 a week for delivering morning papers, $.75 for evening papers, $.95 for Sunday papers. He also makes $1.30 a week by selling the Saturday Evening Post. What does he earn a week? In a year?

4. John's sister Mary earns 16¢ an hour for wheeling a neighbor's baby. She works 1½ hours a day, missing only 18 days in the year. What are her earnings?

5. Their brother Walter started a vegetable garden. He borrowed $1.75 from John to pay for ploughing and seeds, and $1.35 from his sister to pay for tools. He raised $4.20 worth of cabbage, $3.75 worth of tomatoes, $7.50 worth of potatoes, and $1.40 worth of other vegetables. How much money did he earn after deducting what he owed?

6. The following year, John, Mary, and Walter went into the poultry business. They bought 9 hens costing $1.80 each, and a rooster for $2.00. They paid 85¢ for nails, screws, etc., and 95¢ for building materials for a hen house. What was the total output?

7. The chickens' weekly rations were 1 pk. of oats costing 60¢ a bu., 8½ lb. of bran costing $1.50 for 100 lb., and table scraps. What was the chickens' weekly food bill? What was the cost of their keep for one year?

8. The hens laid 28 eggs a week, which sold for 65¢ per dozen. What was their weekly profit after deducting the food costs? The yearly profit?

9. If a dozen hens lay 312 eggs apiece in a year, and eggs sell for 75¢ a dozen, what will a year's supply of eggs bring? If the food for each hen averages 1½¢ a day, what is the yearly profit?

10. Compare the returns in examples 8 and 9. Which is the better paying proposition? Why?

11. If you buy three hives of bees for $5.20 a hive, paying $1.95 for supplies, sell your honey for 35¢ a lb., and obtain 28 lb. of honey from each hive, what is your profit at the end of the year?

12. Two boys leave school. One works for 75¢ a day with board, the other borrows $450 and goes to a trade school for one year to learn a trade, which pays him $2.50 a day with board. Counting 275 working days a year what will each have earned at the end of 10 yrs.? How much more will one have earned than the other?

13. Albert Ring paid $5 for a young pig. The pig's feed and care cost him $21.75. When the pig weighed 275 pounds he sold it for 28¢ a lb. How much money did he make on the pig?

14. There are 60 lbs. of potatoes in 1 bu. If $\frac{1}{8}$ of the edible part of a potato is wasted by careless paring, what is the loss in pounds on a bushel of potatoes? Estimate the money loss on these potatoes at the present day price quotations. If a family's weekly supply of potatoes is 1 peck, what is the money loss in a year?

15. There are 80 lumps of sugar in a pound of lump sugar. The present price of this sugar is 17¢ a pound.

(1) How long will a pound of this sugar last a family of four people, each drinking 2 cups of coffee daily, using 3 lumps to a cup? At this rate, what amount of sugar would be used in 1 year? (365 da.)

(2) How long would a pound last if each used 1 lump per cup?

(3) How many pounds of sugar would be saved in a year, if each used 1 lump per cup of coffee?

(4) How much money would be saved for other purposes?

(5) If each of the 100 million persons in the United States saved 1 lump of sugar daily, how many pounds of sugar could be saved in 1 year? How many tons is this?

(6) What would be the money value of this sugar at the above quoted price per pound?

16. The Department of Agriculture reports that there are 700 million dollars a year wasted in the 20 million families of the United States by throwing away food which might be used. What is the average waste for each family per year? Per month? Per day?

(1) If the average family contains 5 persons, how much is the average waste for each person each day?

(2) What would be the yearly income of this food waste ($700,000,000) if it were invested in $100 Liberty Bonds at 4¼%?

(3) How many boys and girls could receive a college education (4 years @ $800 a year) on this income?

17. By not eating his crusts of bread, a boy wastes an average of ½ a slice of bread a day.

(1) If there are 12 slices to a loaf, how many loaves of bread does he waste in a year? (365 da.)

(2) What is the cost of the bread wasted at the present price a loaf?

(3) How many thrift stamps could be purchased with the money wasted in this way?

18. A pound of butter cost 79¢. If a family of 5 used ¼ pound daily, what is the cost daily? Monthly? Yearly? How much butter is used in a year?

19. If this family should cut down their supply of butter to army rations (½ oz. daily for each person) how much butter would be saved during a year? How much money?

20. How can one economize on the amount of money spent for clothes?

21. What effect will increased production of goods in the United States have upon the prices of goods?

22. How does the extravagance of other people in your community concern you?

23. Prepare a blank similar to the one shown at the top of page 87, and fill in the various items.

24. Give as many reasons as you can why it will help you to know how much it costs to support you.

(1) What help will it be to your parents?

(2) Would it help you to help other children?

25. Answer fully the last two questions on the plan on page 87.

26. Make a spending plan (see page 87) and fill in the items. How will the keeping of a monthly spending plan help you?

27. Did you ever stop to think how much it costs your parents to keep you well and comfortable and to educate you to be a useful man or woman? Ask them to help you fill in the items in this table. The total may surprise you.

ESTIMATE OF MY COST OF MAINTENANCE: Date,..... 1920

For food materials.....	Per week, $.....; per year, $.....
New clothing purchased..	Per month, $.....; per year, $.....
My share of house rent or expense.....	Per month, $.....; per year, $.....
My share of laundry cost.	Per week, $.....; per year, $.....
My share of heat and light bills.....	per year, $.....
My share of reading, recreation, and my education.....	per year, $.....
My share of doctor's and dentist's bills.....	per year, $.....

Total estimated cost per year.....$.....

Estimated cost per week (cost per year divided by 52).....$.....

In addition to this money cost there are also the benefits secured by the unpaid labor of mother, father, and children. Can you help pay your share? With money? By hearty coöperation in the home?

28. Make a plan similar to the one below, extending it so as to include every month of the year; at the end of each month, fill in the items in the blank and total your accounts for the year. What advantage will the keeping of such an account be to you? To your family?

What the Children Can Do

Make a Spending Plan. It might be arranged like this:—

I expect to receive		I expect to save		I expect to spend	
On hand.....	$	For		For	
From earnings..		W. S. S.....	$0.05	School.....	$0.05
From gifts.....		For.....		Self.....	.10
From parents...	0.25	For.....		Church.....	.05
Total.....	.25		.05		.20

Keep an account and compare with the spending plan.

29. Are you a wage earner? If so what have you saved from your weekly earnings? How have you invested your savings?

MONTHLY SUMMARY OF MONEY RECEIVED, SAVED, SPENT

Receipts				Saved		Expenditures	
Month	Earnings	Profits from project	Gifts	W. S. S., Liberty Bond payments	Other savings	Clothing	School miscellaneous
Jan.							
Total for year							

30. Make an account of your possessions on a plan similar to the one given. Consult your "Monthly Summary," and from it fill in the items for this year, and for last year. Compare the results. What does the study of the comparison of the two years "Inventory" show you?

A boy or girl will understand better how to value and manage property if once a year a list is made of everything he or she owns; if the actual value of such things as clothing and the furniture in one's own room is not known, it may be estimated. Investments like War Savings Stamps that increase in value each year and livestock should be listed at their present value. The total should be found and its increase or decrease from year to year noted.

ANNUAL INVENTORY OF PROPERTY OWNED, MONEY, ETC.

Items—Values in	1919	1920	1921	1922	1923
Money in hand or bank.	$.	$.	$.	$.	$. . . . .
W. S. S. owned.	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .
Liberty bonds owned.	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .
Value of animals owned.	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .
Value of jewelry owned.	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .
Value of books owned.	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .
Value of personal clothing. . .	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .
Value of furnishings of my room.	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .
Total value.	. . . . .	. . . . .	. . . . .	. . . . .	. . . . .

Get the habit of keeping careful track of everything you own; it will help to increase your property and teach you business methods.

31. A seventh grade boy made and sold during his manual training course the following articles:

1	hat rack	$1.75
1	book shelf	.98
1	card board	.55
3	picture frames	.65 (each)
1	foot stool	.70
1	utility box	2.95

How much did he earn during his manual training course?

32. How much does a man lose who idles away 185 days each year when wages are $2.75 a day, with board?

35. Education and efficiency applied to the income. The following table, made from facts recently obtained, of the wages of two groups of pupils from Brooklyn Continuation Schools, shows the value of education and efficiency. The lowest wage in each case is paid the youngest, the poorly prepared, and the uneducated; the higher wage is received by the educated, well-prepared, and efficient worker.

WHAT FOUR YEARS IN SCHOOL PAID

Pupils who left school at 14 yrs.	Yearly Salary	Pupils who left school at 18 yrs.	Yearly Salary
When 14 yrs. of age	$200		
When 16 yrs. of age	250		
When 18 yrs. of age	350	When 18 yrs. of age	$500
When 20 yrs. of age	475	When 20 yrs. of age	750
When 22 yrs. of age	575	When 22 yrs. of age	1000
When 24 yrs. of age	600	When 24 yrs. of age	1150
When 25 yrs. of age	688	When 25 yrs. of age	1550

1. How much greater is the yearly income due to education and efficiency in the case of each of the following persons?

(1) The 18 year old persons?
(2) The 20 year old persons?
(3) The 22 year old persons?
(4) The 24 year old persons?
(5) The 25 year old persons?

2. What is the total salary for 11 years of the person who leaves school at the age of 14 years?

3. What is the total salary for 7 years of the person who leaves school at the age of 18 years?

4. Do you think it would be wise for a young man or young woman to borrow money, if necessary, in order to continue his education? Discuss the details of such an arrangement.

5. What relation is there between the family budget and the keeping of household accounts?

6. How does the keeping of family accounts differ from the keeping of personal accounts?

7. A business man who has studied the production problems in one large state reports that the man with a common school education is able to produce one and one-half times as much wealth as the illiterate man. Apply this principle to some one whom you know.

36. Personal budgets. A plan for spending is what business men call a budget. Budgets may be made for the year, for the month, or for a week. Each year your city or state government estimates under certain definite heads the amount of money needed properly to carry out the activities of its various departments. The Budget is simply planning, in advance of spending, as opposed to the more frequent method of spending first.

A personal budget is a detailed plan of anticipated receipts and proposed savings and expenditures for a definite future period.

MONTH OF DECEMBER, 1920

Income	$1.75	Savings	$1.00
Allowance	1.50	Expenditures	
Work	3.50	Books	2.00
		Paper	.50
		Collars	.25
		Pin	.75
		Necktie	.75
		Theatre	.50
		Car-fares	.40
		Extras	.25
		Balance	.35
Total	$6.75	*Total*	$6.75

1. Make a similar Personal Budget of your own receipts and expenditures for the previous month or week. Estimate future savings and spendings for next week or month.

A PLAN FOR REGULAR SAVING AND FOR SPENDING

	$15 a week or $65 a month		or $780 a year	$17.30 a week or $75 a month		or $900 a year
	A week	*1 mo.*	*A year*	*A week*	*1 mo.*	*A year*
War Savings Stamps and other savings	.25	?	?	?	?	$ 52.00
Room and board	8.00	?	?	?	?	416.00
Lunches	1.50	?	?	?	?	78.00
Carfare to business	.60	?	?	?	?	31.20
Clothing	3.00	?	?	?	?	182.00
Laundry	.45	?	?	?	?	26.00
Church, charities, gifts	.20	?	?	?	?	26.00
Health, recreation, education	.75	?	?	?	?	52.00
Miscellaneous	.25	?	?	?	?	36.40
	$15.00	?	?	?	?	$899.60

Balance for savings $?

2. Complete the "Plan" filling out amounts under "A Year" and "A Week."

3. To change weekly costs to monthly costs: Multiply by $4\frac{1}{3}$.

4. Do you keep an expense account? What can you learn from keeping such an account?

1920

Income	Planned expenses per year		Per month	Per week
$1200	Board	$400.00	?	?
	Clothing	300.00	?	?
	Laundry	35.00	?	?
	Advancement	300.00	?	?
	General Expenses	100.00	?	?
	Carfare	25.00	?	?
	Emergencies	40.00	?	?
Total $1200	*Total*	$1200	?	?

Complete the above table. Prepare a table similar to the one given, and plan out your expenses.

37. Budgets based on various incomes.

In making budgets based on various incomes, the percentage of the income appropriated to each of the respective headings of the budget varies with the circumstances,—number in family, family tastes, occupations, climate, whether living in city or country, etc. Under advancement should be included savings (bank, insurance), education, books, newspapers, etc. Under general expenses include amusements, travel, recreation, vacation, church, charity.

1. Copy and complete the Budget on page 93.

2. Of what value is the family budget in planning expenditures?

3. Do you think it possible for a girl earning $12 a week and paying for her board and room to keep her expenses for the different divisions within the limits of the ideal division for a family given on page 94? Give reasons for your answer.

	Food		Shelter		Clothing		Operating Expenses		Advancement		General	
Income per yr.	Amt.	Pc.	Amt.	Pc.	Amt.	Pc.	Amt.	Pc.	Amt.	Pc.	Amt.	Pc.
$600	$270	45	120	20	75	$12\frac{1}{2}$	60	10	30	5	45	$7\frac{1}{2}$
900	?	40	?	20	?	$13\frac{1}{2}$	?	12	?	7	?	$7\frac{1}{2}$
1200	384	?	240	?	180	?	120	?	100	?	126	?
1500	?	28	?	18	?	15	?	11	?	$16\frac{2}{3}$	?	$11\frac{1}{3}$
1800	477	?	306	?	270	?	225	?	342	?	180	?
2100	?	23	?	17	?	14	?	14	?	24	?	9
2400	?	20	408	?	312	?	400	?	600	?	200	?

Per cent means a certain number of hundredths: $15\% = \frac{15}{100}$

15% of $300 $= \frac{15 \times 300}{100} = \$45$: or $\$300 \times .15 = \$45$.

4. In the making out of budgets an average family of five will divide each $100 received about as follows:

(a) When the income is less than $2,000: savings, one-tenth, or $10 in each $100 (less in smaller incomes and larger families); rent, one-sixth, or $15 to $20 in each $100; food, two-fifths, or $40 (more with smallest income); clothing, one-sixth, or $15; housekeeping expenses, one-tenth, or $10; personal expenses, one-tenth, or $10.

(b) When the income is $3,000 or over, each $100 spent may divide somewhat as follows: savings, about one-seventh, or $15 in each $100; rent, one-seventh, or $15 in each $100; food, two-sevenths, or about $20 to $30 in each $100; food, clothing about one-seventh or $15; housekeeping expenses, about one-seventh, or $14; personal expenses, about one-seventh, or $15.

Of course, every family must make its own plans for spending, and these standards are only suggestive in a most general way.

Ask members of the group to bring in accounts and suggestions to the next meeting to throw light on how it is possible to increase the amount saved systematically each week or month.

5. Make out a budget of your last year's expenses using the schedule given in "Suggested Budgets."

6. Make out a budget for this year's expenses up to the present date.

7. Compare the budget of last year's expenses with the present year budget.

8. Can you see any unnecessary expenditures in either budget?

SUGGESTED BUDGETS [1]

FAMILY INCOME	PERCENTAGE FOR—				
	Food	Rent	Operating expenses (wages, fuel, light, etc.)	Clothes	Higher life (books, travel, church, charity, savings, insurance)
Two adults and two or three children (equal to four adults):—					
Any income (ideal division) .	25	20±	15±	15±	25
$2000 to $4000	25	20±	15±	20±	20
$800 to $1000	30	20	10	15	25
$500 to $800	45	15	10	10	20
Under $500	60	15	5	10	10

EXERCISE

(1) Extend the above budget to include the amounts of money each percent represents, based on the different incomes given.

(2) Using the following incomes per year and the percentages of the next lower income in the table above, construct a table similar to "Budgets Based on Various Incomes." (§37). $700, $1100, $1400, $1600, $2000, $2200.

(3) Write down all the advantages you can think of in favor of the budget plan.

(4) What is the principal object of the budget?

(5) Explain the object of the various subdivisions of the budget.

(6) What changes would you make in the model budget to make it conform to your own requirements?

(7) What should be included under the term "Savings?"

[1] Richards, Ellen H., *The Cost of Living*, 1905, published by J. Wiley & Sons.

38. Keeping accounts. Study the following record of receipts and expenditures.

Here is a simple form that classifies everything—try it. Enter your savings and expenditures daily in a form similar to the following, Form I, using a page a month. At the end of each month carry the totals forward on a monthly summary sheet similar to Form II, and watch the amounts. Compare them with your plan for saving and spending.

PERSONAL CASH BOOK

February 1, 1920		Received	Paid
Feb. 1	Balance on hand	$ .35	
2	Received allowance	1.25	
3	Earnings	2.00	
4	Paid for car fares		$ .50
5	Paid for theatre		.75
6	Deposited in bank		1.00
8	Paid for stamps		.20
9	Paid for lunch		.35
10	Paid for laundry		.65
12	Earnings	1.50	
13	Paid for lunch		.35
15	Paid for hdkfs.		.75
15	Balance on hand		.55
		$5.10	$5.10

Rule a sheet of paper similar to the Model Personal Cash Book, and enter the following transactions on it.

Balance the Cash Book weekly, and bring the balance down on the same sheet.

Mar. 1.	Received allowance for one week	$1.50
2.	Paid for handkerchiefs	.50
3.	Paid for car fare	.30
4.	Earnings	2.00
	Balance the Cash Book	

FORM I

Balance on hand at first of month: $.....

Date	Items	Money received	W. S. S.	Other savings	Rent	Food	Clothes	House-keeping	Health	Recreation	Education	Personal, mis-cellaneous	Money given
1													
2													
3													
4													
5													
6													
7													
8													
9													
10													
11													
12													
13													
14													
15													
16													
17													
18													
19													
20													
21													
22													
23													
24													
25													
26													
27													
28													
29													
30													
31													
Totals.													

FORM II

Monthly Summary of Savings and Expenditures

(To be filled in at end of each month, bringing totals from monthly records on preceding pages. Compare monthly amounts for similar items.)

Items	January	February	March	April	May	June	July	August	September	October	November	December	Total
Income or earnings													
Savings;													
Payments on War Savings Stamps and Liberty bonds													
Other savings and investments													
Total savings													
Expenditures:													
Rent or house expense (taxes, interest, fire insurance, repairs)													
Food													
Clothing													
Housekeeping expenses (supplies fuel, light, repairs, laundry, wages, etc.)													
Personal expenses—													
Church and charity													
Health, recreation, education (reading, etc.)													
Personal and miscellaneous (fares, gifts, toilet, etc.)													
Total personal expenses													
United States income tax													
Total expenditures for month													

CHAPTER VII

FACTORS, DIVISORS, MULTIPLES

39. Factors.

1. Name two numbers whose product is 24; 15; 32; 63; 21; 75; 96; 84; 36; 45; 54; 28; 72.

2. Since the multiplier and multiplicand make the product, they are called the factors (makers) of the product.

3. Name the factors of 28, 33, 32, 65, 66, 72, 81, 84, 91, 96, 99, 42, 54, 108, 132, 144.

4. Name four factors or divisors common to 24, 36, 72, and 40.

5. Name two or more factors or divisors that are common to 60, 42, 18, 54, 84, and 48.

40. Greatest common divisor. A factor or divisor common to two or more integers is called a common factor, or common divisor of the integers. The greatest common divisor is the greatest factor that is common to two or more given numbers (expressed g. c. d.). Thus 9 is the g. c. d. of 18, 27, 36, and 81, because no larger integer is a common divisor of them all.

1. Which of the following numbers have no factors except themselves and one? 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.

41. Prime and composite numbers and factors. A number that is divisible only by itself and one, is a *prime number;* if divisible by any other number, it is a *composite number.* A factor that is a prime number is a *prime factor.* An integer of which two is a factor is an *even* number. All integers that are not even are *odd* numbers.

1. Make a list of numbers from 1 to 100 that are prime

numbers; that are composite numbers; that are odd numbers; that are even numbers.

42. Tests of divisibility. A number is divisible:

By 2, if it ends in 0, 2, 4, 6, 8, or when it is even.

By 3, if the sum of its digits is divisible by 3.

By 4, if the number expressed by its two right hand digits, is divisible by 4.

By 5, if it ends in 0 or 5.

By 6, if divisible by 2 and 3, or if it is an even number and the sum of its digits is divisible by 3.

By 8, if the number represented by the three right hand digits is divisible by 8.

By 9, if the sum of its digits is divisible by 9.

By 10, if its right hand figure is zero.

By 12, if it is divisible by 3 and 4.

43. Factoring. Name one or more factors of each of the following numbers:

a. 125	*d*. 276	*g*. 4200
b. 284	*e*. 648	*h*. 3725
c. 657	*f*. 936	*i*. 2945

The process of separating a number into its prime factors is called factoring; factors that are prime numbers are prime factors. The prime factors of 21 are 3 and 7. When numbers have no common factor they are prime to each other. Ex. 3 and 4; 7 and 8; 2 and 5.

ILLUSTRATIVE EXAMPLE: Find the prime factors of 9360.

Applying the tests in § 42, we find the factors are 5, 9, 8, 2, 13. (9 and 8 were used to save time. The prime factors of 9, are 3×3; of 8, are $2\times2\times2$.) Therefore the prime factors of 9360 are $5\times3\times3\times2\times2\times2\times2\times13$.

5)9360
9)1872
8) 208
2) 26
13) 13
1

WRITTEN EXERCISE

(1) Find the prime factors of:

a. 150	*f.* 315	*k.* 2433
b. 216	*g.* 363	*l.* 4350
c. 1000	*h.* 1124	*m.* 891
d. 777	*i.* 780	*n.* 912
e. 762	*j.* 1600	*o.* 1728

ILLUSTRATIVE EXAMPLE (1): Find the greatest common divisor of 18, 36, 12, 24.

Solution: Separate each of the numbers into prime factors. The factor 2 occurs once in all the numbers. The factor 3 occurs once in all the numbers. Therefore 2×3, or 6, is the greatest common divisor of 18, 36, 12, 24.

$18 = 3 \times 2 \times 3$
$36 = 2 \times 2 \times 3 \times 3$
$12 = 3 \times 2 \times 2$
$24 = 2 \times 2 \times 2 \times 3$

To find the greatest common divisor of two or more numbers: Separate the numbers into their prime factors. The product of the prime factors common to all the numbers is the g. c. d.

ILLUSTRATIVE EXAMPLE (2): Find the g. c. d. of 144, 84, 36, 1728.

Solution:

36—84—144—1728
36—12— 0 — 0
12 is the g. c. d.

Explanation: Divide all the other numbers by the smallest number, writing the remainders as shown in the solution, bringing the divisor down as before. Divide again by the smallest number and we find it is exactly contained in 36. Therefore 12 is the g. c. d.

(2) Find the g. c. d. of the following, using either of the two methods explained above:

a. 108, 54, 72	*d.* 49, 700, 35	*g.* 52, 65, 39, 78
b. 105, 35, 70	*e.* 72, 144, 288	*h.* 38, 76, 95, 133
c. 136, 212, 24	*f.* 29, 58, 87	*i.* 69, 115, 207, 161

44. Least common multiple.

a. Name some numbers that are exactly divisible by 4; by 5; by 7; by 3 and 4; by 2 and 3.

b. What are the smallest numbers that are exactly divisible by 3? By 8? By 5?

c. Instead of saying that one number will contain another

number, we say it is a *multiple* of a number. Hence 24 is a multiple of 2, 3, 4, 6, 8.

d. 45 is a multiple of what two numbers? What is the least number that will contain 3, 5, 6? That will contain 4, 6, 5? Name other multiples of the above numbers.

1. A number that will exactly contain each of two or more numbers is a common multiple of those numbers as: 48 is a common multiple of 2, 4, 6, 8. The least number that will contain each of two or more numbers is the least common multiple (l. c. m.) as: 24 is the l. c. m. of 2, 4, 6, 8. The least common multiple of two or more numbers is equal to the product of all their prime factors, using each factor the greatest number of times that it occurs in any of the numbers.

ILLUSTRATIVE EXAMPLE: Find the l. c. m. of 18, 45, 21, 36.

(1) Separate numbers into prime factors. The factor 3 occurs twice in 18, 45, 36. The factor 2 occurs twice in 36 and once in 18. The factor 3 occurs once in 21, therefore, the l. c. m. is $3\times3\times2\times5\times7\times2=1260$. Or, Method 2—

(1) $18=3\times3\times2$
$45=3\times3\times5$
$21=3\times7$
$36=3\times3\times2\times2$

(2) Arrange the numbers in a horizontal line and divide by any prime number that will exactly divide two or more of them, until quotients are found that are prime to each other. The product of the several divisors and the last quotients, is the least common multiple.

(2)
3) 18, 45, 21, 36
3) 6, 15, 7, 12
2) 2, 5, 7, 4
1, 5, 7, 2

WRITTEN EXERCISE

Find the l. c. m. of:

a. 45, 72, 63, 105
b. 129, 144, 168, 43
c. 96, 192, 84, 72
d. 75, 105, 60, 90
e. 91, 169, 39, 52
f. 18, 54, 27, 9
g. 11, 91, 13, 52
h. 117, 234, 48
i. 26, 65, 91, 78

45. Cancellation.

1. $(5\times12)\div(5\times6)=\frac{1\,2}{6}=2$. Why? How many times is 8×6 contained in 24×6? 6×8 in 12×8? 5×9 in 15×9? 9×6 in 27×6? 15×3 in 45×3? 4×6 in 12×6?

2. In determining the quotient, what factors may be omitted from both dividend and divisor?

3. What effect has rejecting equal factors from both dividend and divisor on the quotient?

4. This process of shortening computations by rejecting or cancelling equal factors from both dividend and divisor is cancellation.

ILLUSTRATIVE EXAMPLE (1): Divide the product of 25, 6, 4, and 12 by the product of 3, 5, 8, 2, and 10.

Solution: The dividend is written above the divisor with a line between as in illustration. Dividing the dividend and divisor by 5 does not change the value of the quotient; hence, the factor 5 may be cancelled from the dividend 25, and from 5 in the divisor, leaving in their places 5 in the dividend and 1 in the divisor.

$$\frac{\overset{5}{\cancel{25}}\times\overset{2}{\cancel{6}}\times\overset{2}{\cancel{4}}\times\overset{6}{\cancel{12}}}{\underset{4}{\cancel{8}}\times\underset{2}{\cancel{2}}\times\underset{2}{\cancel{10}}}=\frac{3}{1}=3$$

a. In like manner the factors: 3, 5, 4, 2, 2, 4, 3, 2, may be rejected; there remains the uncancelled factor 3 in the dividend and 1 in the divisor; hence, the quotient is 3.

b. If all the factors of the dividend are cancelled, the numerator of the quotient is 1. The 1's need not be written unless all other factors cancel.

WRITTEN EXERCISE

(1) Divide, using cancellation:

a. $6\times5\times9\times8\times3$ by $3\times24\times6\times15$

b. $15\times7\times8\times4$ by $32\times30\times14\times6\times3$

c. $28\times35\times54$ by $42\times21\times18\times3$

d. $75\times49\times28\times16$ by $100\times91\times7\times56\times2$

e. $150\times168\times42\times38$ by $75\times56\times21\times96$

(2) If 28 yd. cloth cost \$168, what will 36 yd. cost at the same rate?

(3) How many days' work at \$1.75 will pay for 30 days' board at \$1.25 a day?

CHAPTER VIII

FRACTIONS. UNITS OF MEASURE

1. How are fractions used in the tailoring business? In dressmaking? In some other trades or businesses? What are the measures most used?

2. What is the unit of value in United States money? How many cents in one dollar? What part of one dollar is 50 cents? How many half dollars in one dollar?

3. If we speak of one-half yard, we must think of the yard as being divided into how many parts? How many halves in one yard? How do the parts compare in size?

4. If we speak of one-fourth yard, we must think of the yard as being divided into how many parts? How many fourths in one yard? How do the parts compare in size?

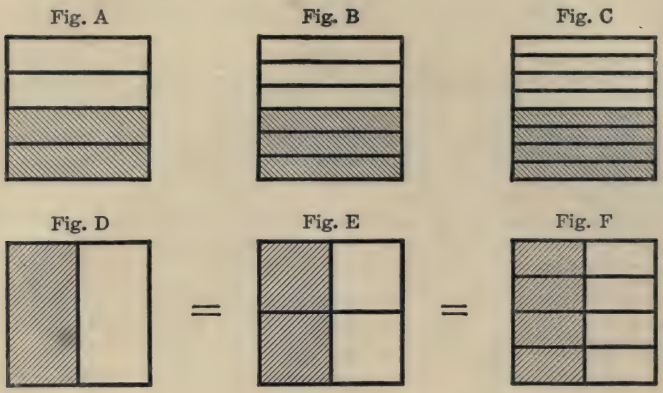
46. Fraction defined. One or more of the equal parts of a unit is a fraction: one-half, three-fourths, etc. Common fractions are expressed by two numbers one written above and the other below a short, horizontal line. Thus, one-half is written $\frac{1}{2}$: three-fourths is written $\frac{3}{4}$, etc.

1. In speaking of $\frac{1}{2}$ yard, or $\frac{3}{4}$ of a yard, which number gives the name to the fraction? Which number shows the size of the parts?

47. Numerator and denominator. The numerator and denominator are the terms of the fraction.

1. The number which shows into how many parts a thing has been divided is the denominator (namer) and is written below the line. Thus, in the fraction $\frac{1}{2}$, 2 is the denominator. It shows that something has been divided into two equal parts, and also tells the size of the parts.

2. The number of parts used or taken is the numerator (numberer) and is written above the line. Thus, in the fraction $\frac{4}{5}$, 5, the denominator, shows that a number has been divided into five equal parts, and 4, the numerator, shows that four of these parts have been taken.



(1) Into how many parts is Fig. A divided? What is the denominator or name of the fractions? The shaded part shows the number of parts taken. What is the numerator of the fraction? Write the fraction represented by Fig. A. Write the fraction represented by Fig. B. Explain each term. Write the fraction represented by Fig. C. Explain each term.

(2) What part of Fig. D is shaded? What part of Fig. E is shaded? Of Fig. F?

(3) Into how many parts is Fig. D divided? How many parts are taken? Write the fraction represented in Fig. D.

(4) Into how many parts is Fig. E divided? How many parts are taken? How does the shaded part of Fig. D compare in size with the shaded part of Fig. E? How does the denominator in the fraction $\frac{1}{2}$ compare in size with the de-

nominator in the fraction $\frac{2}{4}$? How do the sizes of the parts compare? How does the number of parts taken differ?

(5) How many parts are there in Fig. F? How many parts are taken? Which of the three squares has the greatest number of equal parts? Which has the smallest parts? Which has the fewest parts? How does the size of the parts in Fig. D compare with the size of the parts in Fig. E? With the size of the parts in Fig. F? Compare the number and size of the parts in Fig. E with the number and size of the parts in Fig. F.

48. Fractional values. From this comparison it is easily seen that the greater the number of equal parts into which a unit is divided, the smaller is each part; and the fewer the equal parts into which a unit is divided, the greater the size of the parts. Therefore, when two fractions have the same denominator, the one having the greater numerator expresses the greater value. Example: $\frac{5}{8}$ is greater than $\frac{3}{8}$. Also, when two fractions have the same numerator, the one having the smaller denominator expresses the greater value. Example: $\frac{3}{4}$ is larger than $\frac{3}{8}$.

1. Show by a diagram that $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ are equal in value.

2. Name the following fractions in the order of their size, the largest first:

$$\frac{1}{10}\ \frac{1}{6}\ \frac{1}{8}\ \frac{1}{9}\ \frac{1}{5}\ \frac{1}{3}\ \frac{1}{12}\ \frac{1}{2}\ \frac{1}{7}\ \frac{1}{50}\ \frac{1}{25}\ \frac{1}{100}\ \frac{1}{4}$$

3. Name the following fractions in the order of their size, the smallest first:

$$\frac{5}{36}\ \frac{7}{12}\ \frac{1}{18}\ \frac{1}{6}\ \frac{1}{9}\ \frac{1}{36}\ \frac{1}{4}\ \frac{1}{2}$$

4. What is the unit of measure for buttons, napkins, etc.? How many buttons in one dozen? In one-half dozen? What is $\frac{1}{2}$ of $\frac{1}{2}$ a dozen? How many buttons in $\frac{1}{4}$ of a dozen? In $\frac{2}{4}$

a dozen? Compare $\frac{1}{2}$ a dozen with $\frac{2}{4}$ of a dozen. How many buttons in $\frac{3}{4}$ of a dozen? In $\frac{4}{4}$ of a dozen? How many buttons in a half dozen?

5. How many buttons in $\frac{1}{3}$ of a dozen? In $\frac{2}{3}$ of a dozen? In $\frac{3}{6}$ of a dozen? What is $\frac{1}{2}$ of $\frac{1}{3}$ of a dozen buttons? How many buttons in $\frac{1}{6}$ of a dozen? Compare $\frac{1}{3}$ and $\frac{1}{6}$ of a dozen. How many buttons in $\frac{2}{6}$ of a dozen? In $\frac{3}{6}$? In $\frac{4}{6}$? In $\frac{5}{6}$? In $\frac{6}{6}$?

6. How many buttons in $\frac{1}{6}$ of a dozen? What is $\frac{1}{2}$ of $\frac{1}{6}$ of a dozen buttons? How many buttons in $\frac{1}{12}$ of a dozen? Compare $\frac{1}{6}$ and $\frac{1}{12}$. How many buttons in $\frac{2}{12}$ of a dozen? Compare $\frac{2}{12}$ and $\frac{1}{6}$. How many buttons in $\frac{3}{12}$ of a dozen? What is another name for $\frac{3}{12}$? How many buttons in $\frac{1}{4}$ of a dozen? How many buttons in $\frac{4}{12}$ of a dozen? 3 is what part of 12? What is another name for $\frac{1}{3}$ of a dozen? When the fraction $\frac{4}{12}$ is changed to $\frac{1}{3}$, what change is made in the size of the parts? In the number of parts taken? When the fraction $\frac{3}{12}$ is changed to $\frac{1}{4}$, what change is made in the size of the parts? In the number of parts taken?

49. Reduction. A fraction can be changed to larger terms by multiplying both terms by the same number. What effect is produced upon the value of the fraction?

EXERCISE

(1) Read the following fractions, supplying the numerators:

a. $\frac{1}{2} = \frac{}{4} = \frac{}{6} = \frac{}{8} = \frac{}{10} = \frac{}{12} = \frac{}{14} = \frac{}{16} = \frac{}{20}$

b. $\frac{1}{3} = \frac{}{6} = \frac{}{9} = \frac{}{12} = \frac{}{18} = \frac{}{24} = \frac{}{30} = \frac{}{36}$

c. $\frac{1}{4} = \frac{}{8} = \frac{}{16} = \frac{}{20} = \frac{}{32} = \frac{}{28} = \frac{}{12} = \frac{}{40} = \frac{}{48}$

d. $\frac{2}{3} = \frac{}{6} = \frac{}{9} = \frac{}{12} = \frac{}{15} = \frac{}{18} = \frac{}{21} = \frac{}{24} = \frac{}{36}$

e. $\frac{3}{4} = \frac{}{16} = \frac{}{20} = \frac{}{32} = \frac{}{28} = \frac{}{40} = \frac{}{12} = \frac{}{36} = \frac{}{48}$

f. $\frac{1}{8} = \frac{}{24} = \frac{}{32} = \frac{}{40} = \frac{}{48} = \frac{}{56} = \frac{}{64} = \frac{}{72}$

g. $\frac{1}{12} = \frac{}{60} = \frac{}{72} = \frac{}{84} = \frac{}{96} = \frac{}{108} = \frac{}{120} = \frac{}{132}$

h. $\frac{1}{16} = \frac{}{32} = \frac{}{48} = \frac{}{64} = \frac{}{80} = \frac{}{96} = \frac{}{112} = \frac{}{128}$

(2) Change:

a. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{6}$ $\frac{5}{6}$ to twelfths.

b. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{8}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$ to sixteenths.

c. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{1}{10}$ $\frac{3}{10}$ $\frac{5}{10}$ to twentieths.

d. $\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{1}{8}$ $\frac{5}{8}$ to twenty-fourths.

e. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{8}$ $\frac{5}{8}$ $\frac{1}{16}$ $\frac{5}{16}$ $\frac{9}{16}$ to thirty-seconds.

f. $\frac{5}{6}$ $\frac{3}{8}$ $\frac{5}{12}$ $\frac{3}{4}$ $\frac{5}{16}$ $\frac{1}{2}$ $\frac{2}{3}$ to forty-eighths.

g. $\frac{1}{2}$ $\frac{7}{10}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{3}{25}$ $\frac{8}{50}$ to one-hundredths.

(3) By what number must both terms of the fraction be divided to change $\frac{18}{20}$ to $\frac{9}{10}$? To change $\frac{12}{15}$ to $\frac{4}{5}$? To change $\frac{15}{24}$ to $\frac{5}{8}$? When the fraction $\frac{4}{16}$ is changed to $\frac{1}{4}$, what change is made in the size of the parts? In the number of parts taken?

1. Dividing both terms of a fraction by the same number does not change the value of a fraction. A fraction can be changed to smaller terms by dividing both terms by the same number. To change a fraction to its lowest terms, strike out all the factors common to both terms. When the numerator and denominator are prime to each other, a fraction is reduced to its lowest terms.

(1) Change to their lowest terms:

a. $\frac{28}{168}$ $\frac{32}{182}$ $\frac{15}{75}$ $\frac{16}{128}$ $\frac{13}{65}$ $\frac{24}{168}$

b. $\frac{18}{126}$ $\frac{17}{153}$ $\frac{23}{115}$ $\frac{125}{625}$ $\frac{24}{1728}$

c. $\frac{18}{96}$ $\frac{15}{225}$ $\frac{24}{216}$ $\frac{16}{96}$ $\frac{175}{315}$

2. Changing fractions to fractions having the same number for a denominator (or the same kind of parts) is called reducing to a common denominator. Fractions having the same denominator are sometimes called similar fractions. Thus $\frac{8}{12}$, $\frac{6}{12}$, $\frac{5}{12}$, each having 12 for a denominator, are similar fractions.

(1) To what fractions can $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ be changed so that

they will have the same kind of parts, or a common denominator? Change to a common denominator and illustrate:

a. $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$
b. $\frac{2}{3}$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{6}$
c. $\frac{1}{2}$ $\frac{3}{4}$ $\frac{5}{8}$
d. $\frac{2}{3}$ $\frac{5}{6}$ $\frac{1}{12}$

50. Equivalent fractions. These examples show that the same value may be expressed in units of different size, and that when we lessen the size of the units, we increase the number of parts; when we lessen the number of units, we increase their size. Thus we express equivalent values in different forms. Thinking one fraction in terms of another simplifies the whole subject of fractions. Equivalent fractions are equal in value.

1. Example: $\frac{5}{6}$ and $\frac{10}{12}$ are equivalent fractions because they are equal in value. Name other equivalent fractions, and tell why they are equivalent. Draw a diagram to illustrate the facts stated.

51. Improper fractions. A fraction equal to, or greater than a whole one or a whole unit is an improper fraction, as: $\frac{20}{4}, \frac{8}{4}, \frac{6}{5}$.

1. How many half dollars are equal to \$1? To \$2? How many quarter dollars are equal to \$1? To \$2? To how many dollars are 8 quarters equal? 4 quarters? 12 quarters? 20 quarters?

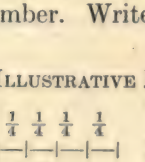
2. How many half dollars in \$3? In $\$3\frac{1}{2}$? How many quarters are there in \$1? In \$2? In $\$1\frac{3}{4}$? In $\$2\frac{1}{4}$? In $\$3\frac{3}{4}$? In $\$2\frac{1}{2}$?

3. Express as fourths: \$1, $\$1\frac{1}{4}$, $\$2\frac{3}{4}$, \$3, $\$5\frac{1}{4}$, \$7.

4. Reduce to eighths: 1, 2, 3, $\frac{1}{2}$, $2\frac{1}{2}$, $5\frac{1}{2}$, $3\frac{1}{4}$, $3\frac{3}{4}$, 6, $6\frac{1}{2}$, $6\frac{3}{4}$, $5\frac{1}{4}$, $4\frac{1}{4}$, $7\frac{1}{4}$, $10\frac{1}{2}$, $9\frac{1}{2}$.

52. Mixed numbers. A number made up of an integer and a fraction is a mixed number, as: $3\frac{5}{8}$, $2\frac{1}{2}$. Write a mixed number. Write an improper fraction.

ILLUSTRATIVE EXAMPLE: Show that $3\frac{3}{4} = \frac{15}{4}$.



$1 = \frac{4}{4}$.
$3 = 3 \times \frac{4}{4}$ or $\frac{12}{4}$.
$3 = \frac{12}{4}$
$\frac{12}{4} + \frac{3}{4} = \frac{15}{4}$.

Explanation: Since 1 equals 4 fourths, three equals 3 times 4 fourths or 12 fourths, and $3\frac{3}{4}$ equals 12 fourths plus $\frac{3}{4}$, or 15 fourths: $3\frac{3}{4} = \frac{15}{4}$.

Reducing 3 to $\frac{12}{4}$ or $3\frac{3}{4}$ to $\frac{15}{4}$ is reducing a whole or mixed number to an improper fraction.

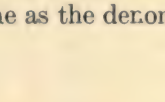
53. Change mixed numbers to improper fractions. To change a mixed number to an improper fraction: Multiply the whole number by the denominator of the fraction and to the product add the numerator; the denominator of the equivalent improper fraction is the same as the denominator of the given fraction.

1. Reduce to improper fractions:

a. $3\frac{3}{5}$	*d.* $12\frac{1}{2}$	*g.* $33\frac{1}{3}$	*j.* $87\frac{1}{2}$	*m.* $125\frac{3}{4}$
b. $5\frac{1}{6}$	*e.* $8\frac{1}{3}$	*h.* $62\frac{1}{2}$	*k.* $56\frac{1}{4}$	*n.* $208\frac{5}{12}$
c. $6\frac{7}{8}$	*f.* $16\frac{2}{3}$	*i.* $66\frac{2}{3}$	*l.* $37\frac{1}{2}$	*o.* $125\frac{5}{16}$

54. Change improper fractions to mixed numbers.

ILLUSTRATIVE EXAMPLE: Show that $\frac{8}{3} = 2\frac{2}{3}$.



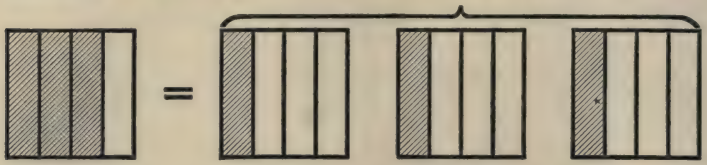
Show that:

a. $\frac{5}{2} = 2\frac{1}{2}$; $\frac{4}{2} = 2$; $\frac{6}{5} = 1\frac{1}{5}$; $\frac{12}{7} = 1\frac{5}{7}$.

b. $\frac{7}{4} = 1\frac{3}{4}$; $\frac{9}{3} = 3$; $\frac{10}{3} = 3\frac{1}{3}$; $\frac{11}{3} = 3\frac{2}{3}$.

Explanation: Reduce $\frac{8}{3}$ to a mixed number. $8 \div 3 = 2\frac{2}{3}$. There

are $\frac{3}{3}$ in one unit: in $\frac{8}{3}$ there are as many units as 3 is contained times in 8, or 2 times, and 2 over. Therefore $\frac{8}{3} = 2\frac{2}{3}$.



1. Changing $\frac{4}{2}$ to 2, or $\frac{5}{2}$ to $2\frac{1}{2}$, is reducing an improper fraction to a whole or mixed number.

2. This illustration shows that $\frac{3}{4}$ of $1 = \frac{1}{4}$ of 3, or $3 \div 4$. Therefore a fraction denotes that the numerator is to be divided by the denominator. $\frac{3}{4}$ represents 3 of 4 equal parts of a whole. It also represents one-fourth of 3, and 3 divided by 4. Show by a diagram what $\frac{2}{3}$ represents: $\frac{5}{6}$; $\frac{4}{5}$; $\frac{5}{8}$; $\frac{3}{11}$; $\frac{5}{7}$; $\frac{13}{15}$.

3. Reduce to a whole or mixed number:

a. $\frac{9}{8}$	*d.* $\frac{18}{3}$	*g.* $\frac{43}{5}$	*j.* $\frac{52}{7}$	*m.* $\frac{105}{11}$	*p.* $\frac{115}{12}$
b. $\frac{12}{7}$	*e.* $\frac{27}{9}$	*h.* $\frac{37}{3}$	*k.* $\frac{108}{12}$	*n.* $\frac{84}{7}$	*q.* $\frac{125}{4}$
c. $\frac{15}{9}$	*f.* $\frac{63}{7}$	*i.* $\frac{36}{12}$	*l.* $\frac{75}{3}$	*o.* $\frac{96}{8}$	*r.* $\frac{150}{8}$

55. Least common denominator.

(1) $\frac{1}{2}$ = how many 4ths, 6ths, 8ths, 10ths, 20ths, 24ths, 50ths?

(2) $\frac{1}{4}$ = how many 8ths, 12ths, 16ths, 20ths, 32ds, 48ths, 36ths, 60ths?

(3) $\frac{1}{8}$ = how many 16ths, 24ths, 32ds, 40ths, 56ths, 72ds, 80ths, 96ths?

(4) $\frac{1}{3}$ = how many 6ths, 12ths, 18ths, 30ths, 42ds, 48ths, 60ths, 72ds?

(5) $\frac{1}{6}$ = how many 12ths, 18ths, 24ths, 48ths, 60ths, 72ds, 84ths?

(6) $\frac{1}{12}$ = how many 24ths, 36ths, 48ths, 60ths, 96ths, 108ths, 144ths?

(7) $\frac{1}{5}$ = how many 10ths, 15ths, 30ths, 45ths, 60ths, 75ths, 90ths?

(8) $\frac{1}{10}$ = how many 20ths, 40ths, 80ths, 200ths, 150ths, 170ths?

(9) $\frac{1}{7}$ = how many 14ths, 28ths, 35ths, 49ths, 56ths, 84ths?

(10) $\frac{1}{9}$ = how many 18ths, 36ths, 27ths, 63ds, 72ds, 96ths, 108ths?

(11) $\frac{1}{20}$ = how many 40ths, 80ths, 100ths, 120ths, 160ths, 200ths?

(12) $\frac{1}{50}$ = how many 600ths, 150ths, 300ths, 350ths, 500ths, 1000ths?

(13) Into what parts of the same size may $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$ be divided?

(14) Into what parts of the same size may $\frac{1}{2}$ and $\frac{1}{3}$ be divided? $\frac{1}{4}$ and $\frac{1}{5}$? $\frac{1}{8}$ and $\frac{1}{16}$? $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$? $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{5}$, and $\frac{1}{6}$?

1. Changing fractions to fractions of like denominators is reducing to a common denominator. Fractions that have the same denominator are called similar fractions or like fractions. Similar fractions expressed in their lowest terms have the least common denominator. Thus, the least common denominator (l. c. d.) of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{12}$, is 12, for 12 is the least number that 3, 4, 6, and 12 will divide without a remainder.

ILLUSTRATIVE EXAMPLE: Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to similar fractions having the least common denominator.

SOLUTION: It will readily be seen that the l. c. d. is 12. $12 = 4 \times 3$, 3×4, or 2×6.

$$\frac{2}{3} = \frac{4}{4} \times \frac{2}{3} = \frac{8}{12}$$
$$\frac{3}{4} = \frac{3}{3} \times \frac{3}{4} = \frac{9}{12}$$
$$\frac{5}{6} = \frac{2}{2} \times \frac{5}{6} = \frac{10}{12}$$

2. The least common denominator is the same as the least common multiple of the denominators. (See § 44, page 100.)

(1) Reduce to similar fractions having their least common denominator:

a.	$\frac{3}{4}$	$\frac{7}{5}$	$\frac{5}{6}$		*e.*	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{15}{20}$		*i.*	$\frac{8}{24}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{7}{8}$
b.	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{7}{12}$		*f.*	$\frac{4}{9}$	$\frac{5}{12}$	$\frac{7}{16}$		*j.*	$\frac{8}{11}$	$\frac{20}{22}$	$\frac{5}{9}$	$\frac{2}{3}$
c.	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{6}$	*g.*	$\frac{4}{15}$	$\frac{3}{5}$	$\frac{6}{7}$	$\frac{9}{10}$	*k.*	$\frac{16}{25}$	$\frac{12}{15}$	$\frac{2}{3}$	$\frac{4}{5}$
d.	$\frac{3}{8}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{9}$	*h.*	$\frac{8}{9}$	$\frac{3}{4}$	$\frac{11}{12}$	$\frac{2}{3}$	*l.*	$\frac{5}{12}$	$\frac{8}{21}$	$\frac{11}{28}$	$\frac{2}{3}$

3. When it is not easy to find by inspection the least common multiple of the given denominator, (that is, the least common denominator) it may be found by factoring the denominators; for the least common denominator is the product of all the different prime factors, each factor used the greatest number of times that it occurs in any denominator. (See Sec. 44, l. c. m.)

56. Addition of fractions.

(1) 6¢+3¢+4¢=? cents. 5 yd.+8 yd.+3 yd.=? yd. What kind of numbers can be added?

(2) How many fifths are $\frac{1}{5}+\frac{3}{5}+\frac{4}{5}$? How many ones and how many fifths over?

(3) What is the sum of: $\frac{1}{2}+\frac{1}{2}$? $\frac{5}{12}+\frac{3}{12}$? $\frac{3}{20}+\frac{9}{20}$? $\frac{8}{24}+\frac{3}{24}$? $\frac{3}{16}+\frac{8}{16}$? $2\frac{1}{2}+5\frac{1}{2}$? $3\frac{5}{8}+2\frac{3}{8}$? $1\frac{1}{6}+3\frac{5}{6}$? $3\frac{5}{16}+1\frac{7}{16}$? $5\frac{1}{10}+3\frac{7}{10}+2\frac{3}{10}$? What kind of fractions can be added?

(4) How are fractions having the same denominator added?

(5) What is the sum of $\frac{5}{6}$ and $\frac{3}{5}$? Solution: $\frac{5}{6}$ and $\frac{3}{5}$ are not similar fractions; they must be reduced to equivalent fractions having a least common denominator.

$\frac{5}{6}=\frac{25}{30}$; $\frac{3}{5}=\frac{18}{30}$; $\frac{25}{30}+\frac{18}{30}=1\frac{13}{30}$.

ILLUSTRATIVE EXAMPLE: What is the sum of $15\frac{3}{4}$, $28\frac{7}{8}$, and $42\frac{5}{6}$?

Solution:

$15\ \frac{3}{4}=18$
$28\ \frac{7}{8}=21$
$42\ \frac{5}{6}=20$
$87\frac{11}{24}\quad\frac{59}{24}=2\frac{11}{24}$

EXPLANATION: Write the numbers as shown in the solution. Since the numbers are composed of both integers and fractions they may be added separately, and their sums united.

By inspection, the least common denominator is seen to be 24. Write the 24 under the sum of the new numerators as shown in the solution. Write each new numerator after the fraction to which it is equivalent. Find the sum of the new numerators, writing the sum, over the common denominator, 24; $\frac{59}{24} = 2\frac{11}{24}$ which, added to the sum of the integers equals $87\frac{11}{24}$, the sum required.

(6) Find the sum of:

a. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{12}$

b. $\frac{7}{8}$, $\frac{5}{8}$, $\frac{5}{9}$, $\frac{2}{3}$

c. $12\frac{1}{2}$, $13\frac{15}{16}$, $3\frac{5}{8}$

d. $15\frac{1}{4}$, $25\frac{7}{15}$, $3\frac{3}{5}$

e. $6\frac{5}{8}$, $18\frac{3}{4}$, $12\frac{4}{5}$

f. $7\frac{5}{9}$, $12\frac{5}{6}$, $18\frac{2}{3}$, $9\frac{5}{12}$

g. $20\frac{8}{9}$, $16\frac{2}{3}$, $42\frac{5}{8}$, $120\frac{11}{12}$

h. $120\frac{15}{16}$, $42\frac{7}{8}$, $250\frac{3}{4}$, $90\frac{9}{10}$

i. $84\frac{19}{25}$, $360\frac{49}{50}$, $218\frac{3}{4}$, $45\frac{4}{5}$

j. $39\frac{9}{10}$, $115\frac{9}{20}$, 65, $204\frac{4}{5}$

k. $365\frac{1}{4}$, $272\frac{11}{12}$, $120\frac{5}{6}$, $308\frac{5}{18}$

l. 72, $309\frac{19}{20}$, $160\frac{7}{10}$, $248\frac{3}{5}$

Problems

(1) A skirt requires $3\frac{3}{4}$ yd., a coat, $2\frac{5}{8}$ yd., a waist, $1\frac{3}{4}$ yd. How many yards are required in all?

(2) If a customer bought $3\frac{5}{8}$ yd. of cloth for $\$16\frac{3}{4}$; $4\frac{2}{3}$ yd. for $\$25\frac{1}{2}$; $8\frac{5}{6}$ yd. for $\$12\frac{3}{5}$, how many yards did she buy, and how much money did the goods cost her?

(3) A merchant bought four pieces of cloth containing $28\frac{5}{6}$, $47\frac{1}{2}$, $35\frac{7}{8}$, and $52\frac{3}{4}$ yd. respectively; how many yards of cloth did he buy?

(4) A silk merchant sold at different times $136\frac{5}{8}$ yd., $275\frac{3}{4}$ yd., $129\frac{1}{2}$ yd., $258\frac{5}{6}$ yd., $305\frac{2}{3}$ yd., and $324\frac{1}{4}$ yd. of silk. How many yards did he sell?

1. Quarters or fourths are very common fractions in the drygoods business and are usually written without denominators, the numerators being placed a little above the integers. Thus $28\frac{1}{4}$ is written 28^1; $28\frac{2}{4}$ or $\frac{1}{2}$, is written 28^2; $28\frac{3}{4}$ is written 28^3.

(1) A drygoods dealer bought 25 pieces of crêpe. If the pieces contained 45^2, 48^3, 45^1, 49^2, 54^1, 60^2, 58^3, 59^2, 57^1, 46^7, 51^2, 41^1, 52^3, 47^2, 42^3, 55^1, 44^1, 59^3, 54^3, 56^3, 62^2, 64^2,

49^3, 57^2, 60^1 yds., respectively, how many yards did he buy?

2. Short methods in addition of fractions.

$\frac{1}{5}+\frac{1}{7}=\frac{12}{35}$ Compare the sum of the denominators in the given numbers with the numerator of the answer. Compare the product of the denominators in the given numbers with the denominator of the answer.

(1) Any two simple fractions whose numerators are 1, may be added by adding the denominators for the numerator of the sum, and multiplying the denominators for the denominator of the sum.

(2) $\frac{2}{9}+\frac{2}{11}=\frac{40}{99}$; $40=(9+11)\times 2$. The sum of the numerators is equal to the sum of the denominators multiplied by the numerator of either of the given fractions. $99=9\times 11$

(3) $\frac{3}{5}+\frac{3}{8}=\frac{39}{40}$ Explain this problem by the short method. State a short way of adding any two fractions whose numerators are alike.

(4) $\frac{1}{5}+\frac{1}{6}+\frac{1}{9}=?$ Solution: $\frac{1}{5}+\frac{1}{6}=\frac{11}{30}$; $\frac{11}{30}+\frac{1}{9}=\frac{129}{270}=\frac{43}{90}$

(5) State the sum of:

a. $\frac{1}{3}, \frac{1}{4}$	*g.* $\frac{1}{7}, \frac{1}{3}$	*m.* $\frac{4}{9}, \frac{4}{5}$	*s.* $\frac{2}{3}, \frac{2}{5}, \frac{2}{7}$
b. $\frac{1}{5}, \frac{1}{6}$	*h.* $\frac{1}{4}, \frac{1}{8}$	*n.* $\frac{6}{7}, \frac{6}{8}$	*t.* $\frac{2}{7}, \frac{2}{9}, \frac{2}{11}$
c. $\frac{1}{7}, \frac{1}{8}$	*i.* $\frac{1}{5}, \frac{1}{11}$	*o.* $\frac{7}{8}, \frac{7}{9}$	*u.* $\frac{1}{6}, \frac{1}{10}, \frac{1}{8}$
d. $\frac{1}{9}, \frac{1}{10}$	*j.* $\frac{2}{3}, \frac{2}{5}$	*p.* $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$	*v.* $\frac{1}{5}, \frac{1}{12}, \frac{1}{4}$
e. $\frac{1}{11}, \frac{1}{12}$	*k.* $\frac{2}{7}, \frac{2}{9}$	*q.* $\frac{1}{5}, \frac{1}{6}, \frac{1}{7}$	*w.* $\frac{1}{4}, \frac{1}{9}, \frac{1}{3}$
f. $\frac{1}{12}, \frac{1}{8}$	*l.* $\frac{3}{4}, \frac{3}{5}$	*r.* $\frac{1}{2}, \frac{1}{8}, \frac{1}{12}$	*x.* $\frac{2}{3}, \frac{2}{4}, \frac{2}{5}$

3. Business fractions are usually small and may be added as easily as integers. Example: $\frac{5}{8}+\frac{3}{4}+\frac{2}{3}+\frac{1}{2}$. Solution: Determine the least common denominator by inspection; then reduce mentally each fraction to a fraction having a common denominator and add the numerators.

$$\frac{5}{8}+\frac{3}{4}+\frac{2}{3}+\frac{1}{2}=\frac{15+18+16+12}{24}=\frac{61}{24}=2\frac{13}{24}.$$

(1) Find the sum of:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{2}{3}$	$\frac{5}{9}$	$\frac{2}{7}$	$\frac{7}{8}$	$\frac{1}{2}$
$\frac{2}{3}$	$\frac{5}{6}$	$\frac{3}{10}$	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{2}{9}$	$\frac{7}{9}$	$\frac{1}{5}$
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{9}$	$\frac{5}{12}$	$\frac{2}{3}$	$\frac{7}{10}$	$\frac{1}{4}$

l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*
$\frac{2}{3}$	$\frac{3}{8}$	$\frac{9}{14}$	$\frac{2}{3}$	$\frac{11}{12}$	$\frac{12}{15}$	$\frac{4}{7}$	$\frac{5}{9}$	$\frac{8}{15}$
$\frac{2}{5}$	$\frac{5}{6}$	$\frac{2}{7}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{3}{7}$	$\frac{5}{8}$	$\frac{9}{10}$
$\frac{2}{9}$	$\frac{1}{15}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{8}{9}$	$\frac{9}{10}$	$\frac{2}{7}$	$\frac{2}{9}$	$\frac{3}{5}$
$\frac{1}{15}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{2}{6}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{7}$	$\frac{1}{2}$

4. The addition of ordinary mixed numbers forms a large part of the work of store employees, accountants, and many other people in the business world. Before adding, arrange the work in columns, in practically the same way as in adding integers; add the fractions first, and then the integers.

ILLUSTRATIVE EXAMPLE: Find the sum of $3\frac{5}{8}$, $10\frac{3}{4}$, $7\frac{5}{6}$.

Solution: Find by inspection that the l. c. d. is 24. Find the sum of the fractions mentally; the result is $2\frac{5}{24}$. Add the result to the integers; the entire sum is $22\frac{5}{24}$.

$$\begin{array}{r} 3\frac{5}{8} \\ 10\frac{3}{4} \\ 7\frac{5}{6} \\ \hline 22\frac{5}{24} \end{array}$$

(1) State the sum of:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
$3\frac{1}{2}$	$6\frac{1}{8}$	$12\frac{1}{6}$	$18\frac{2}{3}$	$6\frac{7}{8}$	$17\frac{6}{7}$	$16\frac{2}{3}$	$8\frac{5}{6}$	$12\frac{1}{2}$
$5\frac{3}{4}$	$7\frac{1}{9}$	$10\frac{1}{5}$	$11\frac{1}{2}$	$10\frac{3}{4}$	$9\frac{1}{2}$	$9\frac{5}{6}$	$11\frac{1}{2}$	$10\frac{2}{3}$
							$4\frac{1}{4}$	$3\frac{5}{6}$
							$10\frac{1}{3}$	$1\frac{1}{4}$

j	*k*	*l*	*m*	*n*	*o*
$24\frac{1}{3}$	$95\frac{3}{10}$	$17\frac{3}{16}$	$11\frac{3}{14}$	$92\frac{5}{8}$	$51\frac{3}{4}$
$6\frac{5}{6}$	$8\frac{1}{8}$	$10\frac{1}{2}$	$8\frac{2}{7}$	$6\frac{1}{3}$	$9\frac{2}{3}$

p	q	r	s	t	u	v	w
$7\frac{5}{7}$	$5\frac{2}{3}$	$8\frac{3}{7}$	$11\frac{4}{15}$	$6\frac{6}{7}$	$12\frac{3}{4}$	$12\frac{1}{2}$	$10\frac{1}{2}$
$9\frac{5}{14}$	$6\frac{3}{4}$	$5\frac{1}{7}$	$10\frac{4}{5}$	$9\frac{6}{11}$	$8\frac{3}{4}$	$8\frac{1}{3}$	$9\frac{1}{8}$
$8\frac{3}{7}$	$7\frac{1}{5}$	$4\frac{1}{3}$	$9\frac{2}{3}$	$3\frac{1}{4}$	$5\frac{2}{5}$	$9\frac{1}{4}$	$3\frac{2}{5}$
$1\frac{1}{2}$	$3\frac{1}{6}$	$5\frac{5}{7}$	$8\frac{2}{5}$	$5\frac{1}{5}$	$10\frac{1}{2}$	$2\frac{1}{5}$	$4\frac{2}{3}$
						$3\frac{1}{6}$	$3\frac{5}{6}$
						$1\frac{1}{7}$	$2\frac{5}{8}$

x	y	z	aa	bb	cc
$11\frac{1}{5}$	$4\frac{1}{2}$	$7\frac{5}{8}$	$10\frac{3}{10}$	$2\frac{3}{8}$	$10\frac{4}{9}$
$11\frac{1}{6}$	$3\frac{3}{8}$	$6\frac{5}{9}$	$7\frac{3}{5}$	$5\frac{3}{4}$	$5\frac{4}{5}$
$5\frac{3}{4}$	$10\frac{2}{3}$	$2\frac{2}{3}$	$6\frac{1}{8}$	$7\frac{1}{6}$	$7\frac{1}{10}$
$4\frac{3}{8}$	$5\frac{2}{5}$	$5\frac{2}{7}$	$5\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{3}{5}$
$1\frac{2}{5}$	$1\frac{5}{8}$	$1\frac{4}{9}$	$2\frac{4}{5}$	$8\frac{5}{8}$	$9\frac{1}{3}$
$1\frac{2}{3}$	$3\frac{1}{2}$	$1\frac{1}{3}$	$3\frac{4}{10}$	$1\frac{5}{6}$	$1\frac{1}{9}$

(2) Silk was sold in the following quantities: $3\frac{5}{8}$ yd., $6\frac{1}{2}$ yd., $3\frac{3}{4}$ yd., $5\frac{1}{6}$ yd. How many yds. of silk were sold?

(3) The following amounts of velvet were sold: $3\frac{1}{2}$ yd., $5\frac{5}{8}$ yd., $3\frac{3}{4}$ yd., $2\frac{5}{6}$ yd. How many yards of velvet were sold?

(4) Some remnants of lace measured $5\frac{5}{6}$ yd., $3\frac{3}{4}$ yd., $2\frac{2}{3}$ yd., $4\frac{1}{2}$ yd., $3\frac{7}{8}$ yd. How many yards in all?

57. Subtraction of fractions.

EXERCISE

(1) $\$\frac{3}{4}-\$\frac{1}{4}=?$ $\$\frac{4}{5}-\$\frac{3}{5}=?$ $\$\frac{9}{10}-\$\frac{7}{10}=?$

(2) How many tenths in $\frac{9}{10}+\frac{7}{10}$? In $\frac{9}{10}-\frac{7}{10}$?

(3) How many eighths are there in $\frac{1}{2}+\frac{3}{8}$? $\frac{1}{2}-\frac{3}{8}$? In $\frac{7}{8}+\frac{1}{4}$? In $\frac{7}{8}-\frac{1}{4}$? In $\frac{3}{4}+\frac{3}{8}$? In $\frac{3}{4}-\frac{3}{8}$?

What kind of fractions can be subtracted?

(4) How much is $\frac{5}{7}+\frac{1}{3}$? $\frac{5}{6}-\frac{1}{3}$? $\frac{1}{2}+\frac{1}{4}$? $\frac{1}{2}-\frac{1}{4}$? $\frac{3}{4}+\frac{2}{3}$? $\frac{3}{4}-\frac{2}{3}$? $\frac{1}{4}+\frac{1}{6}$? $\frac{1}{4}-\frac{1}{6}$?

1. Fractions must be made similar before they can be subtracted or added; only like numbers, and parts of like units, can be added, or subtracted.

ILLUSTRATIVE EXAMPLE (1): Find the difference between $\frac{6}{7}$ and $\frac{3}{5}$.

$\frac{6}{7}=\frac{30}{35}$
$\frac{3}{5}=\frac{21}{35}$
$\frac{9}{35}$

Solution: Change the given fractions to equivalent fractions having a least common denominator. By inspection the l. c. d. is found to be 35. Instead of adding the new numerators, subtract the one from the other, obtaining $\frac{9}{35}$, the required remainder.

ILLUSTRATIVE EXAMPLE (2): From $25\frac{3}{8}$ take $15\frac{5}{6}$.

$25\frac{3}{8}=24\frac{3}{24}$
$15\frac{5}{6}=15\frac{20}{24}$
$9\frac{13}{24}$

Solution: Explanation: $\frac{5}{6}$ is larger than $\frac{3}{8}$. Borrow one from 25 (leaving 24), change it to eighths, making $\frac{8}{8}$, and add it to the $\frac{3}{8}$ making $\frac{11}{8}$. $\frac{11}{8}=\frac{33}{24}$. $\frac{5}{6}=\frac{20}{24}$. $\frac{33}{24}-\frac{20}{24}=\frac{13}{24}$. $24-15=9$. Ans. $9\frac{13}{24}$.

EXERCISE

Find the value of:

a. $\frac{3}{4}-\frac{3}{8}$ *b.* $\frac{9}{10}-\frac{3}{4}$ *c.* $\frac{7}{8}-\frac{1}{2}$ *d.* $\frac{8}{9}-\frac{5}{6}$ *e.* $2\frac{3}{4}-1\frac{1}{2}$ *f.* $3\frac{1}{4}-\frac{7}{8}$ *g.* $2\frac{5}{8}-\frac{11}{16}$ *h.* $3\frac{5}{6}-\frac{5}{12}$ *i.* $6\frac{1}{2}-2\frac{2}{3}$ *j.* $8\frac{1}{4}-5\frac{7}{12}$ *k.* $9\frac{3}{8}-6\frac{9}{10}$ *l.* $5\frac{2}{5}-3\frac{3}{4}$ *m.* $25\frac{1}{3}-10\frac{4}{5}$ *n.* $30\frac{5}{9}-20\frac{2}{3}$ *o.* $50\frac{5}{8}-25\frac{5}{16}$ *p.* $100\frac{1}{8}-50\frac{5}{12}$

q. $25\frac{1}{2}$ yards of crêpe were sold from a piece containing 52 yards. How many yards were left?

r. How much change should a customer receive from a \$10 bill if she buys shoes costing $\$5\frac{3}{4}$ and stockings costing $\$3\frac{1}{2}$?

s. I buy $8\frac{2}{3}$ yards of goods for one dress, and $9\frac{3}{4}$ yards for another. How many yards more in the second dress than the first?

t. If $1\frac{7}{8}$ yd. is cut from a piece of lace $2\frac{1}{3}$ yd. long, what part of a yard is left?

u. A piece of woolen cloth measured $3\frac{3}{4}$ yd. before it was washed; after washing it measured only $3\frac{3}{16}$ yd. How much was lost by shrinkage?

v. From a piece of crêpe measuring $40\frac{5}{8}$ yards there were sold the following amounts to different customers: $13\frac{2}{3}$ yards, $5\frac{5}{6}$ yards, $11\frac{3}{4}$ yards, and $6\frac{5}{8}$ yards. How many yards were left in the piece?

WRITTEN EXERCISE

Find the value of:

a. $15\frac{5}{6}-13\frac{7}{8}$	*e.* $100\frac{1}{6}-37\frac{5}{8}$	*i.* $\frac{15}{16}-\frac{1}{6}-\frac{1}{5}$
b. $75-28\frac{5}{9}$	*f.* $150\frac{7}{9}-98\frac{7}{8}$	*j.* $\frac{23}{24}-\frac{5}{12}-\frac{1}{3}$
c. $100-69\frac{5}{8}$	*g.* $204\frac{7}{12}-125\frac{7}{16}$	*k.* $7\frac{3}{4}-2\frac{5}{8}-1\frac{1}{2}$
d. $85\frac{2}{3}-20\frac{17}{18}$	*h.* $198\frac{8}{9}-136\frac{7}{8}$	*l.* $10\frac{5}{6}-3\frac{5}{12}-2\frac{2}{3}$

2. Short methods of subtraction.

ILLUSTRATIVE EXAMPLE (1): From $\frac{1}{3}$ take $\frac{1}{12}$.

Solution: $12-3=9$, the new numerator.

$12\times3=36$, the new denominator.

Therefore, the required remainder is $\frac{9}{36}$ or $\frac{1}{4}$.

ILLUSTRATIVE EXAMPLE (2): From $\frac{7}{8}$ take $\frac{7}{12}$.

Solution: $(12-8)\times7=28$, the new numerator.

$12\times8=96$, the new denominator.

Therefore, the required remainder is $\frac{28}{96}$ or $\frac{7}{24}$.

(1) Any two fractions may be subtracted in this way when the numerators are alike.

(2) State a short way of subtracting two fractions when the numerators are alike.

ORAL EXERCISE

State the value of:

a. $\frac{1}{5}-\frac{1}{6}$	*g.* $\frac{1}{2}-\frac{1}{3}$	*m.* $\frac{2}{3}-\frac{2}{5}$	*s.* $8\frac{1}{5}-3\frac{1}{2}$
b. $\frac{1}{6}-\frac{1}{7}$	*h.* $\frac{1}{2}-\frac{1}{9}$	*n.* $\frac{2}{5}-\frac{2}{9}$	*t.* $12\frac{1}{2}-7\frac{1}{4}$
c. $\frac{1}{7}-\frac{1}{8}$	*i.* $\frac{1}{2}-\frac{1}{8}$	*o.* $\frac{2}{7}-\frac{2}{11}$	*u.* $15\frac{1}{6}-10\frac{1}{5}$
d. $\frac{1}{8}-\frac{1}{9}$	*j.* $\frac{1}{2}-\frac{1}{7}$	*p.* $\frac{5}{8}-\frac{5}{12}$	*v.* $25\frac{8}{9}-11\frac{1}{7}$
e. $\frac{1}{9}-\frac{1}{10}$	*k.* $\frac{1}{2}-\frac{1}{6}$	*q.* $\frac{6}{7}-\frac{6}{17}$	*w.* $32\frac{2}{5}-12\frac{2}{9}$
f. $\frac{1}{3}-\frac{1}{4}$	*l.* $\frac{1}{2}-\frac{1}{5}$	*r.* $\frac{8}{9}-\frac{8}{20}$	*x.* $56\frac{5}{8}-30\frac{5}{6}$

Problems

(1) A piece of flannel containing $56\frac{7}{8}$ yards shrank $1\frac{2}{3}$ yards in dyeing. How much did the cloth then measure?

(2) If $28\frac{5}{16}$ yards are cut from a piece of cloth containing $51\frac{7}{8}$ yards, how many yards will be left?

(3) A milliner uses $2\frac{5}{8}$ yards of ribbon on one hat, $3\frac{5}{6}$ yards on another, and $4\frac{2}{3}$ yards on a third; how many yards of velvet has she left, from a piece containing 15 yards?

(4) Mrs. Hill went shopping with $\$35\frac{3}{4}$ in her purse; she spent $\$\frac{1}{2}$ for car fares, $\$1\frac{1}{4}$ for gloves, $\$5\frac{5}{8}$ for a hat, $\$21\frac{1}{5}$ for a clock, $\$\frac{7}{10}$ for thread. She bought some linen with what she had left. How much did she spend for linen?

(5) Two garments, one requiring $4\frac{5}{16}$ yd. and another $7\frac{5}{8}$ yd. are cut from a piece of cloth containing $15\frac{3}{4}$ yd. How many yards are left?

(6) A towel is to be made from a piece of crash $28\frac{7}{8}$ in. long. If $\frac{5}{16}$ in. is used at each end for a hem, how long will the finished towel be?

(7) If it costs on an average $5\frac{1}{4}¢$ a lb. to raise cotton and market it, what is the profit per lb. when it sells at $9\frac{7}{8}¢$? At $10\frac{3}{16}¢$? At $12\frac{3}{4}¢$? At $13\frac{5}{16}¢$? At $11\frac{1}{2}¢$?

(8) A sales girl at the silk counter sold the following quantities of silk: $13\frac{3}{4}$ yd., $17\frac{5}{9}$ yd., $\frac{3}{4}$ yd., $12\frac{2}{3}$ yd., $16\frac{5}{6}$ yd. How many yards did she sell? If the piece of silk contained 70 yd., how many yards were left in the piece?

(9) At the ribbon counter, a salesgirl sold the following quantities: $12\frac{5}{8}$ yd., $15\frac{3}{4}$ yd., $16\frac{1}{2}$ yd., $28\frac{2}{3}$ yd. How many yards did she sell?

(10) How many yards of ribbon were left from a total of 100 yards? How many more yards of ribbon were sold than of silk?

58. Multiplication of fractions.

EXERCISE

(1) 6 times 2 yards are how many yards? 6 times 2 thirds ($\frac{2}{3}$) are how many thirds? $\frac{1}{3}\frac{2}{}=?$

(2) 24 yards divided by 4 equals how many yards? $\frac{1}{4}$ of 24 yards equals how many yards?

(3) What is the cost of $\frac{1}{2}$ dozen buttons at 16¢ a dozen? At 40¢? At 50¢? At 48¢? How is the result obtained in each

case? Multiplying by $\frac{1}{2}$ is the same as dividing by what integer?

(4) At 20¢ a dozen, what will $\frac{1}{4}$ of a dozen buttons cost? At 40¢? At 60¢? At 48¢? What is $20 \times \frac{1}{4}$?

(5) Show that multiplying by $\frac{1}{4}$ is dividing by 4.

(6) Multiplying by $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, etc., is the same as dividing by what integers?

(7) $\frac{2}{3}$ of \$15 = \$15 multiplied by what? $\frac{3}{4}$ of 12 = 3 times what part of 12? $\frac{3}{4}$ of 12 = 12 multiplied by what fraction?

ILLUSTRATIVE EXAMPLE: Multiply $\frac{5}{6}$ by 726.

Showing that $\frac{5}{6} \times 726 = 726 \times \frac{5}{6}$.

Solution: (1) 726 times 5 sixths = 3630 sixths = $\frac{3630}{6} = 605$.

$$\frac{5}{6} \times 726 = \frac{3630}{6} = 605.$$

(2) This work may be shortened by cancellation if the multiplication is indicated, thus:—

$$\frac{121}{\cancel{726} \times 5}{\cancel{6}} = 605$$

1. The interchange of multiplicand and multiplier does not affect the value of the product.

2. Multiplying the numerator or dividing the denominator of a fraction by the integer, multiplies the fraction by that number.

(1) Show by an example that this is true. Which is the shorter method?

3. The times sign ($\times$) may be read "times," "multiplied by," or "of." Compare these two solutions: (1) $4 \times \frac{5}{6} = \frac{20}{6} = 3\frac{1}{3}$; (2) $\frac{5}{6} \times 4 = \frac{20}{6} = 3\frac{1}{3}$. Number one may be read "4 times $\frac{5}{6} = \frac{20}{6}$; or 4 multiplied by $\frac{5}{6} = \frac{20}{6}$." Number two may be

read, "$\frac{5}{6}$ of $4 = \frac{20}{6}$," or $\frac{5}{6}$ multiplied by $4 = \frac{20}{6}$. It is not correct to say "$\frac{5}{6}$ times 4" or "4 of $\frac{5}{6}$."

ORAL EXERCISE

(1.) One yard of lace costs $\$\frac{5}{8}$. What will 48 yd. cost? 24 yd.? 72 yd.? 108 yd.?

(2.) Crash is worth 24 cents a yard. What will $\frac{2}{3}$ of a yd. cost? $\frac{5}{8}$ of a yd.? $\frac{3}{4}$ yd.? $\frac{7}{8}$ yd.? $\frac{5}{6}$ yd.?

(3). Silk is 96 cents a yard. What is the cost of $\frac{1}{4}$ of a yd.? $\frac{3}{4}$ yd.? $\frac{5}{8}$ yd.? $\frac{2}{3}$ yd.? $\frac{5}{6}$ yd.? $\frac{9}{16}$ yd.?

(4). What is the cost of 1 dozen collars at the rate of 2 for $\$\frac{1}{4}$?

(5.) A girl earned \$.75 ($\$\frac{3}{4}$) a day. How much will she earn in a month of 24 working days? In 48 days? In 72 days? In 168 days?

WRITTEN EXERCISE

(1) Find the product of:

a. $36 \times \frac{5}{6}$	*g.* $\frac{5}{9}$ of 45	*m.* $162 \times \frac{5}{9}$	*s.* $\frac{7}{8}$ of 4200
b. $90 \times \frac{8}{15}$	*h.* $\frac{5}{12}$ of 96	*n.* $456 \times \frac{1}{8}$	*t.* $\frac{7}{16}$ of 3248
c. $60 \times \frac{7}{30}$	*i.* $\frac{9}{15}$ of 42	*o.* $304 \times \frac{2}{19}$	*u.* $\frac{11}{12}$ of 1008
d. $98 \times \frac{12}{49}$	*j.* $\frac{8}{24}$ of 100	*p.* $610 \times \frac{9}{10}$	*v.* $\frac{8}{9}$ of 1080
e. $64 \times \frac{7}{16}$	*k.* $\frac{7}{19}$ of 38	*q.* $850 \times \frac{1}{42}$	*w.* $\frac{14}{15}$ of 6000
f. $100 \times \frac{3}{42}$	*l.* $\frac{11}{15}$ of 105	*r.* $300 \times \frac{13}{16}$	*x.* $\frac{9}{11}$ of 1320

ILLUSTRATIVE EXAMPLE (1): Multiply 36 by $6\frac{3}{5}$.

Solution:

$$\begin{array}{r} 36 \\ \times\ 6\frac{3}{5} \\ \hline 5)108 \\ \hline 21\frac{3}{5} \\ 216 \\ \hline 237\frac{3}{5} \end{array}$$

Explanation: Multiply the fraction $\frac{3}{5}$ by 36. (1) Multiply the numerator of the fraction by 36. $(36 \times 3 = 108)$. (2) Divide 108 by 5, the denominator of the fraction. $(108 \div 5 = 21\frac{3}{5})$. (3) Multiply 36 by 6 $(36 \times 6 = 216)$ and add the results. The process may be shortened by performing the division first, provided the integer is divisible by the denominator without a remainder.

$36 \times 6\frac{3}{5} = 237\frac{3}{5}$.

ILLUSTRATIVE EXAMPLE (2): Multiply $356\frac{3}{4}$ by 79.

Solution: Explanation (1): Multiply the numerator of the fraction, 3, by 79 ($3 \times 79 = 237$). (2) Divide 237 by 4, the denominator of the fraction ($237 \div 4 = 59\frac{1}{4}$). (3) Multiply 356 by 79 ($356 \times 79 = 28124$). (4) Add the results. $356\frac{3}{4} \times 79 = 28183\frac{1}{4}$.

$356\frac{3}{4}$
$\times\ 79$
$4)237$
$59\frac{1}{4}$
3204
2492
$28183\frac{1}{4}$

(2) Compare the solution in illustrative example 2, with the solution in illustrative example 3.

ILLUSTRATIVE EXAMPLE (3): Multiply 79 by $356\frac{3}{4}$.

Solution: Explanation (1): Multiply 79 by the numerator of the fraction. ($79 \times 3 = 237$). (2) Divide the product by the denominator of the fraction ($237 \div 4 = 59\frac{1}{4}$). (3) Multiply 79×356, and add the results. $79 \times 356\frac{3}{4} = 28183\frac{1}{4}$.

79
$\times 356\frac{3}{4}$
$4)237$
$59\frac{1}{4}$
474
395
237
$28183\frac{1}{4}$

4. To multiply an integer by a fraction or mixed number: multiply the integer and the fraction separately, and find the sum of the products.

ORAL EXERCISE

Find the cost of:

a. $12\frac{3}{4}$ yd. ribbon @ 8¢
b. $11\frac{5}{6}$ yd. ribbon @ 9¢
c. $2\frac{2}{3}$ yd. velvet @ $4.00
d. $10\frac{5}{12}$ yd. braid @ 6¢
e. 20 yd. lace @ $12\frac{1}{2}$¢
f. $9\frac{1}{4}$ yd. muslin @ 36¢
g. $2\frac{1}{2}$ yd. cloth @ 72¢
h. $18\frac{1}{2}$ yd. tape @ 10¢
i. $12\frac{1}{2}$ yd. taffeta @ 20¢
j. $9\frac{3}{4}$ yd. fringe @ 24¢

WRITTEN EXERCISE

Find the cost of:

a. 1260 pairs of shoes @ $\$3\frac{5}{8}$

b. 680 prs. of slippers @ $\$2\frac{7}{8}$

c. 165 suits @ $\$12\frac{3}{4}$

d. 650 skirts @ $\$2\frac{1}{4}$

e. $56\frac{7}{8}$ yd. silk @ 98¢

f. $150\frac{3}{4}$ doz. buttons @ 88¢

g. $156\frac{7}{8}$ yd. ribbon @ 49¢

h. $65\frac{1}{2}$ doz. towels @ 72¢ (each)

i. $96\frac{1}{2}$ doz. napkins @ 98¢ (each)

j. 480 pr. stockings @ $\$\frac{3}{4}$

Find the total cost of:

$78\frac{2}{3}$ lb. sugar @ $\$.12\frac{3}{4}$

$24\frac{3}{5}$ lb. tea @ $\$.58\frac{1}{3}$

$28\frac{1}{3}$ lb. mixed spices @ $\$.19\frac{1}{2}$

$33\frac{1}{3}$ lb. of butter @ $\$.64\frac{1}{2}$

$56\frac{1}{4}$ lb. coffee @ $\$.72\frac{2}{5}$

Find the total value of each of the following groups:

$72\frac{1}{2}$ yd. muslin @ $\$.14\frac{1}{2}$

$23\frac{1}{4}$ yd. muslin @ $.08\frac{1}{2}$

$17\frac{1}{4}$ yd. muslin @ $.18\frac{3}{4}$

$62\frac{3}{4}$ yd. muslin @ $.28\frac{1}{2}$

$19\frac{1}{4}$ yd. muslin @ $.16\frac{1}{3}$

5. To multiply a mixed number by a mixed number.

ILLUSTRATIVE EXAMPLE: $15\frac{5}{9} \times 29\frac{4}{7} = \frac{\overset{20}{\cancel{140}}}{\underset{1}{\cancel{9}}} \times \frac{\overset{23}{\cancel{207}}}{\underset{1}{\cancel{7}}} = \frac{460}{1} = 460.$

To multiply a mixed number by a mixed number: change the mixed numbers to improper fractions.

(1) At a January mark-down sale the following remnants were offered to purchasers at a reduced price. Write the amounts saved per yard, and the total amounts saved, by customers purchasing goods at this sale.

Customer No.	Goods Purchased	Yds. Purchased	Former Price per yd.	Reduced price per yd.	Amount saved per yard	Total amount saved
1.	Embroidered Batiste Insertion	$17\frac{3}{4}$	$1.00	$ .25	?	?
2.	Filet Lace	$11\frac{7}{8}$	2.00	.45	?	?
3.	Venise Edge	6	8.00	1.50	?	?
4.	Novelty Lace Band	$11\frac{3}{4}$	2.50	.65	?	?
5.	Black Lace Galoon	$29\frac{1}{2}$	.65	.15	?	?
6.	Black and White Lace Band	$6\frac{1}{2}$	6.50	2.95	?	?
7.	Chiffon Flouncing	$15\frac{1}{2}$	5.00	1.95	?	?
8.	Net Trimming	$10\frac{3}{4}$	2.00	1.37	?	?
9.	Marabou Trimming	$27\frac{1}{2}$	1.50	.98	?	?
10.	Gold Net Flouncing	$11\frac{1}{8}$	3.00	1.75	?	?
11.	Silk Net	$19\frac{1}{2}$	1.75	.95	?	?
12.	Point Applique Flounce	$12\frac{3}{4}$	8.50	4.95	?	?

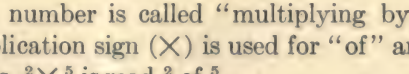
(2) A merchant bought 15 pieces of cotton crêpe, containing 56^1, 52^3, 49^2, 60^3, 58^2, 51^1, 48^1, 43^2, 54^1, 53^2, 57^1, 59^3, 48^2, 46^2, 44^3, at $62\frac{1}{2}$¢ ($\$\frac{5}{8}$) a yard. He sold the first 7 pieces

at 75¢ a yard and the remainder at $87\frac{1}{2}$¢ ($\$\frac{7}{8}$) a yard. What did he receive?

6. Multiplying a fraction by a fraction.

Finding a part of a number is called "multiplying by a fraction." The multiplication sign ($\times$) is used for "of" and must be read "of": thus, $\frac{2}{3}\times\frac{5}{8}$ is read $\frac{2}{3}$ of $\frac{5}{8}$.

(1) What is $\frac{2}{3}$ of $\frac{4}{5}$?



In the figure, the line A B is divided into five equal parts, and each of the five parts into three equal parts. Therefore, each part is $\frac{1}{15}$ of the line A B which is divided into 15 equal parts, or fifteenths. This diagram shows that,

a. $\frac{1}{3}$ of $\frac{1}{5}=\frac{1}{15}$ of the whole line.

b. $\frac{1}{3}$ of $\frac{4}{5}$ must be 4 times as much, or $\frac{4}{15}$.

c. $\frac{2}{3}$ of $\frac{4}{5}$ must be twice $\frac{4}{15}$, or $\frac{8}{15}$.

Therefore $\frac{2}{3}$ of $\frac{4}{5}=\frac{8}{15}$.

ILLUSTRATIVE EXAMPLE (1): Multiply $\frac{2}{3}$ by $\frac{3}{4}$.

a. This means find $\frac{2}{3}$ of $\frac{3}{4}$ · $\frac{1}{3}$ of $\frac{3}{4}=\frac{1}{4}$. $\frac{2}{3}=2\times\frac{1}{4}=\frac{2}{4}$ or $\frac{1}{2}$.

Operation.

(1) $\frac{1}{3}$ of $\frac{3}{4}=\frac{1}{4}$: $\frac{2}{3}$ of $\frac{3}{4}=\frac{1}{2}$

b. The same result is obtained by multiplying the numerators together and denominators together.

(2) $\frac{2}{3}\times\frac{3}{4}=\frac{2\times3}{3\times4}=\frac{6}{12}=\frac{1}{2}$

(2) To find $\frac{3}{4}$ of any number is to divide the number into how many equal parts, and to take how many of these parts?

ILLUSTRATIVE EXAMPLE (2): What is the product of $3\frac{4}{6}\times\frac{2}{3}\times\frac{3}{4}$?

Solution: Change the mixed number, $3\frac{4}{6}$, to an improper fraction. Cancel factors common to numerators and denominators and multiply the remaining factors in the numerator for the numerator of

$3\frac{4}{6}=\frac{22}{6}$

Operation.

$\frac{\overset{11}{\cancel{22}}}{\underset{3}{\cancel{6}}}\times\frac{2}{\underset{1}{\cancel{3}}}\times\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}}=\frac{11}{6}=1\frac{5}{6}$

the product, and the remaining factors in the denominator for the denominator of the product, and obtain $\frac{11}{6}$ or $1\frac{5}{6}$.

7. To multiply a fraction or a mixed number by a fraction or a mixed number.

(1) Reduce the mixed numbers and integers to improper fractions:

(2) Cancel all factors common to the numerators and denominators:

(3) Find the product of the remaining numerators for the required numerator, and the product of the remaining denominators for the required denominator and simplify.

ORAL EXERCISE

(1) What is the cost of $\frac{1}{2}$ yd. cloth @ $\$\frac{1}{2}$ a yard? @ $\$\frac{1}{4}$?

(2) What is the cost of $\frac{3}{4}$ yd. material @ $\$\frac{1}{2}$ a yard? Of $\frac{3}{8}$ yard? Of $\frac{5}{6}$ yd.? Of $\frac{7}{8}$ yd.? Of $\frac{2}{3}$ yd.? Of $\frac{5}{16}$ yd.? Of $\frac{9}{16}$ yd.? Of $\frac{3}{16}$ yd.?

Problems

(1) What is the cost of $1\frac{1}{2}$ yd. lace at $37\frac{1}{2}$¢? Of $3\frac{3}{4}$ yd? Of $2\frac{2}{3}$ yd.? Of $15\frac{5}{6}$ yd.? Of $6\frac{7}{8}$ yd.? Of $9\frac{3}{16}$ yd.?

(2) Ribbon is $62\frac{1}{2}$¢ a yard. What is the cost of $1\frac{5}{8}$ yd.? Of $2\frac{5}{6}$ yd.? Of $15\frac{2}{3}$ yd.? Of $56\frac{3}{4}$ yd.? Of $165\frac{9}{16}$ yd.?

(3) Silk is $87\frac{1}{2}$¢ a yard. Find the cost of $17\frac{5}{6}$ yd., $24\frac{5}{8}$ yd., $63\frac{3}{4}$ yd., $52\frac{1}{2}$ yd., $108\frac{2}{3}$ yd.

(4) Find the cost of:

a. $37\frac{1}{2}$ yd. carpet @ $\$1.87\frac{1}{2}$.

b. $42\frac{3}{4}$ yd. carpet @ $\$1.12\frac{1}{2}$.

c. $38\frac{3}{4}$ yd. carpet @ $\$1.75$.

d. $72\frac{7}{8}$ yd. carpet @ $\$1.82\frac{1}{2}$.

e. $156\frac{5}{6}$ yd. carpet @ $\$2\frac{1}{4}$.

f. $165\frac{3}{16}$ yd. carpet @ $\$2\frac{7}{8}$.

(5) A clerk spends $\$6\frac{3}{4}$ a week for his board, and $\frac{2}{3}$ as much for his other expenses. How much more money does he spend for board, than for his other expenses in one year?

(6) A dealer bought $56\frac{1}{2}$ yd. cloth at $\$1\frac{3}{4}$ a yd. He sold $\frac{5}{8}$ of it at $\$1.87\frac{1}{2}$ a yard, and the remainder at $\$1.62\frac{1}{2}$ a yard. Did he gain or lose? How much?

8. When mixed numbers are large it may be found shorter to multiply directly, as shown in the following example:

Multiply $156\frac{3}{4} \times 38\frac{5}{6}$.

$156\frac{3}{4}$
$38\frac{5}{6}$

$\frac{5}{8} = \frac{5}{6} \times \frac{3}{4}$
$130 = \frac{5}{6} \times 156$
$28\frac{1}{2} = \frac{3}{4} \times 38$
$5928 = 156 \times 38$

$6087\frac{1}{8} = 38\frac{5}{6} \times 156\frac{3}{4}$

WRITTEN EXERCISE

h. Find the total value of each of the following groups:

(1)	(2)
$83\frac{1}{3}$ yd. @ $52\frac{1}{2}¢$ a yd.	$84\frac{1}{2}$ yd. @ $5\frac{5}{13}¢$ a yd.
$45\frac{5}{8}$ yd. @ $5\frac{1}{3}¢$ a yd.	$31\frac{7}{8}$ yd. @ $24\frac{4}{5}¢$ a yd.
$24\frac{4}{5}$ yd. @ $15\frac{5}{9}¢$ a yd.	$53\frac{1}{3}$ yd. @ $83\frac{1}{4}¢$ a yd.
$43\frac{3}{4}$ yd. @ $28\frac{4}{5}¢$ a yd.	$61\frac{2}{3}$ yd. @ $57\frac{3}{5}¢$ a yd.
$26\frac{1}{4}$ yd. @ $42\frac{4}{5}¢$ a yd.	$71\frac{3}{4}$ yd. @ $62\frac{6}{7}¢$ a yd.

a. Multiply $305\frac{1}{8}$ by $27\frac{3}{4}$ *c*. $695\frac{5}{6} \times 42\frac{5}{6}$ *e*. $429\frac{5}{16} \times 18\frac{3}{4}$

b. $1008\frac{2}{3}$ by $39\frac{5}{8}$ *d*. $549\frac{1}{2} \times 34\frac{2}{3}$ *f*. $34\frac{2}{3} \times 38\frac{9}{16}$

9. Short methods in multiplication.

(1) Multiplication of any number ending in $\frac{1}{2}$ or .5 by itself.

ILLUSTRATIVE EXAMPLE (1): What is the cost of $12\frac{1}{2}$ yd. cloth at $12\frac{1}{2}¢$ a yard?

Solution: $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$; $\frac{1}{2}$ of the integer in the multiplicand plus $\frac{1}{2}$ of the integer in the multiplier is equal to the integer in either the multiplicand or multiplier. Hence, add 1 to

$12\frac{1}{2}$
$12\frac{1}{2}$
$156\frac{1}{4}$

the integer in the multiplicand and multiply by the multiplier. Thus, $12 \times 13 = 156$. Therefore $12\frac{1}{2} \times 12\frac{1}{2} = 156\frac{1}{4}$; and $12\frac{1}{2}$ yd. cloth at $12\frac{1}{2}¢$ a yard, will cost $156\frac{1}{4}¢ = \$1.56\frac{1}{4}$.

ILLUSTRATIVE EXAMPLE (2): To find the cost of 25 pr. of stockings at 25¢ a pair.

Solution: Find the product, as explained in Ex. (1), considering the 5's as fractions, and the numbers to the left of the 5's as the integers. $5 \times 5 = 25$; $2 \times 3 = 6$. 25 pairs of stockings at 25¢ a pair will therefore cost \$6.25.

$$\begin{array}{r} 25 \\ 25 \\ \hline 25 \\ 6 \\ \hline 625 \end{array}$$

ILLUSTRATIVE EXAMPLE (3): Find the cost of 6.5 yards of velvet at \$6.50 a yard.

Solution: The principle involved in these three examples is practically the same. $5 \times 5 = 25$. $6 \times 7 = 42$. Therefore, 6.5 yards of velvet at \$6.50 a yard will cost \$42.25.

$$\begin{array}{r} 6.5 \\ 6.5 \\ \hline 42.25 \end{array}$$

State a short way of squaring numbers ending in $\frac{1}{2}$ or 5.

ORAL EXERCISE

Multiply:

a. $3\frac{1}{2}$ by $3\frac{1}{2}$
b. $5\frac{1}{2}$ by $5\frac{1}{2}$
c. $8\frac{1}{2}$ by $8\frac{1}{2}$
d. $9\frac{1}{2}$ by $9\frac{1}{2}$
e. $12\frac{1}{2}$ by 12.5
f. 3.5 by 3.5
g. 9.5 by 9.5
h. 10.5 by 10.5

WRITTEN EXERCISE

Find the total cost of:

a. 15 pieces braid @ 15¢
b. 35 pieces chiffon @ 35¢
c. 45 pairs gloves @ 45¢
d. 65 waists @ 65¢
e. 25 doz. buttons @ \$2.50
f. $4\frac{1}{2}$ doz. prs. gloves @ \$4.50
g. 250 pairs of shoes @ \$2.50
h. 85 pieces braid @ 85¢

(2) Multiplication of two numbers ending in $\frac{1}{2}$ or .5.

ILLUSTRATIVE EXAMPLE (1): Multiply $8\frac{1}{2}$ by $3\frac{1}{2}$.

Solution: $\frac{1}{2}$ of the integer in the multiplicand plus $\frac{1}{2}$ of the integer in the multiplier is equal to $\frac{1}{2}$ of $(8+3)$ or $5\frac{1}{2}$; $5\frac{1}{2}$ added to $\frac{1}{2}$ of $\frac{1}{2}$ equals $5\frac{3}{4}$. Write $\frac{3}{4}$ as shown in the illustration and carry 5. $(3\times8)+5=29$. Therefore, $8\frac{1}{2}\times3\frac{1}{2}=29\frac{3}{4}$.

$$\begin{array}{r} 8\frac{1}{2} \\ 3\frac{1}{2} \\ \hline 29\frac{3}{4} \end{array}$$

ILLUSTRATIVE EXAMPLE (2): Multiply $6\frac{1}{2}$ by $8\frac{1}{2}$.

Solution: $\frac{1}{2}$ of $(6+8)=7$ with no remainder. $\frac{1}{2}$ of $\frac{1}{2}=\frac{1}{4}$. Write the product of 6 and 8, and carry 7; $6\times8+7=55$. $55+\frac{1}{4}=55\frac{1}{4}$; $6\frac{1}{2}\times8\frac{1}{2}=55\frac{1}{4}$.

$$\begin{array}{r} 6\frac{1}{2} \\ 8\frac{1}{2} \\ \hline 55\frac{1}{4} \end{array}$$

In finding $\frac{1}{2}$ of an even number there is no remainder; in finding $\frac{1}{2}$ of an odd number there is always a remainder of 1; therefore, to find the product of any two numbers ending in $\frac{1}{2}$, or .5:

(a) When the sum of the integers is an even number, add $\frac{1}{2}$ of their sum to their product, and annex $\frac{1}{4}$ (.25 or 25) to the result, as the case may require.

(b) When the sum of the integers is an odd number, add $\frac{1}{2}$ of their sum (disregarding the fraction) to their product and annex $\frac{3}{4}$ (.75, or 75), to the result, as the case may require.

Multiply: *a.* $2\frac{1}{2}\times3\frac{1}{2}$ *c.* $16\frac{1}{2}\times10\frac{1}{2}$ *e.* $11\frac{1}{2}\times10\frac{1}{2}$ *g.* 2.5×1.5
b. $5\frac{1}{2}\times6\frac{1}{2}$ *d.* $12\frac{1}{2}\times9\frac{1}{2}$ *f.* $20\frac{1}{2}\times8\frac{1}{2}$ *h.* 3.5×4.5

Find cost of:

i. 25.5 yds. lace @ \$56.5 per yd.
j. $17\frac{1}{2}$ " " @ \$.$12\frac{1}{2}$ " "
k. 65 " " @ \$.85 " "
l. 15.5 " " @ \$3.50 " "
m. 8.5 doz. h'dk'fs @ \$2.25 per doz.
n. $4\frac{1}{2}$ " " @ \$.25 each

WRITTEN EXERCISE

Page from a Catalogue

Find the total cost of:

65 pairs white gloves @ $1.25 (No. 89)

35 pairs cape skin gloves @ $1.75 (No. 90)

15 pairs button glacé gloves @ $2.25 (No. 91)

25 pairs doe skin gloves @ $1.55 (No. 92)

5 pairs dog skin gloves @ $1.75 (No. 93)

15 pairs emb. stockings @ $2.55 (No. 94)

5 pairs silk stockings @ $2.75 (No. 95)

5 Umbrellas @ $4.50 (No. 96)

5 Umbrellas @ $2.95 (No. 97)

5 Umbrellas @ $5.95 (No. 98)

5 Umbrellas @ $6.95 (No. 99)

5 Tam O'Shanter caps @ $1.55 (No. 100)

15 Scarfs @ $1.95

15 Sweaters @ $6.95

Find the total cost of articles below:

5 Fern stands @ $2.45	15 book cases @ $35
5 Sofas @ $75	15 desks @ $25
25 Sofa tables @ $45	25 desk chairs @ $25

Furniture Sale

4. Short methods of finding the cost of goods at popular prices: 17¢, 79¢, 37½¢, 49¢, 19¢.

ILLUSTRATIVE EXAMPLE (1): What is the cost of $\frac{2}{3}$ yd. ribbon @ 17¢ a yard?

Solution: The nearest number to 17, that is divisible by 3, is 18;

$$\overset{6}{\cancel{18}}\text{¢} \times \frac{2}{3} = 12\text{¢}.$$ Store answer—what a customer would have to pay.

$\frac{2}{3} \times 1 = \frac{2}{3}$. $12 - \frac{2}{3} = 11\frac{1}{3}$¢ accurate answer.

ILLUSTRATIVE EXAMPLE (2): Find the cost of $\frac{5}{8}$ yd. at 79¢.

Solution: The nearest number to 79 that is divisible by 8, is 80:

$\frac{5}{\cancel{8}}$ of $\overset{10}{\cancel{80}}$¢ = $.50 store answer.

$\frac{5}{8} \times 1 = \frac{5}{8}$. $50 - \frac{5}{8} = \$49\frac{3}{8}$ accurate answer.

5. Stores usually get the benefit of the fraction.

ILLUSTRATIVE EXAMPLE (1): Find the cost of $\frac{1}{2}$ yd. muslin at $37\frac{1}{2}¢$ a yard.

Solution: The nearest number to $37\frac{1}{2}$ that is divisible by 2 is 38. $\frac{1}{2}$ of $38 = 19$. $19¢ =$ store answer. $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$. $19 - \frac{1}{4} = 18\frac{3}{4}¢$, accurate answer.

$$\overset{19}{\cancel{38}} \times \frac{1}{\cancel{2}} = 19¢ \text{ (store answer).}$$

ILLUSTRATIVE EXAMPLE (2): Find the cost of $\frac{5}{8}$ yd. at $49¢$.

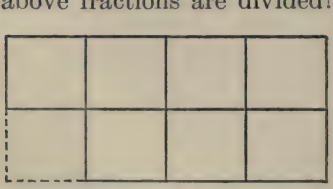
Solution: The nearest number to 49 that is divisible by 8 is 48. $\frac{5}{8}$ of $48 = 30$; $\frac{5}{8}$ of $1 = \frac{5}{8}$; $30\frac{5}{8}$, or $31¢$ (store answer).

ILLUSTRATIVE EXAMPLE (3): Find the cost of $\frac{3}{4}$ yd. at $19¢$; $\frac{3}{4}$ of $20 = 15¢$; $\frac{3}{4}$ of $1¢ = \frac{3}{4}¢$. $15 - \frac{3}{4} = 14\frac{1}{4}¢$ (accurate answer). $15¢ =$ store answer.

59. Division of fractions.

(1) $8 \div 2 = ?$ 8 ninths $(\frac{8}{9}) \div 2 = ?$ $\frac{6}{7} \div 3 = ?$ $\frac{12}{13} \div 6 = ?$ $\frac{15}{16} \div 5 = ?$

To divide a fraction by an integer. What part of the above fractions are divided? Dividing the numerator of a fraction does what to the fraction itself? Show by a diagram that $\frac{4}{5} \div 2 = \frac{2}{5}$.



Comparison of $^1/_2$, $^1/_4$, $^1/_8$

(2) If you divide $\frac{1}{2}$ of this rectangular form into four equal parts, each is what part of the rectangle? How much is $\frac{1}{2} \div 4$? How does the fraction $\frac{1}{8}$ compare in value with the fraction $\frac{1}{2}$?

(3) If you divide $\frac{1}{8}$ of the rectangle into two equal parts, each is what part of the rectangle? How much is $\frac{1}{8} \div 2$? What have you done to the denominator? How does the fraction $\frac{1}{16}$ compare in value with the fraction $\frac{1}{8}$?

(4) If you divide $\frac{3}{4}$ of the rectangle into two equal parts, how many eighths of the rectangle do you have? How much is $\frac{3}{4} \div 2$? What have you done to the denominator? How does the fraction $\frac{3}{8}$ compare in value with the fraction $\frac{3}{4}$?

(5) Multiplying the denominator of a fraction by the integer does what to the fraction itself?

(6) State two ways of dividing a fraction by an integer.

(7) $\frac{6}{7} \div 2 = ?$ $\frac{1}{2}$ of $\frac{6}{7} = ?$ Dividing by 2, is finding what part of a number?

(8) Show that dividing by 3, 4, 5, etc., is multiplying by $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc.

(9) Since $\frac{6}{7} \div 2 = \frac{1}{2}$ of $\frac{6}{7}$, therefore $\frac{1}{7} \div 2 = \frac{1}{2}$ of $\frac{1}{7}$ or $\frac{1}{7} \times \frac{1}{2}$; $\frac{1}{7} \times \frac{1}{2} = ?$

(10) How much is $\frac{5}{6} \div 3$? $\frac{3}{8} \div 5$? $2\frac{1}{2}$ $(\frac{5}{2}) \div 3$? $3\frac{1}{4}$ $(\frac{13}{4}) \div 5$?

ORAL EXERCISE

(1) Find the quotient:

a. $\frac{5}{7} \div 3$	*d.* $\frac{6}{9} \div 6$	*g.* $\frac{18}{5} \div 6$	*j.* $\frac{1}{9} \div 10$
b. $\frac{8}{15} \div 5$	*e.* $\frac{7}{12} \div 8$	*h.* $\frac{32}{8} \div 4$	*k.* $\frac{5}{12} \div 6$
c. $\frac{12}{13} \div 4$	*f.* $\frac{9}{12} \div 3$	*i.* $\frac{5}{7} \div 12$	*l.* $\frac{18}{25} \div 9$

ILLUSTRATIVE EXAMPLE (1): Divide $56\frac{8}{9}$ by 8.

Solution: Divide the integers. $56 \div 8 = 7$. Then divide the fraction by 8 and the result is $\frac{1}{9}$; therefore, $56\frac{8}{9} \div 8 = 7\frac{1}{9}$.

$$8\overline{)56\frac{8}{9}} \quad 7\frac{1}{9}$$

ILLUSTRATIVE EXAMPLE (2): Divide $42\frac{3}{4}$ by 9.

Solution: Divide 42 by 9: result, 4 with a remainder of 6. Join the remainder 6 with the fraction $\frac{3}{4}$, making $6\frac{3}{4}$. $6\frac{3}{4}$ reduced to an improper fraction equals $\frac{27}{4}$. $\frac{27}{4} \div 9 = \frac{3}{4}$. Therefore, $42\frac{3}{4} \div 9 = 4\frac{3}{4}$.

$$9\overline{)42\frac{3}{4}} \quad 4\frac{3}{4}$$

(2) Divide:

a. $15\frac{1}{3}$ by 6	*d.* $24\frac{5}{6}$ by 5	*g.* $7\frac{3}{4}$ by 6
b. $17\frac{2}{3}$ by 8	*e.* $32\frac{5}{8}$ by 6	*h.* $28\frac{3}{4}$ by 7
c. $21\frac{1}{2}$ by 3	*f.* $20\frac{5}{6}$ by 12	*i.* $56\frac{1}{2}$ by 10

(3) How many fifths in one? $1 \div \frac{1}{5} = ?$ $1 \div \frac{1}{10} = ?$

(4) $2 \div \frac{1}{5} = ?$ $5 \div \frac{1}{5} = ?$ $8 \div \frac{1}{5} = ?$ $36 \div \frac{1}{10} = ?$ $360 \div \frac{1}{10} = ?$

1. To divide an integer by a unit fraction, multiply the integer by the denominator of the fraction.

(1) What is the value of $36 \div \frac{1}{2}$? Dividing by $\frac{1}{2}$ is the same as multiplying by what number?

$48 \div \frac{1}{6} = ?$ $1 \div \frac{1}{2} = ?$

(2) $1 \div \frac{3}{4} = \frac{4}{3}$ $(1\frac{1}{3})$. Therefore, $2 \div \frac{3}{4} = \frac{8}{3}$. Why?

(3) $1 \div \frac{3}{4} = \frac{4}{3}$; $\frac{1}{2} \div \frac{3}{4} = \frac{2}{3}$. Explain.

(4) Compare:

$\frac{2}{3} \div \frac{1}{3} = 2$ $\qquad$ $\frac{3}{4} \div \frac{1}{4} = 3$

$\frac{2}{3} \times \frac{3}{1} = 2$ $\qquad$ $\frac{3}{4} \times \frac{4}{1} = 3$

2. To divide a fraction by a fraction, multiply it by the reciprocal of the divisor.

(1) The reciprocal of a fraction is 1 divided by that fraction; thus, the reciprocal of $\frac{3}{4}$ is $1 \div \frac{3}{4}$ or $\frac{4}{3}$; therefore, the reciprocal of a fraction is the fraction inverted.

(2) Integers and mixed numbers should be reduced to improper fractions, and cancellation used whenever possible, before applying the rule.

WRITTEN EXERCISE

(1) Divide:

a. $\frac{5}{9}$ by $\frac{6}{7}$ $\qquad$ *d.* $5\frac{1}{3}$ by $2\frac{2}{5}$ $\qquad$ *g.* $5\frac{5}{6}$ by $\frac{5}{9}$

b. $7\frac{1}{2}$ by $8\frac{1}{3}$ $\qquad$ *e.* $6\frac{1}{8}$ by $1\frac{3}{4}$ $\qquad$ *h.* $12\frac{3}{4}$ by $2\frac{1}{3}$

c. $7\frac{1}{5}$ by $3\frac{3}{10}$ $\qquad$ *f.* 150 by $6\frac{3}{5}$ $\qquad$ *i.* $7\frac{2}{3}$ by $7\frac{1}{2}$

ILLUSTRATIVE EXAMPLE (1): Divide 365 by $42\frac{3}{4}$.

Solution: Multiplying both dividend and divisor by the same number does not affect the quotient. Change

$$\begin{array}{rr} 42\frac{3}{4}) & 365 \\ \times\ 4) & \times\ 4 \\ \hline 171\ ) & 1460\ (8\frac{92}{171} \\ & 1368 \\ & \hline & 92 \end{array}$$

ILLUSTRATIVE EXAMPLE (2): Divide $204\frac{4}{5}$ by $23\frac{5}{8}$.

Solution: Change both dividend and divisor to like fractions; multiply dividend and divisor by lowest common denominator of both fractions determined by inspection. This will change numbers to integers and not alter their quotient. Changing both dividend and divisor to fortieths, the result is $\frac{945}{40}$ and $\frac{8192}{40}$.

$$
\begin{array}{r}
23\frac{5}{8})204\frac{4}{5} \\
\times 40\ ) \times 40 \\
\hline
945)8192(8\frac{5}{9}\frac{3}{4}\frac{2}{5} \\
7560 \\
\hline
632
\end{array}
$$

(2) Divide:

a. $150\frac{1}{3}$ by $16\frac{2}{3}$	*d*. $1032\frac{2}{5}$ by $32\frac{1}{3}$	*g*. $1065\frac{5}{6}$ by $87\frac{1}{2}$
b. $365\frac{2}{3}$ by $20\frac{5}{6}$	*e*. $670\frac{5}{8}$ by $19\frac{1}{4}$	*h*. $2000\frac{4}{5}$ by $12\frac{5}{16}$
c. $1728\frac{5}{8}$ by $25\frac{3}{4}$	*f*. $1008\frac{8}{9}$ by $37\frac{1}{2}$	*i*. $4350\frac{5}{12}$ by $156\frac{8}{9}$

Problems

(1) How many towels can be made from 58 yd. of crash, putting $\frac{2}{3}$ of a yd. into each towel?

(2) It takes $1\frac{1}{8}$ yd. of ribbon to make one bow. How many bows can be made from $47\frac{1}{4}$ yd.?

(3) It takes $2\frac{1}{4}$ yd. of ribbon to make one rosette. How many rosettes can be made from $94\frac{1}{2}$ yd.?

(4) How many waists containing $2\frac{3}{4}$ yd. can be made from $42\frac{5}{8}$ yd. chiffon? How large a piece of chiffon will remain?

(5) How many skirts each containing $4\frac{1}{2}$ yd. can be made from a piece of cloth containing $64\frac{1}{8}$ yd.? How large a piece of cloth will remain?

(6) Waists valued at: \$6.00, \$3.50, \$4.75, \$3.98, \$2.50, \$3.75, \$5.00, \$7.50, \$10.00, \$12.50, were marked down to one-fourth of their value. What was the selling price of each, after the reduction was made?

(7) Suits valued at \$75.00, \$60.98, \$47.50, \$100.00, \$127.50, \$115.00, \$122.50, \$125.00, \$133.00, \$137.50, \$140.00, \$155,00 \$175.00, are marked down to one-third of their value. What is their present selling price?

(8) It takes $1\frac{5}{6}$ dozen buttons for one suit. How many suits could be trimmed with 22 dozen buttons?

(9) If $3\frac{3}{4}$ yards of lace are required for one centre-piece, how many yards are required for 12 centre-pieces? For 6? For 9? For 11? For 24?

(10) What is the selling price of each of the following goods after a $\frac{3}{5}$ reduction has been made in their price?

a. Ready-made dresses,	\$41.50,	\$32.00,	\$22.50
b. Silk poplin "	\$65.00,	\$25.50,	\$20.75
c. Taffeta "	\$45.50,	\$28.50,	\$24.75
d. Crêpe de chine "	\$35.00,	\$57.75,	\$49.75
e. Sport silk suits,	\$58.50,	\$62.50,	\$74.75
f. Gabardine coats,	\$85.00,	\$98.00,	\$125.00
g. Imported taffeta dresses,	\$51.50,	\$75.00,	\$69.75

(11) How many dress patterns, each containing $11\frac{1}{4}$ yards, can be made from a piece of cloth containing $35\frac{1}{5}$ yards? What will the remainder be worth at \$3.37$\frac{1}{2}$ a yard?

60. Solution of problems.

1. The essential steps in the solution of a problem are:

(1) Reading the problem.

Object *a*. To find out what is given.
b. To find out what is required.

(2) Planning the solution.

a. To determine from what is given how to find out what is required.

(3) Making the calculation.

Object *a*. Outlining the process of computation.
b. Performing it.

(4) Testing the work or
Checking the results.

2. Problems should be thoroughly understood before an attempt is made to solve them. When the relation of what is given to what is required is known, the plan of the solution should be briefly stated and then performed as briefly as possible. The accuracy of the work should always be tested by some manner of checking. A good method of checking, in order to prevent impossible results, is to estimate the answer in advance; thus: 56 overcoats at \$49 is equal approximately to 56×50.

ILLUSTRATIVE EXAMPLE: If 216 yd. silk are used in making 48 waists, how many yards are needed to make 36 waists?

Solution

1. Reading problem.
   a. Given, quantity used in making 48 waists.
   b. Required, quantity needed for making 36 waists.

2. Planning the solution.
   a. One waist requires $\frac{216}{48}$ yards.
   36 waists will require $\frac{36 \times 216}{48}$ yd.

3. Making the calculation.
   a. $\frac{36 \times 216}{48}$ = amount of silk required for 36 waists.
   b. $\frac{\overset{3}{\cancel{36}} \times \overset{54}{\cancel{216}}}{\underset{4}{\cancel{48}}}$ = 162 yd. silk, amount required for 36 waists

4. Testing the work or checking the results.
   $\frac{216}{48} = 4\frac{1}{2}$ yd.; $\frac{162}{36} = 4\frac{1}{2}$ yd. or
   $36 = \frac{3}{4}$ of 48. Therefore, the work
   $162 = \frac{3}{4}$ of 216. is probably correct.

Problems

(1) A merchant bought a stock of goods and sold them at $\frac{1}{5}$ above cost. If he received \$250, what was the cost of the goods?

(2) A dealer sold his goods at $\frac{1}{5}$ below cost, and received \$225. What was the cost and what was his loss?

(3) A buyer bought a quantity of dry goods and sold them so as to gain $\frac{1}{6}$ of the cost. The selling price was \$840. What was the gain? What was the cost?

(4) Small wares were bought at the rate of 3 for 2¢ and sold at the rate of 2 for 3¢. How much must be sold to gain \$5.00?

(5) Which is cheaper, to buy 8 yd. linen 27 in. wide at 45¢ a yd., or linen 36 in. wide at 50¢ a yard?

(6) If 8 yd. of material 56 in. wide will make a dress, how many yd. of material 48 in. wide will it take? Of material 54 in. wide?

(7) If $2.00 is the average daily cost for raw material for a family of 5, what is the cost of raw material necessary to supply a family of four grown children and their parents for a year?

61. Review problems.

1. A customer gave a 50-dollar bill in payment for the following goods purchased on a traveler slip:

5 yd. carpet @ $87\frac{1}{2}$¢ a yard. 2 waists @ $4.98 each.
3 yd. flannel @ $33\frac{1}{3}$¢ a yard. 1 gown, $15.00.
5 books @ $1.12\frac{1}{2}$¢ each.

How much change should she receive?

2. A hotel proprietor bought 1376 yd. muslin @ $6\frac{3}{4}$¢ a yard; $42\frac{1}{2}$ yd. matting @ 80¢ a yard; $148\frac{1}{2}$ yd. sheeting @ 19¢ a yard; 200 yd. crash @ $12\frac{1}{2}$¢ a yard; 1250 yd. table linen @ $33\frac{1}{3}$¢ a yard; 32 yd. cretonne @ $37\frac{1}{2}$¢ a yard. What was the amount of his purchases and what was his bill if he received a discount of $\frac{1}{10}$ of the total amount?

3. From a piece of silk containing $66\frac{2}{3}$ yd., $12\frac{1}{2}$ yd. were sold to one customer, and $5\frac{3}{4}$ yd. to another. How many yards were left in the piece?

4. If $13\frac{1}{3}$ yd. ribbon are advertised for sale for 80¢, what will a customer pay for $8\frac{3}{4}$ yd.?

5. What is the total amount on the sales slip of a sales girl, if a customer buys $1\frac{3}{4}$ yd. embroidery @ 48¢ a yard, $2\frac{5}{8}$ yd. @ 56¢ a yard, $4\frac{1}{3}$ yd. @ 27¢ a yard?

6. What is the total cost of a suit if it takes $5\frac{1}{2}$ yd. cloth at $1.25 a yd., 18 skirt buttons at 30¢ a dozen, $2\frac{1}{2}$ yd. lining at $1.00 a yard, 2 spools silk at 10¢ a spool, 4 coat buttons at $.60 a dozen, $1\frac{7}{8}$ yd. velvet at $2.00 a yard, and $30.00 for making the suit?

7. How many dress patterns, each containing $10\frac{1}{2}$ yd., can be

cut from a piece of cloth containing $57\frac{3}{4}$ yd.? What is the remnant worth at $\$.87\frac{1}{2}$ a yard?

8. A tailor used $2\frac{7}{8}$ yd. cloth for a coat, $\frac{5}{6}$ of a yard for a vest, $2\frac{3}{4}$ yd. for a pair of trousers. If he cut the goods from a piece of cloth containing $12\frac{5}{8}$ yd., how many yards would be left?

9. What is the cost of a hat, if a milliner allows $1\frac{7}{8}$ yd. ribbon worth $37\frac{1}{2}¢$ a yard, \$1.50 for a pair of wings and charges \$0.75 for trimming?

10. From a piece of goods containing 60 yards, seven dress lengths of $6\frac{3}{4}$ yd. each, were sold. How many yards remained? The cloth was \$0.75 a yard. What was received for the cloth?

11. The following amounts were cut from a piece of cloth containing $43\frac{7}{8}$ yd.: $2\frac{3}{4}$ yd., $7\frac{1}{8}$ yd., $\frac{1}{2}$ yd., $12\frac{5}{6}$ yd., $2\frac{3}{16}$ yd. How much cloth is left?

12. How many dozen buttons are left from 1 gross, after the following sales: $\frac{1}{3}$ doz., $3\frac{3}{4}$ doz., $1\frac{1}{6}$ doz., $2\frac{1}{2}$ doz., $1\frac{3}{4}$ doz. What was the total amount of the sales?

13. The sales in the suit department of a large store amounted to \$6400 in one month; in the waist department, $2\frac{3}{4}$ as much; in the trimming department, $\frac{3}{8}$ as much; in the furniture department, $3\frac{1}{2}$ times as much; in the dress goods department, $1\frac{5}{8}$ as much; in the stationery department, $\frac{7}{8}$ as much. What was the total amount of the sales?

14. If one waist takes $2\frac{5}{16}$ yd. chiffon, how many yards are required for 48 waists?

15. A girl spends $\frac{1}{2}$ of her weekly salary for board, $\frac{1}{4}$ for clothing, $\frac{1}{12}$ for amusements, and puts the rest in the bank. If her salary is \$12 a week, how much money does she save? If it is \$9 a week? \$18? \$15? \$20?

16. What is the total cost of: 15 yd. cloth @ $\$37\frac{1}{2}$ a yard?
$25\frac{2}{3}$ yd. cloth @ $\$2\frac{1}{4}$ a yard?
$\frac{3}{4}$ dozen buttons @ \$2.00 a dozen?

17. What is the cost of: 16 buttons @ 75¢ a dozen?
9 buttons @ $1.08 a dozen?
8 buttons @ $0.24 a dozen?
15 buttons @ $1.95 a dozen?

18. Mrs. James bought the following goods:
5 yd. cloth @ $1.75 a yard. 3 spools cotton @ $.05 a spool.
$2\frac{1}{2}$ yd. silk @ $0.98 a yard. $1\frac{1}{2}$ yd. lace @ $1.98 a yard.
2 spools silk @ $.10 a spool. $1\frac{5}{6}$ yd. velvet @ $2.21 a yard.
What was the total amount of her bill?

19. Express in dollars and cents the sum of: $\$\frac{3}{8}$; $\$\frac{1}{2}$; $\$\frac{5}{8}$; $\$\frac{2}{3}$; $\$\frac{3}{4}$?

20. What will you pay for: $1\frac{3}{4}$ yd. embroidery @ $.48; $2\frac{5}{8}$ yd. @ $0.56; $4\frac{1}{3}$ yd. @ 27¢?

21. How many breadths of carpet $\frac{3}{4}$ of a yard wide are needed for a room 15 ft. wide?

22. Into how many pieces $\frac{5}{6}$ of a yard long can 15 yards of ribbon be divided?

23. If 27 inches of ribbon are required for one rosette, how many rosettes can be made from 10 yards? From 15 yards?

24. If 3 yd. cloth cost $\$0.37\frac{1}{2}$, how much change would be received from a $5.00 bill if 15 yd. are bought?

25. What is the total cost of the following purchases: $\frac{3}{4}$ yd. velvet @ $3.00 a yard; $3\frac{2}{3}$ yd. velvet @ $1.59 a yard; $1\frac{5}{8}$ yd. velvet @ $4.00 a yard; $11\frac{5}{6}$ yd. velvet @ $\$2.87\frac{1}{2}$ a yard?

26. How many skirts, each containing $3\frac{7}{12}$ yd., can be made from $28\frac{2}{3}$ yd.?

27. Find the total cost of the following purchases:

a. 1 doz. shields @ $.33 each.
$\frac{1}{2}$ doz. boxes pins @ $0.15 each.
$\frac{3}{4}$ doz. pkg. needles @ $.20 each.
$\frac{1}{2}$ doz. pc. tape @ $12\frac{1}{2}$¢ each.
3 tape measures @ 25¢ each.

b. 24 Silk net scarfs @ $4.95 each.
18 Organdie sets @ 95¢ each.
80 Lace vestees @ $1.50 each.

45 Satin collars @ \$1.25 each.
24 Broadcloth collars @ 50¢ each.
36 Linen collars @ \$.50¢ each.
48 Hand emb. collars @ 50¢ each.
50 Broadcloth collars @ 85¢ each.
75 Satin collars @ \$1.45 each.
75 Georgette crêpe sets @ \$1.45 each.

c. $15\frac{1}{2}$ yd. linen @ \$0.98.
$18\frac{3}{4}$ yd. linen @ \$0.62$\frac{1}{2}$.
56 yd. linen @ \$0.87$\frac{1}{2}$.
125 yd. linen @ \$1.12$\frac{1}{2}$.

28. How many yards linen are there in the following remnants: $1\frac{7}{8}$ yd., $2\frac{3}{4}$ yd., $1\frac{5}{8}$ yd., $2\frac{2}{3}$ yd., $3\frac{5}{12}$ yd., $2\frac{5}{16}$ yd.?

29. How many bags requiring $1\frac{7}{8}$ yd. each can be made from 15 yd. of silk?

62. Fractional forms. Division of fractions is often expressed in the form of a fraction, thus:

a. $\dfrac{\frac{5}{6}}{8}$ *b.* $\dfrac{5\frac{1}{10}}{\frac{7}{12}}$ Such fractions are usually called complex fractions, and are solved like other examples in division.

Thus, *a.* $\dfrac{\frac{5}{6}}{8} = \frac{5}{6} \div 8$;

b. $\dfrac{5\frac{1}{10}}{\frac{7}{12}} = 5\frac{1}{10} \div \frac{7}{12}$

EXERCISE

(1) Reduce to simple fractions:

a. $\dfrac{5\frac{1}{2}}{\frac{6}{7}}$ *b.* $\dfrac{9}{2\frac{1}{5}}$ *c.* $\dfrac{5\frac{3}{4}}{8\frac{2}{3}}$ *d.* $\dfrac{6\frac{4}{5}}{3\frac{7}{12}}$ *e.* $\dfrac{15\frac{3}{4}}{12\frac{1}{2}}$

(2) What part of \$2 is \$1? What part of 6 dozen is 1 dozen?

(3) What part of seven-eighths ($\frac{7}{8}$) is one-eighth ($\frac{1}{8}$)?

(4) What part of $\frac{7}{8}$ is $\frac{3}{8}$?

Solution: $\frac{7}{8}$ and $\frac{3}{8}$ are similar fractions and are compared in the same way as integers. 1 is $\frac{1}{7}$ of 7; therefore 3 is $\frac{3}{7}$ of 7; therefore $\frac{3}{8}$ is $\frac{3}{7}$ of $\frac{7}{8}$; for $\frac{3}{8}$ is $\frac{\frac{3}{8}}{\frac{7}{8}}$ of $\frac{7}{8} \cdot \frac{\frac{3}{8}}{\frac{7}{8}} = \frac{3}{8} \times \frac{8}{7} = \frac{3}{7}$

(5) $\frac{5}{6}$ is what part of $2\frac{5}{6}$ $(\frac{17}{16})$? Of $4\frac{5}{6}$? Of $5\frac{1}{6}$?
$\frac{1}{5}$ is what part of $\frac{1}{2}$?
Solution: $\frac{1}{5} = \frac{2}{10}$; $\frac{1}{2} = \frac{5}{10}$; $\frac{1}{10}$ is $\frac{1}{5}$ of $\frac{5}{10}$
Hence, $\frac{2}{10}$ $(\frac{1}{5})$ is $\frac{2}{5}$ of $\frac{5}{10}$ $(\frac{1}{2})$

Or $\frac{1}{5}$ is $\frac{\frac{1}{5}}{\frac{1}{2}}$ of $\frac{1}{2} \cdot \frac{1}{5} \times \frac{2}{1} = \frac{2}{5}$

1. To find what fractional part one number is of another, take the number that denotes the part for the numerator of the fraction, and the number with which it is compared for the denominator of the fraction and simplify.

(1) What part of 100 is:

$6\frac{1}{4}$, $12\frac{1}{2}$, $16\frac{2}{3}$, $8\frac{1}{3}$, $33\frac{1}{3}$, $37\frac{1}{2}$, $62\frac{1}{2}$, $87\frac{1}{2}$, $6\frac{1}{4}$, $6\frac{2}{3}$, $83\frac{1}{3}$, $56\frac{1}{4}$?

(2) What part of \$1 is:

a.	*b.*	*c.*	*d.*	*e.*
25¢	40¢	$83\frac{1}{3}$¢	$87\frac{1}{2}$¢	$8\frac{1}{3}$¢
50¢	60¢	$12\frac{1}{2}$¢	10¢	$6\frac{2}{3}$¢
75¢	80¢	$37\frac{1}{2}$¢	70¢	$6\frac{1}{4}$¢
20¢	$16\frac{2}{3}$¢	$62\frac{1}{2}$¢	90¢	$93\frac{3}{4}$¢

(3) What part of \$10 is:

a.	*b.*	*c.*
$\$2\frac{1}{2}$	$\$6.66\frac{2}{3}$	\$1.25
$\$3.33\frac{1}{3}$	\$6.25	\$7.50

(4) What part of 1000 is:

a.	*b.*	*c.*	*d.*
125	$133\frac{1}{3}$	375	250
625	$666\frac{2}{3}$	875	750

(5) For what fractional part of the original price were the following sold?

MARK DOWN SALE

Original Price	Sale Price	Fractional Reduction
Was \$ 39.75	Now \$26.50	?
" \$ 85.00	" \$56.15	?
" \$100.00	" \$66.67	?
" \$ 29.75	" \$23.80	?
" \$ $0.37\frac{1}{2}$	" \$ 0.25	?
" \$ 7.50	" \$ 3.75	?
" \$ 3.90	" \$ 2.60	?
" \$ 1.50	" \$ 0.90	?
" \$ 1.75	" \$ 1.25	?
" \$ 20.00	" \$17.50	?

(6) A merchant sold goods amounting to \$1026.83 on Monday, which was $\frac{7}{8}$ of the amount received for the goods on Tuesday. What was the amount received on Tuesday?

(7) A merchant marked goods for $\frac{2}{3}$ more than they cost, then reduced the price $\frac{1}{10}$. He gained \$368.00. Find the cost of the goods, the price asked, and the selling price.

(8) A shoe dealer reduced the marked price of his shoes $\frac{1}{8}$ and then sold them so as to gain $\frac{1}{5}$ of the first cost. If he gained \$1.75 on each pair of shoes, what was the original cost and the marked price before any reduction was made?

(9) A merchant who had 450 yards of crêpe sold $\frac{1}{3}$ of it to a wholesale dealer, and $\frac{3}{4}$ of what was left to a customer. How many yards had he left?

(10) Copy and complete the following sales slips and check results:

		DOLLARS	CENTS
24	Towels @ $4.50 a dozen	.....	.....
18	Towels @ $7.50 a dozen	.....	.....
16	Towels @ $6.00 a dozen	.....	.....
9	Towels @ $3.00 a dozen	.....	.....
8	Towels @ $2.00 a dozen	.....	.....
	TOTAL HERE		

Every sales check must be dated by sales person
Two totals must always be written

DATE		DOLLARS	CENTS
	TOTAL HERE		

		DOLLARS	CENTS
$5\frac{1}{2}$ yd.	Silk @ $1.64 a yd.	.....	.....
$3\frac{3}{4}$ yd.	Silk @ $1.96 a yd.	.....	.....
$5\frac{7}{8}$ yd.	Silk @ $2.72 a yd.	.....	.....
$4\frac{9}{16}$ yd.	Silk @ $4.80 a yd.	.....	.....
.....	.....	.....	.....
	TOTAL HERE		

Every sales check must be dated by sales person
Two totals must always be written

DATE		DOLLARS	CENTS
	TOTAL HERE		

		DOLLARS	CENTS
$1\frac{1}{4}$ doz.	Napkins @ $9.28 per dozen	.....	.....
$3\frac{3}{4}$ doz.	Napkins @ $13.60 per dozen	.....	.....
$2\frac{1}{2}$ doz.	Napkins @ $7.50 per dozen	.....	.....
$4\frac{2}{3}$ doz.	Napkins @ $9.00 per dozen	.....	.....
.....	.....	.....	.....
	TOTAL HERE		

Every sales check must be dated by sales person
Two totals must always be written

DATE		DOLLARS	CENTS
	TOTAL HERE		

(11) Copy and complete the following sales slips and check results:

		DOLLARS	CENTS
2	Petticoats @ $3.98	.....	.....
3	Silk skirts @ $7.95	.....	.....
2	Negligees @ $7.50	.....	.....
2	Crêpe de Chine Gowns @ $14.50	.....	.....
.....	.....	.....	.....
	TOTAL HERE		

Every sales check must be dated by sales person
Two totals must always be written

DATE		DOLLARS	CENTS
	TOTAL HERE		

		DOLLARS	CENTS
16 pr.	Blankets @ $3.50 a pair	.....	.....
14 "	Blankets @ $3.85 a pair	.....	.....
18 "	Blankets @ $6.85 a pair	.....	.....
50 "	Blankets @ $3.75 a pair	.....	.....
.....	.....	.....	.....
	TOTAL HERE		

Every sales check must be dated by sales person
Two totals must always be written

DATE		DOLLARS	CENTS
	TOTAL HERE		

		DOLLARS	CENTS
5 pr.	Stockings @ $ .35 a pair	.....	.....
7 pr.	Stockings @ $1.50 a pair	.....	.....
3 pr.	Stockings @ $1.75 a pair	.....	.....
12 pr.	Stockings @ $.25 a pair	.....	.....
.....	.....	.....	.....
	TOTAL HERE		

Every sales check must be dated by sales person
Two totals must always be written

DATE		DOLLARS	CENTS
	TOTAL HERE		

63. Decimal and fractional equivalents.

EXERCISE

(1) Is there any difference in value between $\frac{3}{10}$ and .3? Between $\frac{3}{100}$ and .03?

(2) What is the denominator of the decimal .8 expressed as a common fraction? Of .25?

(3) What is the numerator of .8 expressed as a common fraction? Of .25? Of .06? Of .025? Of .005? Of .0005?

(4) Express as a common fraction: .5, .56, .028, .0056.

1. A decimal is changed to a common fraction by expressing the denominator; the fraction should then be reduced to its lowest terms.

ILLUSTRATIVE EXAMPLE: Reduce .875 to a common fraction. Solution: .875 means $\frac{875}{1000}$; changing to lowest terms by dividing both terms of the fraction by 125, the result is $\frac{7}{8}$.

$$\frac{875}{1000} = \frac{7}{8}$$

(1) Reduce to a common fraction, or to a mixed number:

a. 0.25	*e.* 6.8	*i.* 45.125	*m.* 250.250
b. 0.375	*f.* 8.48	*j.* 10.4230	*n.* 18.0018
c. 0.75	*g.* 6.08	*k.* 45.085	*o.* 654.004
d. 0.8165	*h.* 5.625	*l.* 6.0012	*p.* 1000.001

2. Common fractions may be written as decimals.

ILLUSTRATIVE EXAMPLE (1): Reduce $\frac{7}{8}$ to a decimal. $\frac{7}{8}$ is an expression of division:

$$8\overline{)7.000}\quad 0.875$$

7 is the dividend and 8 is the divisor. This means, then, find $\frac{1}{8}$ of 7 units. 8 is not contained in 7 units; 7 units equals 7000 thousandths; $\frac{1}{8}$ of 7000 thousandths equals 875 thousandths (.875).

EXERCISE

(1) Reduce to equivalent decimals:

a. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{12}$, $\frac{1}{16}$.

(2) Reduce to equivalent decimals:

a. $\frac{1}{5}$, $\frac{7}{16}$, $\frac{3}{25}$, $\frac{9}{50}$, $\frac{7}{12}$, $\frac{13}{20}$, $\frac{21}{25}$, $\frac{31}{32}$.

b. $7\frac{3}{8}$, $28\frac{5}{6}$, $325\frac{9}{16}$, $4200\frac{7}{8}$, $142\frac{3}{128}$, $\frac{156}{200}$.

Reduce to equivalent decimals:

a. $\frac{2}{3}$, $\frac{1}{5}$, $\frac{7}{16}$, $\frac{3}{25}$, $\frac{9}{50}$, $\frac{7}{12}$, $\frac{5}{12}$, $\frac{13}{20}$.

b. $7\frac{5}{8}$, $28\frac{5}{6}$, $325\frac{9}{16}$, $4200\frac{7}{8}$, $142\frac{5}{128}$, $\frac{156}{200}$.

3. To change a common fraction to a decimal fraction, annex ciphers to the numerator and divide by the denominator. All fractions cannot be reduced to exact decimal equivalents; carrying the result to thousandths is usually sufficient.

CHAPTER IX

ALIQUOT PARTS, BUSINESS FRACTIONS, SHORT METHODS IN MULTIPLICATION

64. Aliquot parts.

The aliquot parts of a number are the parts that are contained in the number an integral number of times. Thus, 50, 25, and $6\frac{1}{4}$ are aliquot parts of 100. The knowledge and use of these parts not only facilitates the work, but makes for accuracy and rapidity in all business computations. Thus, in finding by the usual method, the cost of 24 yd. of cloth at $12\frac{1}{2}$¢, $12\frac{1}{2}$ is instantly recognized as $\frac{1}{8}$ of \$1.00. 8 yd. would cost \$1.00; therefore 24 yd. would cost as many dollars as 8 is contained times in the number. The quantity (24) is divided by 8 and the result obtained is \$3.00.

(1) How many cents in $\$\frac{1}{4}$? In $\$\frac{1}{2}$? In $\$\frac{3}{4}$? In $\$\frac{1}{8}$?

(2) What aliquot part of \$1 is 50¢? 25¢? 75¢? $12\frac{1}{2}$¢?

1. Cost, price, and quantity being the elements of a business transaction, the following use of aliquot parts in the solution of business problems is given:

(1) To find the cost, when the price of 1 article is 50¢, consider the quantity as dollars, and divide by 2.

(2) To find the cost, when the price of 1 article is 25¢, consider the quantity as dollars, and divide by 4.

(3) To find the cost, when the price of 1 article is 75¢, from the quantity considered as dollars, taken $\frac{1}{4}$ of itself.

ORAL EXERCISE

(1) Using method given in Sec. 64 state the cost of:

a. 64 yd. cloth @ 50¢; at 25¢; at 75¢.

b. 128 yd. cloth @ 50¢; at 25¢; at 75¢.

c. 160 neckties @ 50¢; at 25¢; at 75¢.

d. 5600 pairs of stockings @ 50¢; at 25¢; at 75¢.

e. 360 tennis balls @ 50¢; at 25¢; at 75¢.

(2) Compare the cost of 64 yd. cloth at 50¢ a yard with the cost of 50 yd. cloth at 64¢.

(3) Twenty-five cents is $\frac{1}{4}$ of \$1.00. Therefore, $\frac{1}{2}$ of \$.25 is $\$\frac{1}{8}$, or $\$.12\frac{1}{2}$.

$$\begin{array}{r} 4)\$1.00 \\ \hline 2)\$\ .25 \\ \hline \$\ .12\frac{1}{2} \end{array}$$

$$\begin{array}{rrrrrrr} \$.12\frac{1}{2} & \$.25 & \$.37\frac{1}{2} & \$.50 & \$.62\frac{1}{2} & \$.75 & \$.87\frac{1}{2} \\ +.12\frac{1}{2} & +.12\frac{1}{2} & +.12\frac{1}{2} & +.12\frac{1}{2} & +.12\frac{1}{2} & +.12\frac{1}{2} & +.12\frac{1}{2} \\ \hline \$.25 & \$.37\frac{1}{2} & \$.50 & \$.62\frac{1}{2} & \$.75 & \$.87\frac{1}{2} & \$1.00 \end{array}$$

(4) How many eighths of a dollar is $\$.12\frac{1}{2}+\$.12\frac{1}{2}$, or \$.25? \$.50? $\$.87\frac{1}{2}$? \$.75? $\$.62\frac{1}{2}$? \$1.00?

(5) Count by eighths of a dollar to \$1.25, thus: $\$.12\frac{1}{2}=\$\frac{1}{8}$; $\$.25=\$\frac{2}{8}$, or $\$\frac{1}{4}$; $\$.37\frac{1}{2}=\$\frac{3}{8}$, etc.

From the above exercise it will be readily seen that:

a. To find the cost, when the price of 1 article is $\$.12\frac{1}{2}$, consider the quantity as dollars, and divide by 8.

b. To find the cost, when the price of 1 article is $\$.37\frac{1}{2}$, consider the quantity as dollars, and find $\frac{3}{8}$ of it.

c. To find the cost, when the price of 1 article is $\$.62\frac{1}{2}$, consider the quantity as dollars, and find $\frac{5}{8}$ of it.

d. To find the cost, when the price of 1 article is $\$.87\frac{1}{2}$, consider the quantity as dollars, and find $\frac{7}{8}$ of it, or, from the quantity, considered as dollars, take $\frac{1}{8}$ of itself.

ORAL EXERCISE

(1) State the cost of:

a. 96 yd. cloth @ $37\frac{1}{2}$¢; 50¢; 75¢; $62\frac{1}{2}$¢; $87\frac{1}{2}$¢; $12\frac{1}{2}$¢.

b. 288 yd. cloth @ $12\frac{1}{2}$¢; 25¢; $37\frac{1}{2}$¢; 50¢; $62\frac{1}{2}$¢; 75¢.

c. 840 yd. cloth @ $12\frac{1}{2}$¢; 25¢; $37\frac{1}{2}$¢; 50¢; $62\frac{1}{2}$¢; 75¢; $87\frac{1}{2}$¢.

d. 1080 yd. cloth @ $12\frac{1}{2}$¢; 25¢; $37\frac{1}{2}$¢; 50¢; $62\frac{1}{2}$¢; 75¢; $87\frac{1}{2}$¢.

e. 3264 handkerchiefs @ $12\frac{1}{2}$¢; 25¢; 50¢; $37\frac{1}{2}$¢; 75¢; $62\frac{1}{2}$¢. $87\frac{1}{2}$¢.

f. Compare the cost of 84 yd. cloth @ $37\frac{1}{2}$¢, with the cost of $37\frac{1}{2}$ yd. cloth @ 84¢.

(2) What part of \$1 is $\$.33\frac{1}{3}$? $\frac{1}{2}$ of $\$.33\frac{1}{3}$, or $\$.16\frac{2}{3}$?

What part of one dollar is $\$.33\frac{1}{3}+\$.33\frac{1}{3}$, or $\$.66\frac{2}{3}$?

3)\$1.00
2)\$ .33⅓
\$ .16⅔

(3)

$\$.16\frac{2}{3}$	$\$.33\frac{1}{3}$	\$.50	$\$.66\frac{2}{3}$	$\$.83\frac{1}{3}$
$+.16\frac{2}{3}$	$+.16\frac{2}{3}$	$+.16\frac{2}{3}$	$+.16\frac{2}{3}$	$+.16\frac{2}{3}$
$\$.33\frac{1}{3}$	\$.50	$\$.66\frac{2}{3}$	$\$.83\frac{1}{3}$	\$1.00

(4) How many sixths of a dollar is $\$.33\frac{1}{3}$? \$.50? $\$.66\frac{2}{3}$? $\$.16\frac{2}{3}$? $\$.83\frac{1}{3}$? \$1.00?

(5) Count by thirds of a dollar to $\$1.33\frac{1}{3}$, thus: $\$.33\frac{1}{3}=\$\frac{1}{3}$; $\$.66\frac{2}{3}=\$\frac{2}{3}$, etc.

(6) Count by sixths of a dollar to $\$1.33\frac{1}{3}$, thus: $\$.16\frac{2}{3}=\$\frac{1}{6}$; $\$.33\frac{1}{3}=\$\frac{2}{6}$ or $\$\frac{1}{3}$; $\$.50=\$\frac{3}{6}$ or $\$\frac{1}{2}$, etc.

From the above it will be readily seen that:

a. To find the cost when the price of 1 article is $\$.16\frac{2}{3}$; divide the quantity, considered as dollars, by 6.

b. To find the cost when the price of 1 article is $\$.83\frac{1}{3}$; from the quantity, considered as dollars, take $\frac{1}{6}$ of itself.

c. To find the cost, when the price of 1 article is $\$.33\frac{1}{3}$; consider the quantity as dollars, and divide by 3.

d. To find the cost, when the price of 1 article is $\$.66\frac{2}{3}$; from the quantity considered as dollars, take $\frac{1}{3}$ of itself.

ORAL EXERCISE

(1) State the cost of:

a. 72 yd. cloth @ $16\frac{2}{3}$¢; at $33\frac{1}{3}$¢; at 50¢; at $12\frac{1}{2}$¢; at $37\frac{1}{2}$¢; at 75¢; at $62\frac{1}{2}$¢; at $66\frac{2}{3}$¢; at $83\frac{1}{3}$¢.

b. 240 pairs stockings @ $12\frac{1}{2}$¢; at 25¢; at $33\frac{1}{3}$¢; at $37\frac{1}{2}$¢; at 50¢; at $62\frac{1}{2}$¢; at $66\frac{2}{3}$¢; at $16\frac{2}{3}$¢; at 75¢; at $83\frac{1}{3}$¢.

c. 1440 pieces of braid @ $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 25¢; at $33\frac{1}{3}$¢; at $37\frac{1}{2}$¢; at 50¢; at $62\frac{1}{2}$¢; at $66\frac{2}{3}$¢; at 75¢; at $83\frac{1}{3}$¢; at $87\frac{1}{2}$¢.

d. 1080 baskets @ $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 25¢; at $33\frac{1}{3}$¢; at $37\frac{1}{2}$¢; at 50¢; at $62\frac{1}{2}$¢; at $66\frac{2}{3}$¢; at 75¢; at $83\frac{1}{3}$¢; at $87\frac{1}{2}$¢.

e. 3600 collars @ $12\frac{1}{2}$¢; at $16\frac{2}{3}$¢; at 25¢; at $33\frac{1}{3}$¢; at $37\frac{1}{2}$¢; at 50¢; at $62\frac{1}{2}$¢; at $66\frac{2}{3}$¢; at 75¢; at $87\frac{1}{2}$¢; at $83\frac{1}{3}$¢.

(2) *a.* How many cents in $\frac{1}{5}$ of \$1? In $\frac{2}{5}$? In $\frac{3}{5}$? In $\frac{4}{5}$?

b. What aliquot part of \$1 is 20¢? 40¢? 60¢? 80¢?

c. State a short way of finding the cost, when the price of 1 article is 20¢; 40¢; 60¢; 80¢.

(3) What is the cost of:

a. 200 yd. cloth @ 20¢? At 40¢? At 60¢? At 80¢?

b. 450 yd. lace @ 20¢? At 40¢? At 60¢? At 80¢?

c. 800 dozen buttons @ 20¢? At 40¢? At 60¢? At 80¢?

(4) *a.* Since $\$.16\frac{2}{3}$ is $\frac{1}{6}$ of a dollar, what part of a dollar is $\frac{1}{2}$ of $\$.16\frac{2}{3}$, or $\$.08\frac{1}{3}$?

$$2)\overline{\$.16\frac{2}{3}}$$
$$\$.08\frac{1}{3}$$

b. How many twelfths of a dollar is $\$.08\frac{1}{3}+\$.08\frac{1}{3}$ or $\$.16\frac{2}{3}$? $\$.16\frac{2}{3}+\$.08\frac{1}{3}$ or \$.25? $\$.33\frac{1}{3}$? \$.50? $\$.66\frac{2}{3}$? \$.75? $\$.83\frac{1}{3}$?

c. State a short way of finding the cost, when the price of 1 article is $8\frac{1}{3}$¢.

(5) *a.* How many times is 15¢ contained in \$1.00?

b. $6\frac{2}{3}$¢ is what part of \$1.00?

c. State a short way of finding the cost of goods, when the price of 1 article is $6\frac{2}{3}$¢.

(6) State the cost of:

96 yd. tape @ $8\frac{1}{3}$¢
90 yd. tape @ $6\frac{2}{3}$¢
288 spools silk @ $8\frac{1}{3}$¢
3060 doz. buttons @$6\frac{2}{3}$¢

(7) *a.* How much more than \$1.00 is $\$1.12\frac{1}{2}$? What fractional part of 100 is $12\frac{1}{2}$?

ILLUSTRATIVE EXAMPLE: Find the cost of 56 yd. of silk @ $\$1.12\frac{1}{2}$.

Solution: At \$1 a yard, the cost would be \$56.00, but $\$1.12\frac{1}{2}$ is $\frac{1}{8}$ more than \$1. Adding $\frac{1}{8}$ of \$56 to itself, the result is \$63.00, the required cost.

b. State a short way of finding the cost when the price of 1 article is $\$1.16\frac{2}{3}$; $\$1.25$; $\$1.33\frac{1}{3}$; $\$.66\frac{2}{3}$; $\$1.75$; $\$1.87\frac{1}{2}$; $\$1.83\frac{1}{3}$.

(8) State the cost of:

a. 36 yd. @ 50¢; 75¢; $1.25; $1.75; $\$1.16\frac{2}{3}$

b. 75 yd. @ $33\frac{1}{3}$¢; $\$1.66\frac{2}{3}$; $\$1.83\frac{1}{3}$; $\$.06\frac{2}{3}$

c. 160 yd. @ $1.25; $1.75; $\$1.12\frac{1}{2}$; $\$1.87\frac{1}{2}$

d. 1080 handkerchiefs @ $\$.12\frac{1}{2}$; $\$.06\frac{2}{3}$; $\$.08\frac{1}{3}$; $\$.66\frac{2}{3}$; $\$.83\frac{1}{3}$

e. 432 yd. velvet @ $\$1.12\frac{1}{2}$; $\$1.16\frac{2}{3}$; $\$1.08\frac{1}{3}$; $1.75

65. Table of business fractions.

Parts of $1	Parts of $1	Parts of $1	Parts of $1
$.50 = \frac{1}{2}$	$.80 = \frac{4}{5}$	$.66\frac{2}{3} = \frac{2}{3}$	$.10 = \frac{1}{10}$
$.25 = \frac{1}{4}$	$.12\frac{1}{2} = \frac{1}{8}$	$.16\frac{2}{3} = \frac{1}{6}$	$.90 = \frac{9}{10}$
$.75 = \frac{3}{4}$	$.37\frac{1}{2} = \frac{3}{8}$	$.83\frac{1}{3} = \frac{5}{6}$	$.05 = \frac{1}{20}$
$.20 = \frac{1}{5}$	$.62\frac{1}{2} = \frac{5}{8}$	$.08\frac{1}{3} = \frac{1}{12}$	
$.40 = \frac{2}{5}$	$.87\frac{1}{2} = \frac{7}{8}$	$.06\frac{1}{4} = \frac{1}{16}$	
$.60 = \frac{3}{5}$	$.33\frac{1}{3} = \frac{1}{3}$	$.06\frac{2}{3} = \frac{1}{15}$	

1. Memorize parts of $1 given in the above table, so that in whatever form they occur, they will be familiar to you.

EXERCISE

(1) Find the total cost of the following (without copying):—

56 yd. table damask @ $1.25

48 yd. linen finish damask @ $33\frac{1}{3}$¢

160 yd. heavy damask @ $87\frac{1}{2}$¢

42 yd. crash @ $16\frac{2}{3}¢$
75 bath towels @ 25¢
96 bath towels @ $12\frac{1}{2}¢$

(2) Find the total cost of the following (without copying):—

5 doz. sheets @ $87\frac{1}{2}¢$ each
8 doz. pillow cases @ $12\frac{1}{2}¢$ each
10 doz. sheets @ $83\frac{1}{3}¢$ each
10 doz. pillow cases @ $16\frac{2}{3}¢$ each

(3) Find the total cost of the following (without copying):—

48 yd. Hamburg edging @ $12\frac{1}{2}¢$
56 yd. insertion @ $37\frac{1}{2}¢$
84 yd. Madeira edging @ $16\frac{2}{3}¢$
1080 yd. lace edging @ $8\frac{1}{3}¢$
320 yd. lace insertion @ $6\frac{1}{4}¢$
24 yd. lace @ $1.25

(4) Find the total cost of the following (without copying):—

160 satin spreads @ $5.75
250 satin spreads @ $2.50
360 bed comforters @ $1.25
180 summer blankets @ $1.50
160 summer blankets @ $1.12\frac{1}{2}$
10 fine white blankets @ $2.50

(5) Find the total cost of the following (without copying):—

268 waists @ $1.25
344 waists @ $1.75
564 waists @ $1.83\frac{1}{3}$
1074 waists @ $1.16\frac{2}{3}$
840 waists @ $0.75

(6) Find the total cost of:

640 pairs of gloves @ $87\frac{1}{2}¢$
960 embroidered collars @ $1.25
1080 wash skirts @ $1.50
320 taffeta silk petticoats @ $3.75

66. Short methods. Pupils will find pleasure in seeking out other short methods, and in this way, finding the cost of goods at popular prices not mentioned before,—as in the following example:—

(1) Find the cost of 250 suits @ \$48 each.

Solution: The cost of 250 suits at \$48 each = the cost of 48 suits at \$250. The cost of 48 suits at \$1000 = \$48,000. \$250 is $\frac{1}{4}$ of \$1000; therefore, $\frac{1}{4}$ of \$48,000, or \$12,000, is the required cost.

(2) Find the cost of 320 cards of hooks and eyes @ $2\frac{1}{2}$¢.

Solution: $2\frac{1}{2}$¢ = $\frac{1}{4}$ of 10¢; therefore, point off one place in the quantity and take $\frac{1}{4}$ of the result.

(3) State a short way of finding the cost when the price of 1 article is $3\frac{1}{3}$¢; $1\frac{2}{3}$¢; $1\frac{1}{4}$¢; \$2.50; \$12.50.

ORAL EXERCISE

State the price of:

(1) 84 coats @ \$2.50; @ \$12.50
(2) 25 pairs tennis shoes @ 72¢
(3) 125 girls' dresses @ 96¢
(4) 25 yd. ribbon @ $33\frac{1}{3}$¢
(5) 112 pairs boots @ \$6.25
(6) 25 skirts @ \$7.20
(7) 25 boys' suits @ \$5.60
(8) 18 crêpe waists @ \$6.25
(9) 188 sport dresses @ \$25
(10) 92 fibre silk sweaters @ \$6.25
(11) 250 yd. ribbon @ 96¢
(12) 250 yd. ribbon @ 72¢
(13) 25 net dresses @ \$6.68
(14) 250 skirts @ \$1.16
(15) 192 misses' suits @ \$12.50
(16) 168 lbs. paper @ $33\frac{1}{3}$¢
(17) 10 yd. velvet @ 96¢
(18) 72 pc. braid @ \$1.25
(19) 125 silk cravats @ \$1.12$\frac{1}{2}$
(20) 1250 suits @ \$36
(21) 72 yd. chiffon @ \$2.50
(22) $2\frac{1}{2}$ yd. crêpe @ \$.88
(23) 320 yd. gingham @ $16\frac{2}{3}$¢
(24) 72 silk skirts @ \$12.50

EXERCISE

(1) Find the total cost of:

a.
360 yd. @ 10¢
620 yd. @ 20¢
100 yd. @ 15¢
1000 yd. @ 24¢
1000 yd. @ $5\frac{1}{2}$¢
1650 yd. @ 40¢

b.
3200 yd. @ 50¢
1800 yd. @ 25¢
2136 yd. @ $3\frac{1}{3}$¢
2400 yd. @ 50¢
2475 yd. @ 60¢
16 yd. @ $2\frac{1}{2}$¢

c.
32 yd. @ $1\frac{1}{4}$¢
$3\frac{1}{3}$ yd. @ 72¢
$2\frac{1}{2}$ yd. @ 92¢
96 yd. @ $12\frac{1}{2}$¢

(2) Find the total cost of:

a.
92 cards DeLong Press Buttons @ $1\frac{1}{4}$¢
36 " " " " @ $3\frac{1}{3}$¢
208 " " " " @ $2\frac{1}{2}$¢
240 " " " " @ $8\frac{1}{3}$¢
180 " " " " @ $6\frac{1}{4}$¢
200 " " " " @ $2\frac{1}{2}$¢

b.
48 boxes pins @ $12\frac{1}{2}$¢
75 " " @ $6\frac{2}{3}$¢
330 " " @ 20¢
156 " " @ $3\frac{1}{3}$¢
320 " " @ $1\frac{1}{4}$¢
384 " " @ $33\frac{1}{3}$¢

c.
348 papers needles @ 10¢
150 " " @ 20¢
420 " " @ 25¢
125 " " @ 10¢
176 " " @ $12\frac{1}{2}$¢
30 " " @ 25¢

(3) Find the total cost of:

a.
240 yd. braid @ $1\frac{1}{4}$¢
400 " " @ $1\frac{1}{4}$¢
600 " " @ $3\frac{1}{3}$¢
640 " " @ $6\frac{1}{4}$¢
6488 " " @ 25¢
3639 " " @ 30¢
2560 " " @ 40¢

b.
498 yd. braid @ 50¢
360 " " @ $3\frac{1}{3}$¢
600 " " @ $2\frac{1}{2}$¢
225 " " @ $6\frac{2}{3}$¢
375 " " @ $33\frac{1}{3}$¢
61 " " @ 50¢
80 " " @ 40¢

c. 184 yd. braid @ $2\frac{1}{2}$¢
$37\frac{1}{2}$ " " @ 24¢
16 " " @ 25¢
$6\frac{1}{4}$ " " @ 80¢
125 " " @ 36¢
320 " " @ $6\frac{1}{4}$¢
960 " " @ $3\frac{1}{3}$¢

(4) Find the total cost of:

a. 720 doz. buttons @ $8\frac{1}{3}$¢ a doz.
240 " " @ $6\frac{2}{3}$¢ " "
480 " " @ $6\frac{1}{4}$¢ " "
388 " " @ 25¢ " "
352 " " @ 50¢ " "
144 " " @ $12\frac{1}{2}$¢ " "
360 " " @ $66\frac{2}{3}$¢ " "
732 " " @ $16\frac{2}{3}$¢ " "

b. 66 doz. buttons @ 30¢ " "
120 " " @ $6\frac{2}{3}$¢ " "
100 " " @ 75¢ " "
50 " " @ 80¢ " "
$12\frac{1}{2}$ " " @ 96¢ " "
75 " " @ $33\frac{1}{3}$¢ " "
30 " " @ $1.80 " "
60 " " @ $2.50 " "

(5) How many 50¢ articles can be bought for $1? $3? $5? $1.50? $5.50? $7.50? $12.00? $16.50?

(6) How many 25¢ articles can be bought for $1? $5? $.50? $2.25? ($2+$.25) $3.50? $8.50? $11? $.75? $3.75? $6.25?

(7) When the price is 25¢, the quantity is 4 times the cost. Hence, to find the quantity when the price is 25¢, multiply the cost by 4.

(8) How many $12\frac{1}{2}¢$ articles can be bought for: \$1? \$3? \$.25? \$1.50? \$2.25? \$5.75? \$8.25?

(9) Give a short method for finding the quantity when the cost is given, and the price is $12\frac{1}{2}¢$; $20¢$.

(10) How many $33\frac{1}{3}¢$ articles can be bought for \$1? \$3? \$10? \$14? \$2? \$6? \$12? \$25? \$50?

(11) How many $16\frac{2}{3}¢$ articles can be bought for the same amounts?

(12) Give a short method for finding the quantity when the cost is given and the price is $33\frac{1}{3}¢$, $16\frac{2}{3}¢$, $6\frac{2}{3}¢$, $6\frac{1}{4}¢$.

Illustrative Example: How many yards of cloth may be bought for \$225 at $6\frac{1}{4}¢$ a yard?

Solution: $6\frac{1}{4} = \frac{1}{16}$ of $100 = \frac{100}{16}$; then the quotient found by dividing 225 by $6\frac{1}{4}$ will be 16 times the quotient found by dividing by 100. $\$225 \div 6\frac{1}{4} = 3600$. Therefore to divide by $6\frac{1}{4}$, move the decimal point two places to the left and multiply the result by 16.

(13) Give a short method of finding the quantity of goods when the cost is given, and the price is $75¢$.

Illustrative Example: At $75¢$ a yard, how many yards of silk can be bought for \$288?

Solution: 75 increased by $\frac{1}{3}$ of itself $= 1$. When the divisor is 1 the quotient is the same as the dividend. $288 \div 3 = 96$; $288 + 96 = 384$, 288 increased by $\frac{1}{3}$ of itself, or the number of yards silk which may be bought for \$288 at $75¢$ a yd. Therefore to divide by .75, increase the number by $\frac{1}{3}$ of itself.

(14) Give a short method of finding the quantity when the cost is given and the price is 125 dollars, or cents.

Illustrative Example: How many suits costing \$125 each, may be bought for \$13,000?

Solution: 125 is $\frac{1}{8}$ of 1000; the quotient found by dividing by 125 will be 8 times the quotient obtained by dividing by 1000; $13000 \div 1000 = 13$. $13 \times 8 = 104$. Therefore 104 suits costing \$125 each may be bought for \$13000. Hence: to divide by 125 multiply by 8 and move the decimal point three places to the left.

ORAL EXERCISE

Find the number of articles that can be bought for:

a. \$15 at \$1.25
b. \$20 at \$2.50
c. \$32 at \$.37$\frac{1}{2}$
d. \$96 at \$.33$\frac{1}{3}$
e. \$81.20 at \$.40
f. \$9.00 at \$.25
g. \$240 at \$.50
h. \$60 at 12$\frac{1}{2}$¢
i. \$250 at 20¢
j. \$430 at 2$\frac{1}{2}$¢

WRITTEN EXERCISE

Find the quantity,

	Cost	*Price per Yard*		*Cost*	*Price per Yard*
a.	\$285.00	75¢	*e.*	\$860.50	33$\frac{1}{3}$¢
b.	\$306.00	75¢	*f.*	\$921.25	25¢
c.	\$140.75	12$\frac{1}{2}$¢	*g.*	\$897.50	62$\frac{1}{2}$¢
d.	\$432.25	12$\frac{1}{2}$¢	*h.*	\$1360.25	16$\frac{2}{3}$¢

The accompanying drill table may be used in any way the teacher may desire. The following suggestion is made; (1) Using the first quantity (45 yd.), make the oral extensions across the page. (2) Find the cost of each quantity by each price given, in lines, a, b, c, d, e. (3) Compare results.

	Drill Table				Time Exercise			
A	33$\frac{1}{3}$¢	16$\frac{2}{3}$¢	50¢	66$\frac{2}{3}$¢	8$\frac{1}{3}$¢	\$1.33$\frac{1}{3}$	\$1.50	\$1.66$\frac{2}{3}$
B	75¢	\$1.25	12$\frac{1}{2}$¢	87$\frac{1}{2}$¢	\$1.75	62$\frac{1}{2}$¢	6$\frac{1}{4}$¢	37$\frac{1}{2}$¢
C	20¢	80¢	60¢	40¢	\$1.20	10¢	5¢	\$1.60
D	\$2.00	6$\frac{2}{3}$¢	1$\frac{1}{4}$¢	3$\frac{1}{3}$¢	2$\frac{1}{2}$¢	1$\frac{2}{3}$¢	\$2.50	83$\frac{1}{3}$¢
E	25¢	\$1.10	90¢	$\frac{1}{2}$¢	30¢	\$1.16$\frac{2}{3}$	\$1.25	\$1.20

(1) Find the cost of:

a.	*b.*	*c.*	*d.*	*e.*
45 yd.	18 yd.	36 yd.	15 yd.	81 yd.
75 yd.	54 yd.	42 yd.	60 yd.	12 yd.
90 yd.	96 yd.	84 yd.	30 yd.	108 yd.

(2) Find the cost of:

a.	*b.*	*c.*	*d.*	*e.*
72 yd.	88 yd.	24 yd.	8 yd.	128 yd.
48 yd.	32 yd.	16 yd.	80 yd.	112 yd.
64 yd.	56 yd.	40 yd.	104 yd.	160 yd.
f.	*g.*	*h.*	*i.*	*j.*
50 yd.	144 yd.	250 yd.	360 yd.	600 yd.
270 yd.	150 yd.	350 yd.	240 yd.	200 yd.
120 yd.	100 yd.	300 yd.	180 yd.	800 yd.

(3) Copy the following invoice, filling in the missing amounts:

Toledo, Ohio, Jan. 1, 1915.

The Halle Bros. Co.
Cleveland, Ohio.

Terms, Net 30 Days. Bought of Lasalle & Koch Co.

126	68	yd.	French Cretonne	$1.25			
301	36	"	French Moire Stripe	2.50			
214	48	"	Novelty Crêpe	1.50			
198	56	"	Crêpe Ratine	$0.37\frac{1}{2}$			
405	108	"	Novelty Suiting	$0.87\frac{1}{2}$			
112	64	"	Plaid Eponge	$0.62\frac{1}{2}$			
115	45	"	Novelty Voile	0.25			
125	72	"	English Serge	1.75			
113	64	"	Wash Silk	$0.87\frac{1}{2}$			
118	96	"	Anderson Gingham	$0.33\frac{1}{3}$			

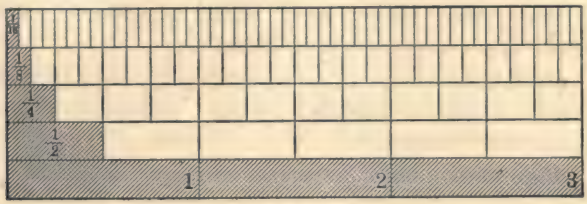
a. The figures to the left of the ruling show the catalog numbers.

CHAPTER X

UNITS OF MEASURE AND THEIR APPLICATION

What is the unit of measure for cloth, ribbons, silk, etc?

Why is exact measuring of great importance to the employer? To the customer? To the employee?



Fractional Lengths

67. How to measure goods. Goods should be measured right side out, the loose threads being left on the right side —not on the wrong side. Use the yard stick with the inches marked on the outside—not on the back.

1. Double width goods should be measured on the fold: flounces, laces, etc., should be measured on the selvedge edge: avoid stretching.

2. One and one-quarter yards make a fold or an "awn."

68. Dress goods. Directions for measuring dress goods accurately:

1. Gauge amount of material to be unrolled.

2. Straighten out merchandise.

3. Hold the length of the goods firmly with the thumb of the left hand on the end of the yard stick, and measure to the next yard with the right hand thumb.

4. Place thumb to thumb, and measure the next yard, etc. Make no allowance for damage in goods, unless under proper authority.

5. In cutting dress goods, use filling threads as a guide to avoid waste.

EXERCISE

(1) How many inches in 1 yd.? 2 yd.? 3 yd.?

(2) How many inches in a half yard? One-quarter of a yard? Three-quarters of a yard?

(3) How many inches in one-third of a yard? What is one-third of a yard sometimes called?

(4) How many feet in one yard? Is the foot measure used in the department store? When is it used? How many inches in 2 ft.? In 6 ft.?

(5) Eighteen inches is what part of 1 yard? Of 2 yards?

(6) What part of 1 yard is 9 inches? 27 inches? 6 inches? 12 inches? 24 inches? 2 inches? 4 inches? 8 inches?

(7) How many inches in $1\frac{1}{2}$ yd.? In $1\frac{3}{4}$ yd.?

(8) Cloth 54″ wide is how many yards wide? 72″? 50″?

Do you know how to estimate the number of yards of carpet required for rooms of various sizes?

How would such knowledge be of service to you? Would it add to your value as an employee of a store? How would it help a customer?

69. Carpeting. Rugs are sold by the piece; all other carpeting is sold by the yard from the roll. Carpeting is usually 27″ wide (Brussels or Wiltons) or 36″ wide (Ingrains).

Linoleum is generally sold by the square yard.

1. The number of yards needed for a given room depends upon the way the strips are to run. There may be more

waste if they are laid one way, rather than another; that is, lengthwise or across the room. There is waste in matching; as in wall paper, a fractional part of a width cannot be bought.

2. To estimate the number of yards required to carpet a room: determine which way the strips shall run and estimate their length and number; add to the length of each strip after the first allowance for waste in matching. The combined length of the strips in feet divided by 3, will give the number of yards required to cover the floor.

Problems

(1) Measure your schoolroom floor and determine how many yards of carpet 1 yd. wide it would take to cover it, placing the strips lengthwise; crosswise. Which is the more economical way of laying the carpet?

(2) What will it cost to carpet a room 16 ft. long, 12 ft. wide, with carpet 36″ wide, at $1.19 a yard?

(3) The price of linoleum is $1.75 a square yard. What will it cost to cover a floor 12 ft. long by 10 ft. wide?

(4) How many yards of carpet $\frac{3}{4}$ yd. wide will it take to cover a floor 19 ft. long by 17 ft. wide? What will the carpet cost at $1.12\frac{1}{2}$ a yard?

(5) How many yards of carpet 27″ wide will be required to cover a floor 9 ft. by 12 ft.? What will the carpet cost at $1.20 a yard?

(6) Estimate the cost of carpeting a room 16′ 2″×12′ 4″ with Brussels carpet, at $1.55 per yard.

Illustrative Example (1): Find the amount of carpet and of border that you must buy to carpet a room 15 ft. long and 10 ft. 6 in. wide, with carpet 27 inches wide, the strips running lengthwise and the carpet having a border $\frac{5}{8}$ of a yard wide.

Solution: 15 ft. = 5 yd. 10 ft. 6 in. = $3\frac{1}{2}$ yd.

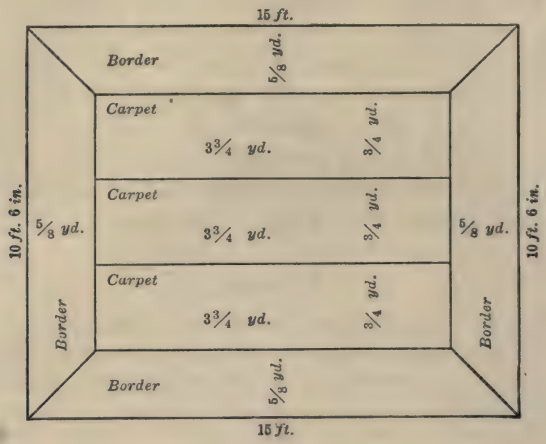
5 yd.—($2\times\frac{5}{8}$ yd.) $1\frac{1}{4}$ yd. = $3\frac{3}{4}$ yd. the length of each strip.

$3\frac{1}{2}$ yd.—($2\times\frac{5}{8}$ yd.) $1\frac{1}{4}$ yd. = $2\frac{1}{4}$ yd. the total width of carpet.

$2\frac{1}{4}\div\frac{3}{4}=3$, the number of strips required.

$3 \times 3\frac{3}{4}$ yd. $= 11\frac{1}{4}$ yd., the amount of carpet required.

$2 \times (5 \text{ yd.} + 3\frac{1}{2} \text{ yd.})$ or 17 yards = the amount of border required.



Diagram

Problems

(1) How many strips of carpet 27″ wide will be required for a floor 12 ft. 6 in. wide, the strips to be laid lengthwise? Draw a diagram.

(2) Find the cost of a carpet for a room measuring 18 ft. long by 14 ft. wide. The carpet is $\frac{3}{4}$ yd. wide and is \$1.85 per yard. The strips run lengthwise, $\frac{1}{2}$ yd. being allowed on each strip for matching the pattern.

(3) Find the amount of carpet and of border which must be bought to carpet a room $15\frac{1}{2}$ ft. long and $14\frac{1}{2}$ ft. wide with carpet and border each 27 in. wide, strips of carpet running lengthwise. Draw a diagram.

(4) How many square yards are there in a piece of linoleum 3 yd. long, if its width is $3\frac{1}{2}$ yd.?

(5) Show by a diagram 4 strips of $\frac{3}{4}$ yd. wide carpet 12 ft. long, surrounded by a border $\frac{5}{8}$ of a yd. wide. What is the length and width of this floor if this carpet and border exactly cover the floor?

Do you know how to estimate the number of rolls of paper that rooms of different dimensions require?

Would the knowledge of how to do this help you in your own home? Would it make you more valuable to your employer? To a customer? Why?

70. Papering. The unit of measure for wall paper is the single roll (8 yd. long and 18″ wide) or the double roll (16 yd. long). There is always so much waste in cutting and matching wall paper that it is better to use double rolls. Double rolls are counted as 2 rolls each. Borders or friezes are sold by the yard and vary in width from 3″ upward. Imported papers differ as to the width and length of the roll. They are usually 12 yd. long and single rolls only. The usual width of 18″ varies in higher priced papers, from 18″ to 22″ and from 30″ to 36″.

1. The approximate cost only can be estimated in papering a room, as allowance must be made for doors, windows, matchings, etc. To estimate the number of rolls of paper required for the walls of a room, multiply the distance around the room in feet by the height in feet, and divide by 72 if the rolls are double, and by 36 if the rolls are single; for each three openings, deduct a double roll.

2. Fractional rolls are not sold; a fractional roll is counted as a whole roll.

Problems

ILLUSTRATIVE EXAMPLE: A customer wishes to purchase paper for a room which is 24 ft. long, 16 ft. 6 in wide, and 12 ft. high. What is her bill if the paper is $12\frac{1}{2}$¢ a roll?

Operation: Wall space to be covered = 972 ft.
972 ft. $= 2(24 \text{ ft.} + 16\frac{1}{2} \text{ ft.}) \times 12$.
$972 \text{ ft.} \div 36 \text{ ft.} = 27 =$ number of rolls required.
$27 \times \$.12\frac{1}{2} = \$3.37\frac{1}{2}$ or \$3.38, the required amount.

(1) What will be the cost of the paper for the walls of a room, the dimensions of which are 36 ft. long, 28 ft. wide, 11 ft. high

(36′×28′×11′), the price of the paper being 38¢ a roll? The room has two windows and one door.

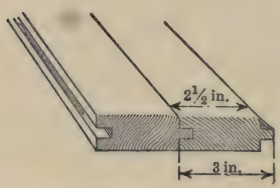
(2) At $33\frac{1}{3}$¢ a roll, what is the cost of paper for a room that is 24 ft. long, 18 ft. wide, and $9\frac{1}{2}$ ft. high? The room has one window and two doors.

(3) Estimate the cost of paper for a room 18 ft. by 22 ft. and 9 ft. high, at 25¢ a roll. The room has two doors and four windows.

(4) A room is 22 ft. long, 20 ft. wide, and 13 ft. 8 in. high. What will paper for this room cost at 19¢ a roll? The room has two windows and one door.

(5) What would it cost to paper your chamber with paper at 20¢ per double roll, the border at the same price, and 25¢ a double roll for hanging? (Although a border is used make no reduction in quantity of paper required.)

71. Flooring. Dimensions are usually written with the length first, then the width, then the height, thus: a room 20 ft. long, 15 ft. wide, and 8 ft. 6 in. high may be written,— 20′×15′×8′6″.



Tongued and Grooved

1. Flooring is estimated by the thousand board feet, M, or by the square. Boards for flooring are tongued, grooved, or matched. This entails a loss on each board; for this waste, carpenters generally allow one-fifth. That is, for 300 sq. ft. of floor space, 360 sq. ft. (board feet) of flooring are needed.

Illustrative Example: How many square feet of flooring are required for a floor which is 16′ 6″×12′ 4″?

Solution: $16'\ 6'' = 16\frac{1}{2}'$

$12'\ 4'' = 12\frac{1}{3}'$

$$\frac{\overset{11}{\cancel{33}}\times 37\times \overset{3}{\cancel{6}}}{\underset{2}{\cancel{2}}\quad \underset{1}{\cancel{3}}\quad 5} = \frac{1221}{5} = 244\tfrac{1}{5}$$

Ans. 245 sq. ft.

The allowance for waste is reckoned by multiplying by $\frac{6}{5}$.

Hence, multiplying the number of feet in the length, by the number of feet in the width, by $\frac{6}{5}$ gives the required result.

73. Measures of time.

1. Seconds—Minutes—Hours

TABLE OF TIME MEASURE

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
30 days,	= 1 calendar month (mo.)
365 days or 12 months	= 1 common year (yr.)

EXERCISE

(1) How many seconds in 5 minutes? In one day?

(2) How many minutes is it from half past seven to eleven o'clock?

(3) How many weeks in 28 days? In 280 days?

(4) John worked from 8.15 A. M. to 5.30 P. M. He was allowed one hour for lunch. He received 65¢ an hour, including his lunch hour. What was his day's pay?

(5) Henry spent from 7.50 A. M. to 11.00 A. M. chopping wood. How long did he work? If he received $33\frac{1}{3}$¢ an hour, how much money did he receive?

(6) The English lesson occupies 39 min.; the geography, 38 min.; sewing, 56 min.; spelling, 19 min.; writing, 28 min.; music, 45 min. How many hours are used in these studies?

(7) How many hours do you spend in bed if you rise at 6.30 A. M. and retire at 8.45 P. M.?

(8) Fred hired a boat from 3.29 to 5.44 P. M., @ 35¢ an hour. How much did it cost?

(9) The tide began to rise at 9.08 A. M. and kept rising till 2.56 P. M. How long did it rise?

(10) Mary hemmed 24 aprons between 8.30 o'clock and 11.30 o'clock. How many aprons per hour did she hem?

If she received 9¢ for her work on each apron, what was her average wage per hour?

(11) When the sun rises at 7.12 A. M. and sets at 4.24 P. M., how many hours are there between sunset and sunrise?

"Thirty days hath September
April, June and November."

Each other month, excepting February, has 31 days. February has 28 days in a common year and 29 days in a leap year.

(12) Beginning with the first, write in order the names of the months of the year, with the number of days in each, after the name.

(13) How many days in May, June, and July? In July, August, and September?

(14) How many days from Nov. 7 to Dec. 23?

(15) If the first day of November is Tuesday, the eighth is what day of the week? The third Tuesday is what day of the month? Christmas will be what day of the week? New Year's day?

2. There are three methods of finding the difference in time between dates.

(1) The exact method is expressed in days, or years and days, and is obtained by counting the exact number of days between two dates.

a. The exact time from June 5 to Oct. 17 is reckoned as follows:

June, 25 days remaining
July, 31 days
Aug., 31 days
Sept., 30 days
Oct., 17 days
Total—134 days

b. Exact time is used by the Government in making interest calculations, when the period of time is less than a

year, also by bankers in computing bank discount, and in some business dealings.

(2) The Bankers' time is expressed in days, or months and days, and is obtained by counting months for the whole months, and exact days for any remaining part of a month,—thus: from June 5 to Oct. 17,—

June 5—July 5 =	1 mo.
July 5—Aug. 5 =	1 mo.
Aug. 5—Sept. 5 =	1 mo.
Sept. 5—Oct. 5 =	1 mo.
Oct. 5—Oct. 17 =	12 days
Time =	4 mo. 12 days

a. This method is generally used by banks in computing interest on notes, mortgages, etc., for short periods of time; hence it is called the Bankers' time.

(3) The commercial or compound time, is expressed in years, months, and days and is obtained by what is called compound subtraction. The difference in time between July 15, 1916, and June 9, 1921, is reckoned thus:

1921	6	9
—1916	7	15
4	10	24

Explanation: Write the dates as above, putting years under years, months under months, days under days. 15 days can not be taken from 9 days so 1 month or 30 days must be added to 9 days making 39 days in all. The remainder is 24 days. One month being used, there are but 5 months left in the minuend, and 1 year or 12 months must be added, making 17 months. Subtracting 7 months, the remainder is 10 months. One year of 1921 being used, the date is 1920; subtracting 1916, the remainder is 4 years. The total difference in time is 4 years, 10 months, 24 days.

a. This method is based on the commercial year,—360 days,—12 mo. of 30 days each. One-half a month is 15 days;

one quarter of a year is three calendar months; one-half of a year is six calendar months.

Problems

Find the compound time between:

(1) Nov. 15, 1904, and July 8, 1907.
(2) May 6, 1903, and Sept. 15, 1922.
(3) Jan. 11, 1899, and Feb. 24, 1916.
(4) Sept. 15, 1912, and Dec. 5, 1919.
(5) Aug. 25, 1901, and Sept. 1, 1920.
(6) Mar. 9, 1906, and Oct. 20, 1918.

ILLUSTRATIVE EXAMPLE: A note dated June 7 was paid Aug. 3. How many days did it run?

Solution: June, 23
July, 31
Aug., 3
57 days from June 7 to Aug. 3.

3. The day on which the note is given is not counted, but the day on which it is due is counted.

(1) Find exact time and Bankers' time between:

a. June 6 and Sept. 23.
b. Nov. 13 and Jan. 4.
c. Mar. 17 and July 4.
d. Jan. 15 and May 12.
e. May 12 and Nov. 10.
f. Apr. 4 and Sept. 20.

(2) Find the compound time and the exact time between:

a. Jan. 10, 1912, and Apr. 20, 1914.
b. Feb. 28, 1913, and Jan. 10, 1916.
c. Nov. 25, 1910, and Mar. 2, 1917.
d. Dec. 25, 1916, and July 4, 1919.
e. Sept. 10, 1917, and Oct. 31, 1920.

4. To count forward or backward from a given date.

ILLUSTRATIVE EXAMPLE (1): Count forward 48 da. from May 10.

Solution: 48 da.
—21 da. left in May after May 10.
27 da. to count forward after May 31 into June.

Therefore, June 27 is the desired result.

ILLUSTRATIVE EXAMPLE (2): Count backward 72 days from Aug. 8.

Solution: 73 da.
—8 back in Aug.
65
31 da. in July
34
30 da. in June
4 days back in May.

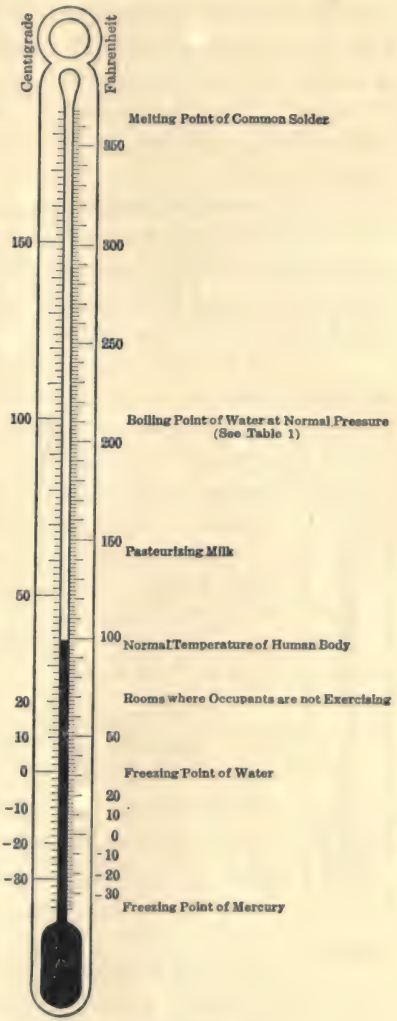
May 31—4 da. = May 27, the desired result.

Problems

Count both forward and backward:

(1) 84 days from July 4.
(2) 96 days from May 30.
(3) 108 days from Mar. 15 (the present year).
(4) 128 days from April 10, 1919.
(5) 203 days from June 17, 1920.

74. Fahrenheit and Centigrade.



The figure shows both the Fahrenheit and the Centigrade scales. Find out where each is used and compare their respective advantages. Which is easier to read?

Study the Fahrenheit scale.

1. How many degrees are represented by each space? How many degrees above or below zero is the freezing point of water? How many degrees above zero is the boiling point of water? How many degrees above zero is blood heat? (Blood heat is the temperature of the interior of the human body in health.)

2. Study the Centigrade scale and by comparison with the Fahrenheit scale try to answer the above questions from the Centigrade scale. On the Centigrade scale the freezing point and normal boiling point of water are respectively 0° and 100°; on the Fahrenheit scale these points are 32° and 212°; thus 1° Centigrade is equal to 1.8° Fahrenheit.

Barometer

3. How much above normal is the temperature of a fever patient recorded at 103°? A patient whose temperature is sub-normal has it recorded at 96.5. How much below normal is this? What is the temperature of a patient whose temperature is 1.5 below normal?

4. If the outdoor temperature goes down to 7° below 0, how many degrees below freezing point is it?

5. The average January temperature of Boston one year was 19°. The same year the average January temperature of Cairo was 45° above 0. What is the amount of difference? By how many degrees is this difference greater or less than the difference between the freezing point and the boiling point of water?

75. Atmosphere—barometer. The picture shows a barometer, an instrument used for measuring the pressure or weight of the atmosphere. The hollow glass tube (marked "c") is sealed air-tight at the top and opens into a

well of mercury at the bottom (marked "w"). The pressure of the air downward on the surface of the mercury in the well outside of the tube, forces the mercury up into the tube, from which the air has been removed. The scale shows in inches how high the mercury stands in the tube. How high does it stand?

1. How many cu. in. of mercury are there in the tube if the cross section of the tube of mercury is 1 sq. in.?

2. If the liquid in the tube were water, how many ounces would it weigh? (One cu. in. of water weighs $\frac{29}{50}$ of an ounce.)

3. How many ounces does the mercury in the tube weigh if mercury weighs $13\frac{3}{5}$ times as much as water?

4. How many pounds does the mercury in the tube weigh? What is the pressure of the atmosphere on a square inch?

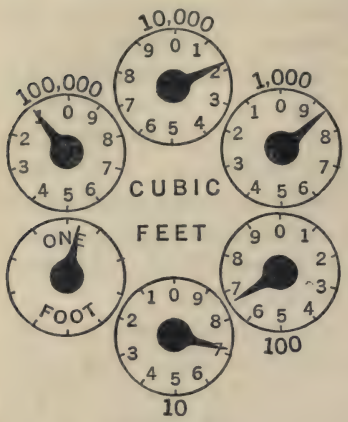
5. When the barometer reading is 30 inches, what is the pressure of the atmosphere? When it is 28 inches? When it is 31 inches?

76. Water. Water is sold by the 1000 cubic feet. Can you read your own water meter? What advantage is it to you to be able to do this in your own home? In your place of business? Meters for measuring water are usually graduated in cubic feet, sometimes in gallons. $7\frac{1}{2}$ gallons equals one cubic foot. Hence to reduce a meter reading in cubic feet, to gallons, multiply the number of cubic feet by $7\frac{1}{2}$.

1. The figure shows the ordinary form of dial, the unit is the cubic foot, and is plainly marked on the dial. The hands revolve around circles, each divided into 10 numbered divisions. The number on the outside of each circle indicates the number of cubic feet for one complete revolution of the hand.

(1) The first dial (at the bottom) is marked 10, and one division measures 1 cubic foot; the second, 100, and one division measures 10 cubic feet; the next is marked 1000, and one division measures 100 cubic feet, and similarly for the rest. The small dial at the left measuring 1 cubic foot for a complete revolution is used for testing the meter, and has nothing to do with the meter reading. One

division of a circle is equal to a complete revolution of the hand on the next lower circle. When a hand is between two figures, the lesser is to be taken. If a hand is very near a figure, whether that figure or the next lower is to be taken, is determined by observing the hand in the next lower circle. Unless the hand on this circle has reached, or just passed 0, the lesser figure is to be taken. The best method of reading is from low to high, that is from right to left. For example, reading the dial in the figure above, and setting down the figures from right to left, we have 7 for units' place, 6 for tens' place, 8 for hundreds' place, and 1 for thousands' place, and 1 for ten thousands' place, or 11,867 cubic feet.



Water Meter

(2) The amount of water which has passed through the meter in a given time is obtained by subtracting the first reading from the last. For example, if the meter were read the thirtieth day of June, and again the thirtieth day of July, the June reading is to be subtracted from that taken in July.

Problems: (Making Out Water-Charges)

(1) Mr. Small's meter readings for the year were as follows: Mar. 1, 18,300 cu. ft.; June 1, 28,600 cu. ft.; Sept. 1, 36,200 cu. ft.; Dec. 1, 38,120 cu. ft.

a. How many cubic feet were used from Mar. 1 to June 1?

b. How many gallons were used?

c. How much did the water cost at 20¢ per 1000 gallons?

Solution: 28,600 cu. ft.—18,300 cu. ft. = 10,300 cu. ft.

10,300 cu. ft. $\times 7\frac{1}{2}$ gallons = 77,250 gal. or 77.25 M. gal.

$77.25 \times 20 = \$15.45$.

(2) Answer the same three questions for each of the remaining quarters.

BILL FOR CITY WATER

Date.....Street.....
Mr.....
To.....Allendale Water Co., Dr.
For water by meter from.....to.....1920

Latest reading—cu. ft.		
Previous reading—cu. ft.		
Total cubic feet used.....		
Total gallons used.....		
Cost at 25¢ per 1000 gal..	?	?
Received payment		
.....		
Per.....		

(3) Water meter readings are made every month in the city of Allendale and billed to the consumer at 25¢ per 1,000 gal. The following card contains the monthly readings of Mr. A. T. Cole's meter.

WATER METER READINGS

No. 13 Salem St.

Month	Monthly Reading
Jan. 4	18,500 cu. ft.
Feb. 6	19,140 cu. ft.
Mar. 10	19,280 cu. ft.
Apr. 12	19,800 cu. ft.
May 10	20,110 cu. ft.
June 8	20,800 cu. ft.
July 10	21,280 cu. ft.

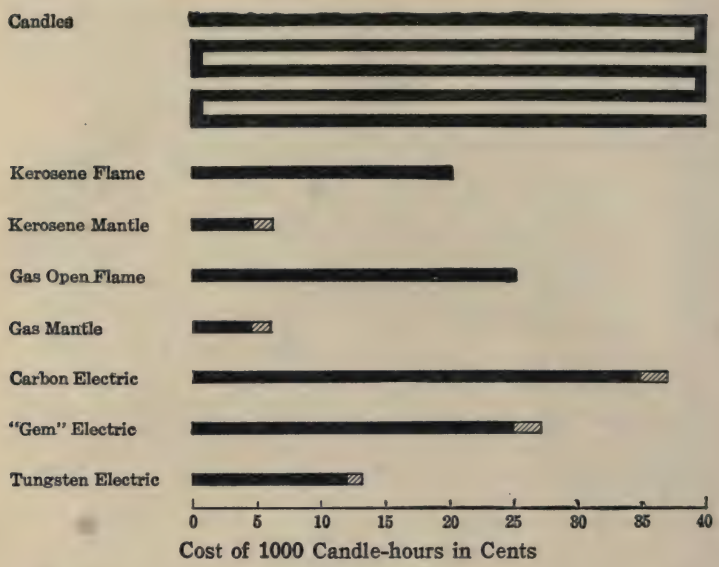
a. Make out the bill for water from Jan. 4 to Feb. 6.

b. Make out the bill for water from Feb. 6 to Mar. 10.

c. Make out the bill for water from Mar. 10 to April 12.

d. Find the cost of water for each of the months recorded on the card.

(4) What is the cost for water for three months at $1.10 per 1000 cu. ft., if a meter registers 21,870 cu. ft. at the beginning, and 29,110 cu. ft. at the end?

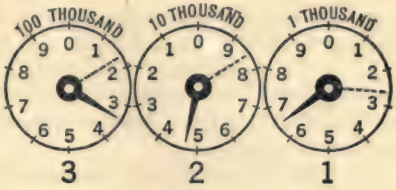


77. Gas. The cost of lighting depends on the candle-power of lamps used and the time the lamps burn. 1000 candle-hours of lighting are consumed by burning a 10-candle lamp 100 hours, or a 50-candle-lamp, 20 hours. The relative costs of producing light by methods usually employed are shown in the figure above.

1. Costs are based on the following prices: candles, 12 cents per pound; kerosene, 15 cents per gallon; gas $1.00 per 1000 cu. ft.; electricity, 10 cents per kilowatt hour. The solid lines represent cost of fuel or of current, the shaded parts the cost of the mantle and bulbs.

2. The gas meter and how to read it. Gas is sold by the thousand cubic feet. The amount consumed is shown by a meter (similar to the one in the figure), upon the face of which are three dials. The dial to the right shows hundreds of cubic feet (the one marked "1 thousand"). This means that during one complete revolution of the hand 1000 cu. ft. of gas has passed through the meter. The dial is divided into 10 equal parts so that the passage of the hand over each part indicates the passage of one-tenth of 1000 cu. ft., or

100 cubic feet. Read from left hand dial to right, always taking the figures which the hands have passed. For example: by above dials the reading of 100,000 dial is 30,000 cu. ft.; of 10,000 dial is 4000 cu. ft.; of 1000 dial is 600 cu. ft. 30000 + 4000 + 600 cu. ft. = 34600 cu. ft., complete reading of the meter. To find out the amount of gas used in a given time, subtract the previous reading from the present; viz.:



Gas Meter

Reading by above dial. 34,600
Reading by previous statement. . 18,200 (indicated by dotted lines)
Gives number of ft. registered. . . . 16,400

Problems

(1) Give the reading when the hand on No. 3 is between 2 and 3; on No. 2, between 3 and 4; and on No. 1 at 6.

(2) Give the reading when the hand on No. 3 is at 0; on No. 2, between 7 and 8; and on No. 1 at 9.

(3) The reading of a meter at successive times was 65,300 and 84,600. How much was the bill for the month at $.90 per thousand cu. ft.?

(4) Determine the position of the hands on the dial for the reading in problem 3.

Elm Hill District — Month ending Dec. 31, 1920.
A. M. Beale — To City Gas Light Co., Dr.
325 Pearl St., City — No. 84 Broadway

	Cost at 1.25 per M.			Net cost	
Present meter reading	62400 cu. ft.				
Last previous reading	59800 cu. ft.				
Cubic feet consumed	2600 cu. ft.	3	25		
Discount of 5 cents per M. on this bill if paid within ten days of date.			10	3	15

Paid, Date, Jan. 2, 1921. Per H. E. B.

5. Make out a similar bill for Mr. Beale, assuming the present reading to be 65800 and the rate 90¢ per M.

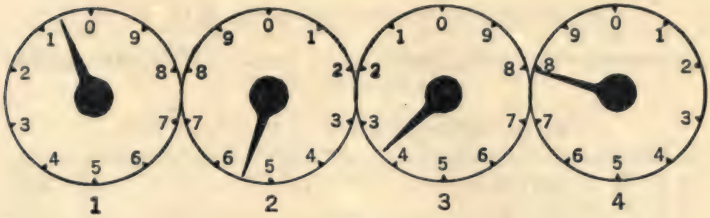
(6) Make out a similar bill for M. F. Dole, whose meter readings for the month are as follows: present reading, 46,200; previous reading 43,800. He pays the bill at the end of the month and receives the discount.

(7) The following card contains the consecutive readings of John A. Loving's gas meter from January to December, 1917. Find the amount of his monthly bill at $.85 per M. and allow a discount of 1¢ per 100 ft. each month for prompt payment.

Meter No. 2850		John A. Loving 12 Spring Street Lowell	
Jan. reading	8,400	July reading	12,800
Feb. reading	8,800	Aug. reading	13,500
Mar. reading	9,600	Sept. reading	14,800
Apr. reading	10,200	Oct. reading	16,200
May reading	10,900	Nov. reading	17,400
June reading	12,200	Dec. reading	18,200

78. Electricity.

A view of the dials of a modern electric meter is given below. The method of reading is similar to that of reading the dial of a gas meter, reading the dials from right to left. The unit is the kilowatt hour instead of the cubic foot. The reading is 538 kilowatt hours. The pointer on the dial at the extreme right points to 8; the number 8 is written down in the units' place; the index of the next dial to the left has passed 3, but has not reached 4; the figure 3 is written in the tens' place. The index of the third dial has passed the 5, and this figure is written in the hundreds' place, giving 538 kilowatt hours as the reading of the meter, since the index of the dial at the extreme left has not reached the figure.



KILOWATT HOURS

Problems

(1) What is the reading when the hand on No. 4 is at 4; on No. 3, at 0; on No. 2, between 7 and 8; and on No. 1, at 5?

(2) What is the reading when the hand on No. 4 is at 2; on No. 3, at 3; on No. 2, between 1 and 2; on No. 1, at 9?

(3) The demand on the electric system of a store wired for electricity varies with the number of lights, consequently stores are given ratings according to the number of lights, etc., used, the owner being charged accordingly.

Mr. Corey's store has a primary demand of 18 K. W. H. (kilowatt hours). If he uses 25 K. W. H. he is charged as follows:

Cost of 18 K. W. H. @ $12\frac{1}{2}$¢ =	\$2.25
Cost of 7 K. W. H. @ 10¢ =	.70
	\$2.95

NOTE: From this it will be seen that the amount he uses above the primary demand costs less.

(4) Mr. Halle's store has a primary demand of 16 K. W. H. and he uses 20 K. W. H. in December. Complete the items in his December bill as follows:

Cost of 16 K. W. H. @ $16\frac{2}{3}$¢ =	\$?
Cost of 4 K. W. H. @ 10¢ =	?
	\$?

(5) The reading of a meter at successive times was 456 and 504. How much was the bill for the month at \$.085 per K. W. H.?

(6) If 15 burners, each consuming 6 cu. ft. of gas an hour, are used on an average of 4 hours per day for 365 days, what will be the gas bill for the year at 80¢ per thousand?

(7) In what ways can one economize in the heat and light bills?

The Edison Electric Illuminating Co. of Peoria, Ill.

In account with Smith, Brown & Co. Date, May 6, 1920.
201 South St., City.

Electric Service from Mar. 5 to May 6, 1920.

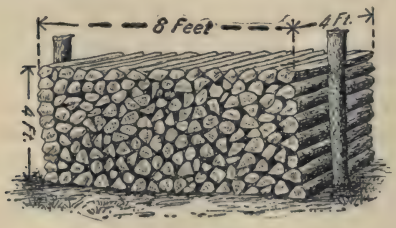
24 K. W. H. @ 12¢	2	88
6 K. W. H. @ 8¢		48
30 K. W. H. used in all	3	36
Discount of ½¢ per K. W. H. used if paid in 10 days		15
	3	21

Received payment 19 .
Signed

(8) Make out a similar bill for M. A. W. Hall, whose store called for a primary demand of 10 K. W. H. and who used 18 K. W. H. from Apr. 2 to May 1, 1920, and paid within 10 days, using the prices quoted in the above bill.

(9) Find the amount paid by each of the following users of electricity if each paid his bill in time to receive the discount.*

	Primary Demand	Used
Mr. Brown	12 K. W. H.	16 K. W. H.
Mr. Swift	16 K. W. H.	18 K. W. H.
Mr. Weld	12 K. W. H.	17 K. W. H.
Mr. Hoover	15 K. W. H.	20 K. W. H.



One Cord

79. Wood and coal. A cord of wood is a pile of 4 foot sticks, piled 8 ft. long and 4 ft. high or any pile containing 128 cu. ft. (4×4×8 cu. ft. = 128 cu. ft.)

* Discount is frequently allowed, if the bill is paid within a specified time. The amount and method of reckoning varies with the type of user (houses, stores, etc.) and the city.

Coal is measured by the ton or bushel. There are 25 bushels in one ton. One ton contains 35 cu. ft. of hard coal.

Problems

(1) A pile of 4 foot wood piled 6 ft. high and 20 ft. long contains how many cu. ft.? How many cords in such a pile?

Solution: $\frac{4\times6\times20}{128}=3\frac{3}{4}$ number of cords. What is the cost of such a pile at $16.50 per cord?

Solution: $\frac{\overset{3}{\cancel{4}}\times\overset{5}{\cancel{6}}\times\overset{8.25}{\cancel{20}}\times\cancel{16.50}}{\cancel{128}\ \cancel{32}\ 8\ 4\ 2}=\$61.87\frac{1}{2}$ or $61.88.

(2) Find the cost of the following piles at prices stated:

a. 4 foot wood, piled 6 ft. high and 18 ft. long, at $ 8.50 a cord.
b. 4 foot wood, piled 8 ft. high and 24 ft. long, at $12.50 a cord.
c. 4 foot wood, piled 4 ft. high and 28 ft. long, at $13.50 a cord.
d. 4 foot wood, piled 10 ft. high and 20 ft. long, at $16.25 a cord.
e. 4 foot wood, piled 9 ft. high and 32 ft. long, at $18.00 a cord.

(3) If a cord of wood for the kitchen stove lasts a family 3 weeks (on the average) how much does the family pay for a year's supply when wood is $8.75 per cord?

(4) Which is cheaper, to buy stove-wood 16 in. long at $10.00 per cord, or to pay $8.00 per cord for cord wood and give a man $3.00 a cord to saw and split it into stove-wood?

(5) A pile of wood 12 ft. high contains 20 cords. How long is it?

(6) How many tons of coal will a coal shed 10 ft. long, 8 ft. wide, and 6 ft. high hold?

(7) A car 32 ft. long, 6 ft. wide, and 4 ft. deep will hold how many tons of coal?

CHAPTER XI

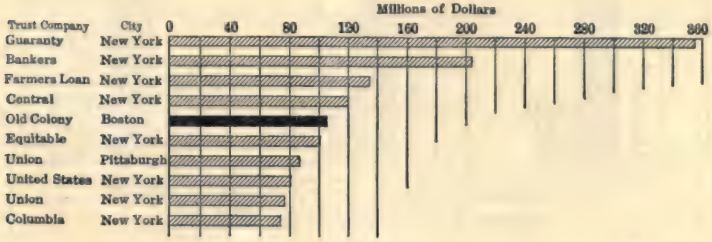
GRAPHS

80. Graphs. Statistical facts are often more clearly represented by pictures called graphs, than by columns of figures. The following is an example:

The length of the line in Fig. I shows the comparative amount of food value in ten cents' worth of each of these foods. How would this knowledge help to increase the family health, and decrease the amount of expenditure?

1. The following diagram taken from a report of the Old Colony Trust Co., Boston, Mass., shows the resources and location of the ten largest trust companies in the United

States, for the year 1915, in a much quicker and clearer way than would be shown by columns of figures. Each vertical line, one-eighth of an inch, represents twenty million dollars. Read the amount of resources of each trust company.



(1) What are the total resources of the ten trust companies?

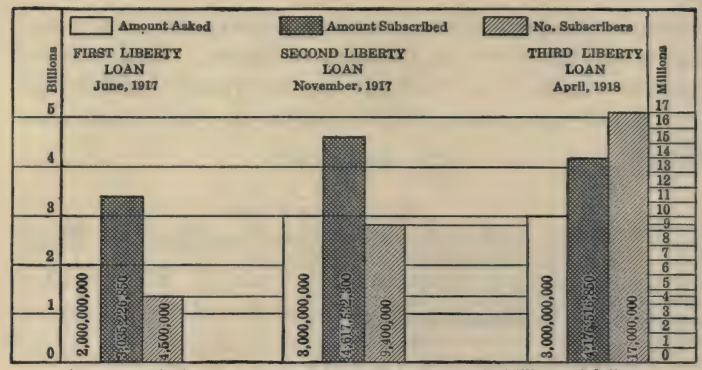
(2) How much does the Guaranty Trust Co. lead over all the others?

(3) What fractional part of these trust companies are located in New York? In Boston? In Pittsburgh?

(4) What are the total resources of the trust companies of New York?

(5) Six departments of a dry goods store produced the following profits. Show by a diagram or graph how the profits made by each department compare with one another:

Department	*Profit*
A	$3650.00
B	4280.00
C	1025.00
D	2160.00
E	6175.00
F	3480.00



Amounts asked and amounts subscribed are scaled in billions of dollars.
The number of subscribers is represented in units of millions.

Liberty Loan Subscriptions, 1917–18

3. From the above graph answer the following questions:

(1) What was the total amount of money subscribed for, in the First, Second, and Third Liberty Loans?

(2) What was the total number of subscribers?

(3) The first Liberty Loan was over-subscribed by how much money? By what per cent?

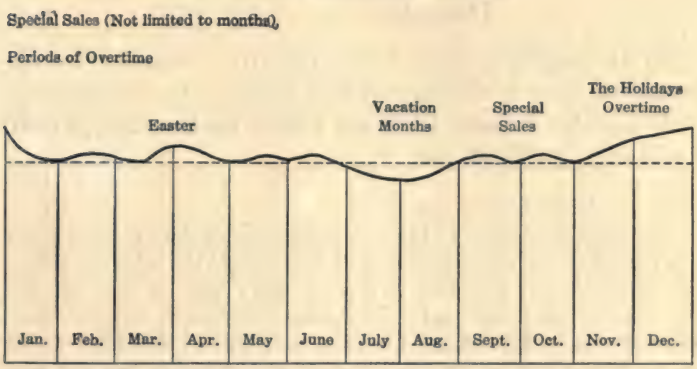
(4) What fractional part was our second Liberty Loan over-subscribed?

4. Illustrate graphically the increased cost of the following products, from 1915 to 1920.

	1915	1920
Eggs (per dozen)	\$.35	\$1.25
Butter (per pound)	.29	.75
Sugar (per pound)	.04½	.23
Milk (per quart)	.07	.17
Beef (per pound)	.35	.85
Lamb (per pound)	.30	.65
Oysters (per quart)	.35	.95
Oranges (per dozen)	.35	.60

5. The following diagram shows the seasonal and special fluctuations in the volume of trade more clearly than a table of statistics:

DIAGRAM OF SEASONAL CHANGES



The curved line represents approximately the usual temporary increase and decrease in the volume of business; the broken line, the normal or average trade.

(1) What month shows uniformity of trade? What are the busy days and the dull months?

(2) During what seasons is there the most trade? Why?

(3) Why is such a graph valuable to a merchant?

6. The graph on page 189 shows the variation in the monthly sales of a department store during the years 1915 and 1916; the dotted line indicates the sales for the year 1916 and the unbroken line indicates the sales for 1915, representing the following statistics:

Sales for January	$1524.
February	1295.
March	1362.
April	1495.
May	1248.
June	1169.
July	100.

Sales for August	$1050.
September	1675.
October	1610.
November	1575.
December	1725.

(1) In preparing data for comparative purposes the approximate sales in dollars only is necessary; thus the approximate sales for January 1916, are $1650; for February, $1550; for March, $1650.00, etc.

(2) Explanation of graph:

The sales for January, 1915, approximated $1500.00, so a point was placed on the January line near that figure; the sales for February approximated $1300.00, so a point was placed on that line and the two points connected. The sales for March approximated $1350.00, and a line extended to meet the point on that line. Other points were placed on the other lines and an unbroken line was then drawn connecting all the points. The dotted line was drawn in the same way to represent the statistics for 1916.

EXERCISE

(1) Answer the following questions from the graph on p. 189:

a. What were the busy months? The dull months?

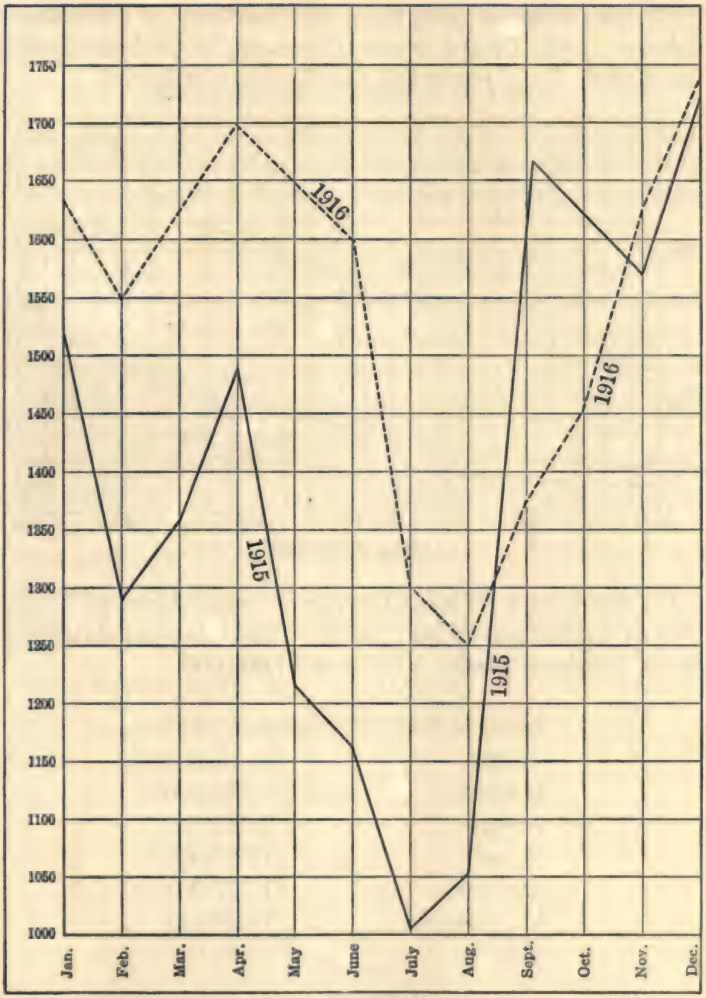
b. What were the busy months of each year? The dull months?

c. How would this knowledge help a merchant to run his store more economically?

d. During what months did this store have its largest trade for the two years?

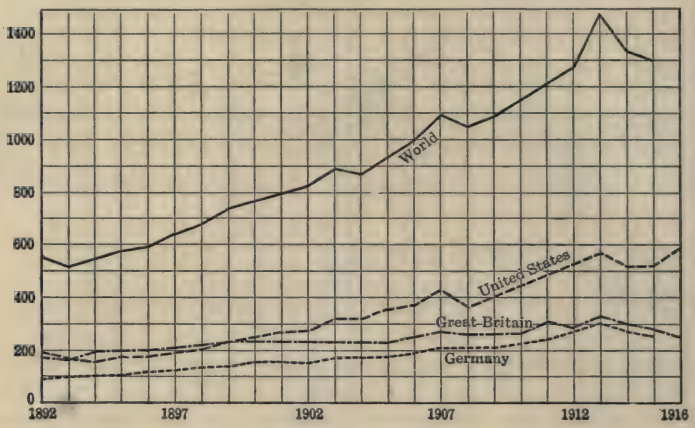
e. How did the business of 1916 compare with that of 1915?

f. How many more dollars' worth of goods were sold during September, October, November, and December of 1915 than were sold for the same months during the year 1916?



Monthly Sales

(7) The accompanying graph tells the story of the mineral industry in the United States, Germany, Great Britain, and the world. From the graph tell the story in figures.



Mineral Growth

(3) Show by a diagram the relative importance of Louisiana as a sugar producing state from the following statistics: Scale, $\frac{1}{4}$ inch represents a value of $1,000,000.

Value of Sugar Produced in 1909.

State	Dollars
Louisiana	$17,752,000
Georgia	2,268,000
Texas	1,669,000
Alabama	1,527,000
Mississippi	1,506,000
Florida	1,089,000

3. There are many different kinds of graphs, but the linear graph, illustrated on page 184, is the simplest and most used.

Graphs are used pictorially to impress startling facts upon the mind.

EXERCISE IN BUSINESS GRAPHS

(1) Make a linear graph showing the following comparative information: Comparison of exports to Europe during two years preceding the late war, and two years during the war:

	Before the War	*During the* War
Iron and Steel	$ 695,000,000	$1,435,000,000
Raw Cotton	1,332,000,000	1,131,000,000
Bread Stuffs	448,000,000	1,218,000,000
Explosives	14,000,000	992,000,000
Meats	367,000,000	636,000,000
Brass Manufactures	21,000,000	407,000,000
Chemicals	67,000,000	276,000,000
Cotton Manufactures	260,000,000	133,000,000
Automobiles for War	79,000,000	225,000,000
Horses and Mules	12,000,000	235,000,000
Sugar	153,000,000	600,000,000
Zinc	2,000,000	107,000,000
Woolen Manufactures	11,000,000	91,000,000

(2) Construct a pictorial graph from each of the following groups of figures:

a. Butter	Water	11.0
	Fat	85.0
	Protein	1.0
	Ash	3.0
b. Milk	Water	87
	Protein	3.4
	Carbohydrate	5
	Fat	4
	Ash	.6

c. English Walnuts	Water	2.5
	Protein	16.6
	Fat	63.4
	Carbohydrate	16.1
	Mineral Matter	1.4

(3) By means of properly proportioned outline, to suggest "skyscrapers," represent the relative amount of new building operations in different cities based on the following for 1911:

Baltimore	$ 9,325,000	Cleveland	$ 17,000,000
Buffalo	10,365,000	Detroit	19,000,000
Chicago	105,270,000	New York	173,500,000
Cincinnati	13,485,000	Pittsburgh	11,700,000

CHAPTER XII

PERCENTAGE AND ITS APPLICATION

How are losses and gains estimated in business houses?

A merchant sold a book-case that cost \$100, for \$99. How many dollars did he lose? What decimal part of the whole did he lose?

81. Percentage. Instead of saying one hundredth we may say 1 per cent, as "per cent" means "hundredths." Any per cent can be written as a fraction whose denominator is 100, or as a decimal, or with the sign (%), which may be read per cent or hundredths. Hence, .15 = 15 per cent; .25 = 25 per cent; $\frac{1}{2} = \frac{50}{100} = .50 = 50\%$.

The common fraction $\frac{1}{2}$ has first been changed to hundredths, next to its decimal form, and last to its per cent form. The per cent form and the decimal form are alike except that the decimal point is used instead of the per cent sign. Percentage problems and fractions are closely related.

EXERCISE

(1) Express each of the following in three ways:

$\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{5}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{50}$, 5%, 25%, $6\frac{1}{4}\%$, 20%, 80%, 75%, $8\frac{1}{3}\%$, $87\frac{1}{2}\%$, $62\frac{1}{2}\%$, 100%, 150%, 125%.

(2) What per cent of Fig. A is shaded? What per cent of Fig. A is not shaded?

(3) What fractional part of Fig. A is not shaded?

Fig. A

(4) If a square is divided into 100 small squares, and 10 of them are shaded, what per cent is shaded? What per cent is not shaded?

(5) Draw a circle and shade 50% of it. Shade 75% of it. What per cent is not shaded?

(6) If you have a dollar, and lose 5¢, what per cent have you lost? What per cent is left?

82. Definition of percentage. The process of computing by hundredths or per cents is percentage.

EXERCISE

(1) Express the following by using the per cent sign, or as per cents:

a. .25	*c.* .18	*e.* .5	*g.* $.33\frac{1}{3}$	*i.* .075
b. .625	*d.* $.37\frac{1}{2}$	*f.* .05	*h.* $.12\frac{1}{2}$	*j.* .75

k. 1.50 *m.* 2.5
l. 1.75 *n.* $.87\frac{1}{2}$

(2) Express as decimal fractions:

a. 5%	*d.* 75%	*g.* 70%
b. 8%	*e.* 150%	*h.* $37\frac{1}{2}\%$
c. 20%	*f.* 33%	

(3) Express as common fractions:

a. 8% *c.* 54% *e.* $62\frac{1}{2}\%$ *g.* $\frac{7}{8}\%$ *i.* $166\frac{2}{3}\%$ *k.* $\frac{1}{4}\%$ *m.* 40%
b. 1% *d.* 3% *f.* 250% *h.* $2\frac{1}{2}\%$ *j.* $187\frac{1}{2}\%$ *l.* $\frac{1}{5}\%$ *n.* 4%

(4) Express as per cents:

a. $\frac{1}{4}$	*c.* $\frac{1}{12}$	*e.* $\frac{5}{6}$	*g.* $\frac{2}{3}$	*i.* $1\frac{7}{9}$	*k.* $\frac{5}{4}$	*m.* $\frac{12}{3}$
b. $\frac{1}{8}$	*d.* $\frac{1}{6}$	*f.* $\frac{3}{5}$	*h.* $1\frac{1}{5}$	*j.* $1\frac{1}{2}$	*l.* $\frac{4}{5}$	*n.* $\frac{50}{2}$

The following table of important business fractions in per cent equivalents, can be used to great advantage in many store problems, and should be memorized. Compare values with business fractions on p. 153, § 65.

Per cent	Fractional Value	Per cent	Fractional Value	Per cent	Fractional value	Per cent	Fractional Value
100%	1 = the whole	$37\frac{1}{2}\%$	$\frac{3}{8} = .375$	$6\frac{2}{3}\%$	$\frac{1}{15} = .06\frac{2}{3}$	20%	$\frac{1}{5} = .20$
50%	$\frac{1}{2} = .50$	$62\frac{1}{2}\%$	$\frac{5}{8} = .625$	$33\frac{1}{3}\%$	$\frac{1}{3} = .33\frac{1}{3}$	40%	$\frac{2}{5} = .40$
25%	$\frac{1}{4} = .25$	$87\frac{1}{2}\%$	$\frac{7}{8} = .875$	$66\frac{2}{3}\%$	$\frac{2}{3} = .66\frac{2}{3}$	60%	$\frac{3}{5} = .60$
75%	$\frac{3}{4} = .75$	$6\frac{1}{4}\%$	$\frac{1}{16} = .0625$	$16\frac{2}{3}\%$	$\frac{1}{6} = .16\frac{2}{3}$	80%	$\frac{4}{5} = .80$
$12\frac{1}{2}\%$	$\frac{1}{8} = .125$	$8\frac{1}{3}\%$	$\frac{1}{12} = .08\frac{1}{3}$	$83\frac{1}{3}\%$	$\frac{5}{6} = .83\frac{1}{3}$	150%	$1\frac{1}{2} = 1.5$
						200%	2. = 2.

The three problems in percentage are:

1. Finding the percent of a number.
2. Finding what per cent one number is of another.
3. Finding the number when a certain per cent of it is known.

EXERCISE

(1) What is $\frac{1}{4}$ of 12; $\frac{3}{8}$ of 16; $\frac{5}{6}$ of 30; $\frac{5}{8}$ of 24; $\frac{3}{16}$ of 48. How do you find a fractional part of a number?

(2) What is $\frac{1}{100}$ of 500? $\frac{3}{100}$ of 600? 3% of 600? $\frac{15}{100}$ of 300? 15% of 300?

(3) Find .2 of 300; .02 of 300; 2% of 300; .04 of 600; 4% of 600%; .25 of 800; 25% of 800.

ILLUSTRATIVE EXAMPLE: What is 38% of $750?

Solution: 38% of a number equals .38 of it. .38 of $750 = $285, the required result.

$750
.38
$285.00

1. To find the per cent of a number, multiply the number by the rate per cent expressed decimally.

EXERCISE

(1) What aliquot part of 1 is .50? What is 50% of $2? Of $8? What is 25% of $2? Of $8? What is 75% of $2? Of $8?

(2) State a short method of finding 50% of a number; 25%; 50%; 75%.

(3) What aliquot part of 1 is .12½?

ILLUSTRATIVE EXAMPLE: What is 12½% of $624?

Solution: 12½% is instantly recognized as ⅛; ⅛ of $624 is $78. 12½% of $624 equals $78.

(4) What is 12½% of:
$84? $96? $108? $32? $72? $144?

(5) What is 37½% of each of the above amounts? 62½%? 87½%?

(6) State a short way of finding 12½% of a number; 37½%; 62½%; 87½%.

(7) What aliquot part of 1 is .33⅓? 66⅔? 16⅔?

(8) State a short way for finding 33⅓% of a number; 66⅔%; 16⅔%.

(9) What aliquot part of 1 is .08⅓? .06⅔? .06¼?

(10) State a short way of finding 8⅓% of a number; 6⅔%; 6¼%.

(11) What is 12½% of $396? 37½% 62½%? 87½%? 25%? 75%? 50%? 8⅓%? 16⅔%? 33⅓%? 66⅔%? 10%? 100%?

(12) *a.* Of $600, find: 100%; 50%; 25%; 75%; 12½%; 37½%; 62½%; 87½%; 16⅔%; 83⅓%; 6⅔%; 10%; 20%; 70%; 60%; 80%.

b. Of $360 find: 100%; 50%; 25%; 75%; 33⅓%; 66⅔%; 8⅓%; 16⅔%.

(13) By inspection, find:

a. 50% of $428.
b. 16⅔% of $960.
c. 12½% of $840.
d. 62½% of $320.
e. 6⅔% of $450.
f. 25% of $5072.
g. 87½% of $3248.
h. 6¼% of $4880.
i. 20% of $13,565.
j. 60% of $8400.
k. 8⅓% of $9684.
l. 66⅔% of $1848.

m. 25% of $861.40.
n. 37½% of $896.32.
o. 33⅓% of $912.54
p. 16⅔% of $1260.24.
q. 62½% of $896.32.
r. 66⅔% of $912.54.

(14) Give answers at sight. What per cent of 1 dozen is:

a. 12 buttons?	*e.* 16 buttons?	*i.* 15 buttons?
b. 6 buttons?	*f.* 3 buttons?	*j.* 8 buttons?
c. 9 buttons?	*g.* 1 button?	*k.* 2 buttons?
d. 4 buttons?	*h.* 18 buttons?	*l.* 24 buttons?

(15) What per cent of 1 hour is:

a. 60 minutes?	*e.* 5 minutes?
b. 30 minutes?	*f.* 20 minutes?
c. 15 minutes?	*g.* 40 minutes?
d. 10 minutes?	*h.* 80 minutes?

i. 75 minutes?
j. 120 minutes?
k. 150 minutes?
l. 90 minutes?

(16) What per cent of 1 yard is:

a. 36 inches?	*e.* 12 inches?	*i.* 30 inches?
b. 18 inches?	*f.* 24 inches?	*j.* 72 inches?
c. 9 inches?	*g.* 6 inches?	*k.* 54 inches?
d. $4\frac{1}{2}$ inches?	*h.* 3 inches?	*l.* 45 inches?

(17) What is 10% of \$840? Of \$36.60? Of 150 yd.?

(18) What aliquot part of 10% is 5%? $2\frac{1}{2}$%? $1\frac{1}{4}$%? $1\frac{2}{3}$%? $3\frac{1}{3}$%?

ILLUSTRATIVE EXAMPLE: What is $1\frac{1}{4}$% of sales amounting to \$320?

Solution: $1\frac{1}{4} = \frac{1}{8}$ of 10; hence, $1\frac{1}{4}$% of \$320 is $\frac{1}{8}$ of 10% of \$320; 10% of \$320 = \$32.00; $\frac{1}{8}$ of \$32 = \$4. Therefore, $1\frac{1}{4}$% of sales amounting to \$320, is \$4.00.

8)\$32.00
\$ 4.

2. To find $1\frac{1}{4}$% of a number, point off one place to the left, and divide by 8.

ORAL EXERCISE

(1) State a short way of finding 5% of a number; $2\frac{1}{2}\%$; $1\frac{2}{3}\%$; $3\frac{1}{3}\%$.

ILLUSTRATIVE EXAMPLE: What is $\frac{1}{2}\%$ of the sales amounting to \$350?

Solution: Since moving the decimal point two places to the left multiplies by .01 (1%), dividing the result by 2 gives $\frac{1}{2}\%$. Therefore, $\frac{1}{2}$ of 1% of \$350 is \$1.75:

$$2)\$3.50$$
$$\$1.75$$

(2) State a short method of finding 1%; $\frac{1}{2}\%$.

(3) Find by inspection:

a. $1\frac{1}{4}\%$ of \$1264
b. 5% of \$3416
c. $1\frac{2}{3}\%$ of \$486
d. $2\frac{1}{2}\%$ of \$236
e. $3\frac{1}{3}\%$ of \$936
f. $3\frac{1}{3}\%$ of 240 yd.
g. $1\frac{2}{3}\%$ of 246 yd.
h. 5% of 480 yd.
i. $1\frac{1}{4}\%$ of 432 yd.

(4) Find $166\frac{2}{3}\%$ of \$4284; of \$3660; of \$7284.

ILLUSTRATIVE EXAMPLE: What is $166\frac{2}{3}\%$ of \$1260?

Solution: $166\frac{2}{3}\% = 1.66\frac{2}{3}$ or $1\frac{2}{3}$; $1\frac{2}{3} = \frac{1}{6}$ of 10; $\frac{1}{6}$ of \$12,600 ($10 \times \$1260$) = \$2100; $166\frac{2}{3}\%$ of \$1260 = \$2100.

(5) State a short method of finding $166\frac{2}{3}\%$ of a number; $33\frac{1}{3}\%$; 150%; 125%; 250%.

(6) Compare 5% of \$60 with 60% of \$5; 5% of \$6000 with 60% of \$500. What is 40% of \$60? Explanation: 40% of \$60 = 60% of \$40 = $\frac{2}{3}$ of \$40 = \$24. Therefore, 40% of \$60 = \$24.

(7) Compare 16% of 12,500 yd. with 125% of 1600 yd.

(8) Find 48% of \$125; of \$1250; of \$12,500; of \$125,000.

(9) Find 250% of \$540; of \$5400; of \$24,600; of \$32,100.

(10) By inspection find:

a. 15% of \$600.
b. 24% of \$200.
c. 20% of \$2500.
d. 120% of 1250 yd.
e. 140% of 1750 yd.
f. 125% of \$256.60.

Problems

(1) A merchant sold 480 yd. of cloth in two lots; he received \$1.25 a yard for $62\frac{1}{2}\%$ of it and 98¢ a yard for the remainder. What was the total amount received?

(2) There are 2864 employees in a store; $62\frac{1}{2}\%$ of them are women, the rest are men. How many women are employed?

(3) The cost of printing a book was \$320; 25% of that amount was spent for labor; $37\frac{1}{2}\%$ of the remainder for paper; and the rest for other expenses. How much was spent for other expenses?

(4) A school which had 640 pupils, now has 125% as many. How many pupils has the school now?

(5) A clerk earns \$1260 a year and spends 68% of it. How much does he spend? How much does he save?

(6) A merchant bought 1272 yd. of cloth; $62\frac{1}{2}\%$ of it was damaged. How many yards were salable?

(7) A dealer sold 56% of his flock of 6700 sheep. How many sheep had he left?

(8) If 16% of a man's wages is \$3.20, what are his wages?

(9) A clerk's salary was increased to \$16 a week. If his former salary was $87\frac{1}{2}\%$ of this amount, how much had be been receiving?

(10) A salesgirl received a salary of \$72 a month and paid $62\frac{1}{2}\%$ of it for a suit. What amount of money had she left?

Finding the Per Cent

(1) 6 is what part of 36? What per cent of 36?

(2) 80 is what per cent of 320? of 400? of 560?

(3) 40 is what per cent of 160? of 200? of 360?

(4) A merchant pays \$3 for shoes and sells them for \$4. How much money does he gain? \$1 is what part of \$3? What per cent? What per cent did the merchant gain?

(5) If you receive \$5 a week and spend $\$2\frac{1}{2}$, what part of your wages do you spend? What per cent? What per cent of 5 is $2\frac{1}{2}$?

(6) An article cost \$3 and was sold for \$2. How much money was lost? What part was lost? What per cent?

(7) 36 is what part of 144? What per cent of 144?

3. To find what per cent one number is of another, first find the fractional part that number is of the other; then change that fraction to hundredths, or per cent.

ILLUSTRATIVE EXAMPLE: What per cent of $231.20 is $46.24?

Solution: *a.* $46.24 is $\frac{4624}{23120}$ or $\frac{46.24}{231.20}=\frac{1}{5}=20\%$

b.
$$231.20\overline{)46.24.}\quad .20 \text{ or } 20\%$$

Divide the percentage given by the amount to get the number of hundredths, or per cent.

EXERCISE

(1) The following midsummer reduction sale was announced in a large shoe store. Find the per cent of reduction on each article of merchandise:

White washable kid lace boots, formerly $10—now $5.00.

White buckskin pumps, formerly $8—now $6.

White canvas pumps, formerly $5.50—now $4.50.

Beaver brown kid lace boots, formerly $10—now $8.

Patent leather calf pumps, formerly $6.00 and $8—now $3 and $4.

Colonial slippers, formerly $5.00 to $7.00—now $2.50, $3, and $3.50.

Tan calf Oxfords, formerly $5.50 and $6.50—now $4 and $5.

Tan or black Oxfords, reduced from $8 to $6.50.

Vici kid Oxfords, reduced from $5.00 to $3.50.

Palm Beach cloth Oxfords, reduced from $4.50 to $3.

Canvas tennis shoes reduced from $4.00 to $3.25.

(2) The following "After Stock-Taking" sale was advertised. What per cent was each garment reduced?

Evening gowns, formerly $45.00—now $15.00.

Evening gowns, formerly $95.00—now $25.00.

Odd silk dresses, formerly \$18.00—now \$10.00.
Beach dresses, formerly \$10.00—now \$5.00.
Knitted cloth suits, formerly \$25.00—now \$15.00.
Cloth suits, formerly \$35.00—now \$18.00.
Linen suits, formerly \$18.00—now \$10.00.
Odd cloth coats, formerly \$45.00—now \$18.00.
Dust coats, formerly \$13.50—now \$10.00.
Odd skirts, formerly \$5.00—now \$2.00.

(3) Find the per cent of reduction on the following merchandise:

\$1.25 night dresses, marked down to 75¢.
\$1.00 night dresses, marked down to 75¢.
\$10.00 night dresses, marked down to \$5.00.
\$1.50 petticoats, reduced to \$1.00.
75¢ union suits, reduced to 59¢.
\$1.25 union suits, reduced to \$1.00.
\$3.50 and \$5.00 combinations, reduced to \$2 and \$2.50.
75¢ corset covers, reduced to 60¢.
\$1.00 shirt waist slips, reduced to 50¢.
\$3.50 shirt waist slips, reduced to \$2.25.

(4) A merchant bought goods for \$7500, and sold them for \$8700. What per cent did he gain?

(5) A salesgirl lost 15 days' pay, on account of illness, from a year of 300 working days. What per cent of the year did she work?

(6) A commission agent sold 35,400 yd. of cloth at 75¢, and received \$796.50 as his share of the profits; what per cent did he receive on the sales made?

(7) An agent earned \$170.04 by selling goods to the amount of \$2834. What per cent did he receive on the sales made?

(8) A salesgirl receives \$1.28 as a commission on one day's sales amounting to \$256. What per cent of her sales did she receive?

Finding the Number When a Certain Per Cent is Known

(1) If 4 pairs of shoes cost \$16, how much will 150 cost?

(2) If 4 per cent of a number is 16, what is 100 per cent of the number?

(3) If 8 yd. cloth cost \$24, how much will 100 yd. cost?

(4) If 8 per cent of a number is 24, how much is 100 per cent of the number?

(5) If 5 per cent of a number is 25, what is 1 per cent? 10 per cent? 100 per cent? What is the number?

ILLUSTRATIVE EXAMPLE: If 12% of a number is \$3648.60, what is the number?

Solution: 12% of the number is \$3648.60.
1% of the number is $\frac{1}{12}$ of 3648.60 = \$304.05.
100% of the number is 100 × \$304.05 = \$30405.
Therefore, \$30405 is the number required.
Check: 12% of \$30405 = \$3648.60.

4. To find a number when a certain per cent of it is known, divide the percentage given by the rate expressed decimally, or in its simplest form.

WRITTEN EXERCISE

(1) If it takes 25% of a clerk's salary to pay his rent, which is \$18 a month, what is his monthly salary?

(2) A manufacturer received 420 lb. of yarn, which was 7% of his order. How much yarn did he order?

(3) A dealer sold 60% of his pieces of broadcloth and had 560 pieces left. How many pieces of broadcloth had he at first?

(4) If \$56 is $87\frac{1}{2}$% of my wages, what are my wages?

(5) A dealer sells his goods for \$581 which is $12\frac{1}{2}$% less than he paid for them. What was the cost of the goods?

(6) A merchant made a profit of \$3245 from his business,

which was $37\frac{1}{2}\%$ of the cost of his goods. What was the cost of his goods?

(7) A dealer sells \$1005 worth of goods today, which is $62\frac{1}{2}\%$ of what he sold the day before. What was the amount of his sales the previous day?

(8) Sold goods so as to make \$360, which was a gain of $12\frac{1}{2}\%$. What was the cost? The selling price?

(9) If a dealer sells goods for \$256 and loses 8%, what did the goods cost?

(10) A dealer sold a suit of clothes for \$24 and lost $16\frac{2}{3}\%$; he sold another at a profit of $37\frac{1}{2}\%$ and gained as much as he had lost on the first. What was the cost of each suit?

Per Cents of Increase

(1) If 5 times a number is 30, what is the number?
(2) If $2\frac{1}{2}$ times a number is 30, what is the number?
(3) If 2.5 times a number is 30, what is the number?
(4) If 250% of a number is 3000, what is the number?
(5) If 250% of a number is 500, what is the number?
(6) If 250% of a number is 150, what is the number?
(7) If 200% of a number is 1800, what is the number?

Illustrative Example (1): A dealer sold goods for \$5500, and gained thereby $37\frac{1}{2}\%$. How much did the goods cost?

Solution: $1.37\frac{1}{2} \times$ the cost = \$5500. The cost = \$5500 ÷ $1.37\frac{1}{2}$ ($\frac{11}{8}$) = \$4000, cost.

Operation:

$1.37\frac{1}{2} = 1\frac{3}{8} = \frac{11}{8}$

\$5500 = $\frac{11}{8}$; $\frac{1}{8} = \frac{1}{11}$ of \$5500 = \$500

$\frac{8}{8} = 8 \times \$500 = \$4000$

Illustrative Example (1): What number increased by $83\frac{1}{3}\%$ of itself equals 1265?

Solution: 100% of the number = the number.
$83\frac{1}{3}\%$ or $\frac{5}{6}$ of the number = the increase.
$183\frac{1}{3}\%$ or $1\frac{5}{6}$ or $\frac{11}{6}$ of the number = 1265.
Therefore the number is $1265 \div \frac{11}{6} = 690$.

ORAL EXERCISE

What number increased by:

(1) 5% of itself is 360?
(2) $1\frac{1}{2}\%$ of itself is 450?
(3) 20% of itself is 420?
(4) $6\frac{2}{3}\%$ of itself is 600?
(5) $6\frac{1}{4}\%$ of itself is 375?
(6) $12\frac{1}{2}\%$ of itself is 450?
(7) 50% of itself is 3600?
(8) 75% of itself is 3500?
(9) $62\frac{1}{2}\%$ of itself is 2600?
(10) $66\frac{2}{3}\%$ of itself is 2500?

Problems

(1) A dealer sold 426 yd. of cloth for $497.00, thereby gaining $16\frac{2}{3}\%$. What did the cloth cost per yard?

(2) A dealer sold a quantity of buttons for $12.36. If he gained $12\frac{1}{2}\%$, what did the buttons cost?

(3) A merchant sold a quantity of silk for $258.75, thereby gaining $3\frac{1}{3}\%$. What did the silk cost him?

Per Cents of Decrease

(1) If you have $1 and spend 20¢, what fractional part of $1 have you spent? What fractional part have you left? What per cent have you spent? What per cent have you left?

(2) Mr. Smith spent $33\frac{1}{3}\%$ of his money and had $10 left. How much had he at first? How much did he spend?

Illustrative Example (1): A dealer sold a rug for $380, thereby losing 24%. What was the cost?

Solution: 100%—24% = 76%. 76% of the cost = $380.
The cost = $380 \div 76 = \$500$.

Illustrative Example (2): What number decreased by $33\frac{1}{3}\%$ of itself equals $560?

Solution: $\frac{2}{3}$ of the number = $560. The number equals

$$560 \div \frac{2}{3} = \frac{\overset{280}{\cancel{560}} \times 3}{\cancel{2}} = \$840.$$

ORAL EXERCISE

What number diminished by:

(1) 25% of itself equals 96?
(2) $62\frac{1}{2}\%$ of itself equals 318?
(3) 75% of itself equals \$60?
(4) $\frac{1}{3}$ of itself equals \$240?
(5) $\frac{1}{6}$ of itself equals 30.5?
(6) 1% of itself equals 36.9?

WRITTEN EXERCISE

(1) A dealer sold 900 suits for \$26,775 which was $16\frac{2}{3}\%$ less than he paid for them. What was the cost per suit?

(2) In selling a piano for \$660 a dealer lost 25% on the cost. If the piano was marked to gain 25%, what was the asking price?

(3) A dealer sold a quantity of shoes at \$4.50 a pair and lost $33\frac{1}{3}\%$ by so doing. If he paid \$162 for the shoes, how many pairs did he buy?

ORAL REVIEW

(1) By inspection, find $6\frac{1}{4}\%$ of the following numbers:

a. \$320	*d*. \$1632	*g*. \$9616
b. 480 dozen	*e*. \$6480	*h*. \$2464
c. 640 chairs	*f*. \$32,108	*i*. \$2176

(2) By inspection, find 10% of each of the above numbers: 25%, 125%, $12\frac{1}{2}\%$, $37\frac{1}{2}\%$

(3) By inspection, find 10% of each of the following:

a. \$560	*d*. \$4225
b. \$2430	*e*. \$5080
c. \$565	*f*. \$2957

(4) What per cent is gained when goods are bought for:

a. \$7 and sold for $\$8\frac{1}{2}$?
b. \$25 and sold for \$50?
c. \$1.50 and sold for \$1.75?

d. \$300 and sold for \$575?
e. \$40 and sold for \$50?
f. \$120 and sold for \$170?
g. \$54 and sold for \$60?
h. \$1300 and sold for \$1469?

(5) What per cent is lost when goods are bought for:
a. \$35 and sold for \$32.50?
b. \$24 and sold for \$22?
c. \$200 and sold for \$140?
d. \$147 and sold for \$72?
e. \$35.00 and sold for \$28.00?

(6) Gain, \$3.75, which is 20%. Find cost.
Loss, \$15.75, which is 20%. Find cost.
Loss, \$1, which is 3%. Find cost.
Gain, \$150, which is 25%. Find cost.

(7) Cost, \$64; selling price, \$72. Find per cent of gain.

(8) What number diminished by:
a. 75% of itself equals 150?
b. 60% of itself equals 320?
c. $12\frac{1}{2}\%$ of itself equals 560?
d. 50% of itself equals 175?
e. 25% of itself equals 909?
f. 10% of itself equals 125?
g. $16\frac{2}{3}\%$ of itself equals 630?

(9) If muslin shrinks $2\frac{1}{2}\%$ of its length, how many yards must a person buy in order that there may be 130 yd. after shrinking?

(10) If you spend 8 hours each day at work, what per cent of 24 hours do you work?

(11) If a clerk had \$72.50 in the savings bank on Jan. 1, and increased this by 30% during the year, what sum would he have in the bank at the end of the year?

(12) By paying my gas bill of \$4.46 before the 10th of the month, I got a reduction of 5%. How much did I save?

(13) If you receive a 25% increase in your salary at the end of the first year; a 20% increase at the end of the second year; and a 25% increase at the end of the third year, what did you receive at first, if your salary at the end of the third year was $1200?

(14) Mr. Finn lost 15% on an investment. If the sum returned to him was $956.25, what was the investment?

(15) A piece of unfinished cloth cost 4¢ a yard. It costs one-half a cent per yard to bleach it, and then it is sold for $7\frac{1}{2}$¢ a yard. The selling price is what per cent above the total cost?

(16) A dealer paid the following bills less the discounts named: $56.50 less 3%; $121.75 less $1\frac{1}{2}$%; $124.32 less 1%; $24.36 less $2\frac{1}{2}$%. What was the total discount allowed?

Some interesting figures on what it cost to outfit a young woman in 1914 as compared with 1919 are afforded by quotations from a number of averaged priced local shops, as follows:

	1914	1919
Shoes	$5.00	$12 to $18
Stockings (silk)	1.25 to 1.50	3.50 and up
Underwear	3.50 to 5.00	5.00 and up
Silk bloomers	2.50 a pair	6.50 a pair
Undervest	1.00 to 1.50	3.50 to 4.00
Petticoat (silk)	4.00 to 5.00	12.00 to 15.00
Waist (georgette)	10.00 to 15	15.00 and up
Suit (woollen)	75.00	95.00 and up
Hat	5.00	10.00 and up
Fur neckpiece	35.00 to 50	60.00 and up
Muff	50.00	75.00 and up
Gloves	1.50	4.00 and up

(17) From the above figures find the total cost of the outfit for a girl in 1914. In 1919.

(18) Figure out the per cent of increase in the cost quoted, in 5 years.

(19) Give as many reasons as you can for this increase in the cost of clothing.

(20) Find out the per cents of increase in the following articles:

	1914	1919
Serge	$1.00 to $1.25	$4.00 to $5.00
Velveteens	1.00 to 1.25	4.50 to 6.50
Babies' buckskin shoes	3.50	4.50 to 6.00

(21) Find the increase and the per cent of increase in the following imports for 1919–1920.

	1919	1920
Silk fabrics	$29,264,585	$35,514,422
Handkerchiefs	1,651,159	1,974,565
Laces and embroideries	2,762,108	5,170,057
Ribbons	182,052	1,064,158
Wearing apparel	4,266,212	6,235,104
Dolls	1,232,344	2,839,532
Veils and veilings	2,299,066	2,361,919
Knitted cotton goods	1,638,534	3,445,460
Women's gloves (doz. pr.)	387,030	861,411
Men's gloves (doz. pr.)	5,185	29,151

(22) What is the decrease and the per cent of decrease in the following price quotations on women's shoes.

	November, 1919		January, 1921	
	High shoes	Low shoes	High shoes	Low shoes
Black calf	$10.00	$9.00	$8.25	$7.25
Russian calf	10.50	9.25	8.50	7.75
Black leather	6.50	6.10	5.50	5.25
Black kid	10.00	9.00	8.50	7.75
Russian kid	11.50	10.00	9.50	8.50

(23) The following table shows the amount of the sales in several of the departments of a large store for the month of October, 1919, and the month of October, 1920.

a. Find the increase or decrease in the amount of the sales for each department for October, 1919, over the sales in the same department for October, 1920.

b. Find the per cent increase or decrease for the store.

COMPARATIVE SALES RECORD

Dept. No.	October, 1919	October, 1920	Increase or Decrease	
1	$ 9659 75	$ 9237 45		
2	15326 42	16758 34		
3	8957 56	7095 48		
4	11324 38	9276 15		
5	19257 45	17165 98		
6	32854 60	29275 49		
7	9745 84	11856 16		
8	10678 53	13724 98		
9	12157 49	15352 12		
10	18324 65	17165 49		
11	27518 34	26028 50		
12	21157 49	31257 61		

c. Find the gross profit of each department; the gross profit of the entire store; the net profit or net loss by departments; the net profit or net loss of the entire store, from the following table: (Gross and net profit, p. 213, § 8.)

Dept. No.	Sales	Cost of Goods Sold	Gross Profit	Expenses	Net Profit or Loss
1	$6873.45	$6054.98		$532.45	
2	8937.28	7583.57		618.98	
3	9546.72	8978.95		748.69	
4	7876.35	6976.42		542.38	
5	9098.24	8932.54		675.94	
6	5739.98	5278.61		456.58	
7	8976.54	8097.37		532.27	
8	7890.12	7684.25		815.49	
9	8305.26	6068.13		675.34	
10	9765.84	7352.16		426.58	

5. A customer buying goods from a merchant usually receives a bill or invoice which states:

(1) The date of the sale.

(2) The names and addresses of persons purchasing the goods.

(3) The name and address of the one from whom the goods are bought.

(4) The terms of the sale.

(5) A detailed list of all the articles sold; the price of each item; the total amount of the purchase or invoice.

(6) The amount of expressage or freight if this is to be paid by the customer; this is added to the invoice.

(7) A statement of the receipt of payment is made by the seller of the goods, if the invoice has been paid.

6. A customer receiving the goods and the invoice:

(1) Sees that the merchandise agrees with the items for which he has been charged on the invoice or bill.

(2) Checks each item in the column at the left if the correct kind and quantity of stock is received.

(3) Inspects and checks to see if the correct prices have been charged.

(4) Inspects to see if any error has been made in the cost of each item or in the total of the invoice.

a. Find what per cent of the suits purchased from the following firms was sold:

Manufacturers	Number Bought	Number Sold	Per Cent Sold
Walker Bros.	516	387	
Rolfe & Sturgis	376	329	
Franklin & Co.	426	355	
Brooks Bros.	852	368	
Moore & Sons	348	319	
E. A. Coon & Co.	825	726	
Jones, Mayo & Co.	1264	1106	
L. F. DeMerritt	896	560	

b. Rule a blank similar to the store record (page 212) of a week's sales in four departments. Enter the amounts on the blank of (1) the total sales for each day made in all de-

partments in the "total column" at the right; (2) the total sales made during the week in each department, entering totals on the line at the bottom marked "total"; (3) the total sales made in all departments for the entire week, entering in space marked "grand total;" (4) find what per cent of the grand total sales was made each day and enter these per cents in the column at the right; (5) find what per cent of the grand total sales was made in each department and enter on the line at the bottom marked "per cent."

Day	Dept. I	Dept. II	Dept. III	Dept. IV	Total	Per Cent
Monday	$560.25	$650.32	$712.65	$324.12		
Tuesday	328.96	398.75	658.32	217.85		
Wednesday	497.57	512.16	824.19	164.19		
Thursday	516.52	487.25	567.35	223.15		
Friday	824.13	756.42	498.16	165.18		
Saturday	568.97	560.98	875.15	196.54		
Total					Grand Total	
Per Cent						

(5) Find the total cost to our government of each of the following great wars, and figure out the per cents of increase or decrease in the cost of each successive war over the preceding one.

Estimated cost of the War of 1812,	$133,703,880
" " " " Mexican War	63,605,621
" " " " Civil War,	3,500,000,000
" " " " World War	24,010,000,000

7. One of the uses which business makes of per cents of increase and decrease is shown by the following table comparing the business for a certain term of the previous year with the corresponding period of the present year—thus:

COMPARATIVE SALES SHEET

Dept. No.	Sales Dec. 1919	Sales Dec. 1920	Increase or Decrease	Per Cent Increase or Decrease
10	$4532 87	$5278 65	$	
11	6057 18	7349 25		
12	5348 20	6527 89		
13	12854 30	12950 72		
14	8408 15	7859 45		
15	5490 12	7256 18		
16	11385 75	12730 95		
17	7698 40	9078 10		
18	7258 36	10283 12		
19	78639 00	81720 00		
20	4975 50	4850 00		
Total				

(1) Find the increase in business in department 10. What was the per cent of increase?

(2) Complete the above table.

(3) What advantage to the merchant is this comparison of sales?

8. The difference between the gross sales and the returned sales equals the net sales. The difference between the net sales and the cost of the goods sold, is the gross profit. The difference between the net sales and the gross profit equals the cost of the goods sold. The difference between the gross profits and the expenses equals the net profit. The percent of net profit on sales is found by dividing the net profit by the net sales.

Problems

(1) The purchases for one month in one department of a store were $7365.42. Some of these goods were imperfect and were returned; the value of the returned goods was $165.98. What per cent was this of the purchases?

(2) Gross sales for August, 1919, were $13,987.98; returned sales amounted to $156.25. This is what per cent of the gross sales for the month?

(3) Gross sales for August, 1920, were $16,315.98; returned sales were $125.16. What per cent of the sales were returned in August, 1920?

(4) What was the increase in the gross sales, and what was the per cent of increase?

(5) What were the net sales for August, 1919? For August, 1920? What was the increase in the net sales, and the per cent of increase?

(6) Merchandise on hand August 1, 1920, was worth $18,212.50 at selling prices. What per cent of the stock on hand August 1, was sold during that month?

(7) The gross profit for August, 1919, was $3,377.08. This is what per cent of the net sales for that month?

(8) The gross profit for August, 1920, was $3,799.21. This is what per cent of the net sales for that month?

(9) The expenses of the department for August, 1919, were $975.50. The expenses for August, 1920, were $1,068.75. What was the per cent of increase in the expenses?

(10) What was the cost of the goods sold in August, 1919? What was the cost of the goods sold in August, 1920?

(11) What was the net profit for August, 1919? What was the net profit for August, 1920?

(12) What was the per cent of net profit for August, 1919? What was the per cent of net profit for August, 1920?

CHAPTER XIII

COMMERCIAL DISCOUNTS

1. What advantage is it for an employee to buy goods in the store where he works? What is a deduction from the regular price called?

2. Name other classes of customers who receive discounts. Can you see a good reason for this?

3. Why are discounts given? Discounts induce trade. How?

4. Discounts encourage buying in large quantities. How?

5. Discounts save expense. To whom? How?

6. State all the reasons you can why discounts are an advantage to the business house, to the employee, or to the customer.

83. Discounts. Manufacturers and dealers in merchandise have a fixed or catalog price of goods, and make deductions from the price listed in the catalog; as these catalogs of goods and prices are often expensive, it is more economical to print a short list of discounts than to issue a new catalog when the prices of goods change.

84. List price. The list price of an article is the price of the article as it appears in the catalog.

85. Trade discounts. A commercial or trade discount is a deduction from the list price of goods. A large purchaser gets better prices and terms than small purchasers. An average customer might be asked the list price, less a trade discount of 25%, while large buyers might be asked the list price less trade discounts of 25%, 10%, and 20%. Trade discount rates are usually aliquot parts of 100%. When two or more discounts are quoted, the first one de-

notes a discount off the list price, the second one a discount off the remainder, and so on.

86. Net price. The net price is the price of the article after the discount has been made.

ILLUSTRATIVE EXAMPLE (1): The list price of 6 coats is \$90. If this price is subject to a series of discounts of 20% and 25% and 10%, what is the selling price?

Solution: 20%, or $\frac{1}{5}$ of \$90 = \$18, the first discount.
\$90 − \$18 = \$72, the price after the first discount.
25%, or $\frac{1}{4}$ of \$72 = \$18, the second discount.
\$72 − \$18 = \$54, the price after the second discount.
10%, or $\frac{1}{10}$ of \$54 = \$5.40, the third discount.
\$54 − \$5.40 = \$48.60, the net selling price.

The work is sometimes shortened by changing the order in which the discounts are subtracted. The net amount is the same.

ILLUSTRATIVE EXAMPLE (2): Goods listed at \$650 were sold to a dealer at a 15% discount. What was the net price?

Operation (1)

\$650		\$650	
× .15		− 97.50	
\$97.50	= the discount	\$552.50	= net price

Operation (2)

\$650
× .85
\$552.50 = net price.

Problems

(1) Dolls selling at \$2.00 during the Christmas holidays, were offered during a January sale at a 25% discount. What was the net price? For dolls whose regular price was \$4.75? \$3.50? \$3.96?

(2) By paying my gas bill of \$3.96 before the tenth of the month, a discount of 10% was allowed. What did I pay?

Find the net price of the following:

	List Price	Discount	Net Price		List Price	Discount	Net Price
1	\$60	$16\frac{2}{3}\%$	?	9	\$298	10%	?
2	80	$12\frac{1}{2}\%$	?	10	500	40%	?
3	56	$12\frac{1}{2}\%$	?	11	369	$66\frac{2}{3}\%$	?
4	960	$8\frac{1}{3}\%$	?	12	824	$37\frac{1}{2}\%$	?
5	450	20%	?	13	1000	30%	?
6	240	$33\frac{1}{3}\%$	?	14	640	$6\frac{1}{4}\%$	?
7	640	25%	?	15	1080	$62\frac{1}{2}\%$	?
8	720	$8\frac{1}{3}\%$	?	16	3060	$16\frac{2}{3}\%$	?

(3) Find the net price:

List price	*Trade discount*	*Net price*
a. \$24	$12\frac{1}{2}\%$ and 10%	?
b. \$90	$33\frac{1}{3}\%$ and 20%	?
c. \$75	20% and 25%	?
d. \$100	25% and $33\frac{1}{3}\%$	?
e. \$200	10% and 20%	?
f. \$500	20%, $12\frac{1}{2}\%$, and $8\frac{1}{3}\%$	?
g. \$800	25%, 20%, and 5%	?
h. \$1000	10%, 10%, and $12\frac{1}{2}\%$	?

(4) A rug listed at \$1200 is sold less $33\frac{1}{3}\%$, 20%, and 10%. What is the net cost to the customer?

(5) What is the selling price per dozen hats, listed at \$48 per dozen and discounted $12\frac{1}{2}\%$ and 10%?

(6) A customer bought a \$48.00 bureau for \$36.00 because it was slightly damaged. What was the rate of discount?

(7) After 1200 lb. wool was scoured it weighed 800 lb. What per cent was lost?

(8) Another lot weighing 1500 lb. weighed 1225 lb. after scouring. What per cent was lost?

(9) A messenger boy received \$2.75, which is $16\frac{2}{3}\%$ more than he received last year. What did he receive last year?

(10) The weekly salary of a clerk is \$16.24. He spent $12\frac{1}{2}\%$ of it for room rent, and $37\frac{1}{2}\%$ of it for board. What does he pay for rent? For board? What has he left?

(11) Suits marked \$12, \$24, \$48, \$32, \$108, \$96, were reduced 25%. What was the selling price of each after the reduction?

(12) If you sleep 9 hours a day, what per cent of your time do you sleep?

87. Short method of finding a single discount equivalent to a discount series.

Illustrative Example: (1) What single rate of discount is equivalent to a discount series of 10% and $16\frac{2}{3}\%$?

(2) What single rate of discount is equivalent to a discount series of 10%, $16\frac{2}{3}\%$, and 25%?

Solution (1)
$.10 + .16\frac{2}{3} = .26\frac{2}{3}$
$.10 \times .16\frac{2}{3} = .01\frac{2}{3}$
Single discount = .25 or 25%, their difference.

Solution (2)
$.25 + .25 = .50$
$.25 \times .25 = .0625$
Single discount = .4375 or $43\frac{3}{4}\%$

1. A short method of finding a single discount equivalent to two discounts:

Subtract the product of the two discounts from their sum, and the remainder will be the equivalent single discount.

2. When a discount series consists of three separate rates, the first two may be combined as in (1), and then the

result and the third discount may be combined in the same way.

ORAL EXERCISE

State a single rate of discount equivalent to a discount series of:

(1) 25% and 20%
(2) 20% and 10%
(3) 25% and 5%
(4) 25% and $33\frac{1}{3}\%$
(5) 20% and 15%
(6) $8\frac{1}{3}\%$ and 10%

Problems

(1) Which is better for the buyer: to have a discount of 40% upon a bill of \$200 worth of goods, or to have successive discounts of 25%, 10%, and 5%?

(2) What is the difference?

(3) What single discount is equivalent to the two successive discounts of $12\frac{1}{2}\%$ and $8\frac{1}{3}\%$?

(4) A merchant is offered discounts of 20% and 20% by one dealer, 30% and 10% by another. Which is the better offer and what difference will there be on a bill amounting to \$600?

(5) A dealer buys desks listed at \$48 each with discounts of 50% and 10%, with a further discount of 2% for cash. What is the net cost?

(6) Find the net price of each of the following:

Selling Price	*Trade Discounts*	*Selling Price*	*Trade Discounts*
a. \$2860	$33\frac{1}{3}\%$ and 25%	*d.* \$3612	$33\frac{1}{3}\%$, 20%, 10%
b. 4872	25%, 20%, 10%	*e.* 4836	$16\frac{2}{3}\%$, 10%, 10%
c. 1248	25%, $16\frac{2}{3}\%$, 10%	*f.* 2416	$12\frac{1}{2}\%$, 10%, 5%

ORAL EXERCISE

(1) A merchant buys goods on 30 da. time or 5% for cash; if his bill is \$85.00, what cash payment will settle the bill?

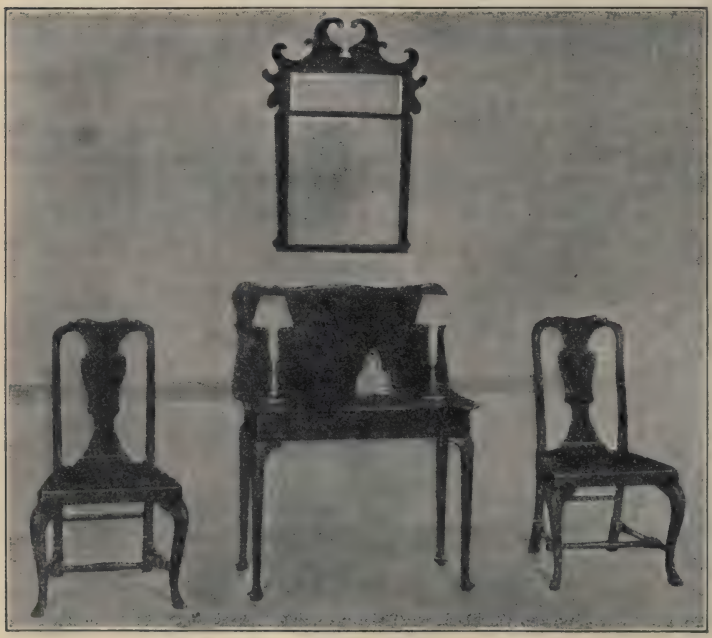
(2) I owe \$800.00 due in 10 da. If I pay cash today I will receive 5% discount. What cash payment will settle the bill?

(3) What is the net amount of a bill of \$720.00, discounts being $12\frac{1}{2}\%$ and $8\frac{1}{3}\%$?

88. Expression of terms. Terms are often expressed thus: $^{3}/_{10}$; $^{N}/_{30}$; the figures at the left of the line show the rate of discount; the figures at the right of the line show the number of days within which payment must be made in order to obtain the discount. Thus $^{3}/_{10}$ means 3% discount if paid in 10 days; $^{N}/_{30}$ means net amount, no discount, bill due in 30 days.

1. Explain the meaning of the following when printed on a bill:

"Terms: 60 days net; 4% off 10 da." "Terms: 30 da. net; 5% 10 da. cash."



Mark Down Sale on Merchandise

2. The following table shows prices and discounts. How much must a customer pay for each of the following articles allowing a further discount of 3% for cash?

	Article	*List Price*	*Discount*	*Net Price*
a.	Desk	$60.00	15%	?
b.	Chairs (each)	24.00	$8\frac{1}{3}\%$	?
c.	Mirror	32.00	$12\frac{1}{2}\%$	?
d.	Coat	75.00	$16\frac{2}{3}\%$	?
e.	Veil	1.50	5%	?
f.	Silk gown	100.00	25%	?
g.	Underwear	4.80	$6\frac{2}{3}\%$	?
h.	Chiffonier	75.00	20%	?

3. I bought silk for $8.75 that was offered previously for $12.50. What was the rate of discount?

4. Goods listed at $7686, sold for $6405 because they were out of style. What was the rate of discount?

5. From a bill of $840.00 for electric supplies, 20% and 10% discounts were allowed. What was the net amount of the bill?

6. The terms of a bill for the following items are a trade discount of 5%, and an additional discount of 2% for cash. What is the gross amount of the bill? The net amount if cash is paid?

250 lb. Paper @ $.23
150 lb. Paper @ .15
200 lb. Paper @ .27

7. Mr. Smith sells a machine on 60 days' time for $72.00. If paid in 30 days the purchaser will receive a discount of $8\frac{1}{3}\%$. If paid in ten days, he will receive a 2% additional discount. What would the "cash in 10 days' payment" be?

8. The following bill shows the discounts and the terms of a sale:

New York, June 3, 1920.

Mr. J. F. Strong,
Portland, Maine.

Bought of M. Altman & Co.,
29–31–33 West 30th Street, New York.

Terms: 60 days; 3% 10 days.

25 pr. Shoes	$6.75	$ 168.75	
50 pr. Gloves	3.50	175.00	
150 yd. Silk	2.75	412.50	
225 yd. Serge	2.50	562.50	
75 yd. French Crêpe	2.25	168.75	
		$1487.50	
Less 10%		148.75	$1338.75
Less 3%			40.16
			$1298.59

Received Payment,
June 4, 1920.
M. Altman & Co.,
per J.

Mr. Strong paid his bill within the 10 da. allowed, and received the discount of $40.16. If he had paid the bill after the 10 da. allowance he would have had to pay $1338.75.

9. Make out bills for the following using the name and address of some firm you are familiar with and the current date.

Terms 30 da. 5% cash.

a. 15 tables @ $12.60; 12 chairs @ $4.75;
6 rockers @ $12.50; 10 desks @ $24.75;
12 bureaus @ $36.75. Discount $12\frac{1}{2}\%$, 5%.

b. 12 chests tea, 50 lb. @ 75¢ per pound.
8 chests tea, 25 lb. @ $1.25 per pound.
6 chests tea, 35 lb. @ 85¢ per pound. Discount 10%; 20%.

c. 12 doz. pkg. rolled oats @ $0.15 per package.
6 doz. pkg. cream of wheat @ $.23 per package.
10 doz. pkg. corn flakes @ $.19 per package.
8 doz. pkg. Graham crackers @ $.19 per package.
Discount 25%; 10%.

State the amount of money saved in each problem by prompt payment of the bill. Receipt each bill, allowing for payment before the final date.

10. Make out a bill for the following purchases:

(1) C. A. Pond, Cleveland, Ohio, bought of The Regal Shoe Co., Boston, Mass., May 5, 1919. Terms $^{3}/_{15}$ $^{N}/_{30}$.
50 cases slippers, 150 pr. @ $4.50 per pair.
25 cases rubbers, 100 pr. @ .85 per pair.
12 cases calf (boots) 125 pr. @ $5.50 per pair.

(2) If a discount of 10% and 20% is allowed on the entire bill, find the amount required to pay the bill if it is paid the day after it is received.

11. Examine the following invoice.

256. Indianapolis, Indiana, Jan. 1, 1919.
Mr. E. L. Hooper
Fort Wayne, Ind.
Bought of George A. Brooks & Co.
Terms, $^{3}/_{10}$; $^{2}/_{20}$; $^{1}/_{60}$

12	bolts black muslin, 12^{2}, 10, 10^{2}, 11^{1}, 8^{3}, 12, 10, 11, 9^{3}, 12^{2}, 11^{2}, 10^{3}	yd. @ $\$.23\frac{1}{2}$
6	bolts black lawn, 10, 10^{2}, 12, 12^{3}, 12^{1}, 11	yd. @ $.17
4	bolts black cambric, 10^{2}, 12, 11^{2}, 11^{3}	yd. @ $.12
5	bolts black Gloria, 10^{2}, 10^{3}, 12^{1}, 10, 12	yd. @ $\$.37\frac{1}{2}$
7	bolts black grenadine, 12, 10, 11^{3}, 10^{2}, 11, 12^{1}, 10^{1}	yd. @ $.70

(1) What is the last day on which payment can be made and the discount secured on the above invoice?

(2) When is the invoice due?

(3) What is the list price of each of the above purchases?

(4) What is the discount upon each of the above purchases?

(5) What is the net price of each article?

(6) Copy and complete the invoice.

12. Business houses that have both a wholesale and retail trade deduct a part of the fixed or catalog price when selling to dealers, which they call a Trade Discount. Catalogs are sent to the dealers inclosing a discount sheet similar to the following:

DISCOUNT SHEET

The following discounts are offered on articles listed in Catalog No. B 15	
Pages 2—7	25%
" 13—20	30%
" 8—12	20%
" 21—50	10%
" 51—100	15%
" 101—115	Net *

* "Net" means that no trade discount is allowed on goods offered for sale on these pages.

(1) Which are larger, trade discounts or cash discounts?

(2) Why?

(3) Why are trade discounts offered to dealers?

CHAPTER XIV

PROFIT AND LOSS

89. Profit and loss. Profits and losses in business transactions are expressed in terms of per cent for purposes of comparison.

ILLUSTRATIVE EXAMPLE: Mr. Smith sold a house for \$7200 and lost 25% by doing so. What did the house cost? What amount of money was lost by the transaction?

Solution: $25\% = \frac{1}{4}$.

$\frac{3}{4}$ of cost = \$7200.

$\frac{1}{4} = \frac{1}{3}$ of \$7200 = \$2400 amount of loss.

$\frac{4}{4} = 4 \times \$2400 = \$9600$ cost of house.

Problems

(1) A dealer bought a horse for \$1600, and sold it for \$1800. What per cent did he gain?

(2) By inspection, find the gain or loss in the purchase of the following lots of land. State the selling price of the land in each purchase.

	Cost of Land	*Per Cent of Gain*		*Cost of Land*	*Per Cent of Loss*		*Cost of Land*	*Per Cent of Gain*
Lot	*a.* \$3200	25%	*i.*	\$1800	20%	*q.*	\$7500	40%
	b. 2400	$16\frac{2}{3}\%$	*j.*	1600	60%	*r.*	4800	$6\frac{1}{4}\%$
	c. 1800	$33\frac{1}{3}\%$	*k.*	6000	$8\frac{1}{3}\%$	*s.*	5100	$3\frac{1}{3}\%$
	d. 4800	$62\frac{1}{2}\%$	*l.*	3000	$6\frac{2}{3}\%$	*t.*	5600	$37\frac{1}{2}\%$
	e. 3600	$8\frac{1}{3}\%$	*m.*	5600	$12\frac{1}{2}\%$	*u.*	1100	50%
	f. 1600	$87\frac{1}{2}\%$	*n.*	1900	50%	*v.*	2350	10%
	g. 2100	$66\frac{2}{3}\%$	*o.*	3500	80%	*w.*	2400	$1\frac{1}{4}\%$
	h. 1500	75%	*p.*	3200	$1\frac{1}{4}\%$	*x.*	6000	$1\frac{2}{3}\%$

(3) By inspection, find the cost on each of the following problems:

Selling Price	*Rate of Gain*	*Selling Price*	*Rate of Gain*
a. $1260	25%	*f.* $840	$8\frac{1}{3}\%$
b. 1632	$12\frac{1}{2}\%$	*g.* 600	$16\frac{2}{3}\%$
c. 4200	$16\frac{2}{3}\%$	*h.* 320	$6\frac{1}{4}\%$
d. 5400	$33\frac{1}{3}\%$	*i.* 720	$33\frac{1}{3}\%$
e. 2448	2%	*j.* 640	$62\frac{1}{2}\%$

(4) Find the per cent gain in the following problems:

Cost	*Marked Price*	*Cost*	*Marked Price*
a. $1.00	$1.20	*d.* $2.00	$5.00
b. 1.00	1.10	*e.* 6.00	8.00
c. 1.00	3.00	*f.* 3.00	5.40

(5) State a short way of finding a gain of $16\frac{2}{3}\%$; of 25%; of 50%; of 75%; of $8\frac{1}{3}\%$; of 10%; of $2\frac{1}{2}\%$.

(6) State a short way of finding a loss of $16\frac{2}{3}\%$; a loss of 20%; a loss of $12\frac{1}{2}\%$; a loss of $37\frac{1}{2}\%$; a loss of $87\frac{1}{2}\%$; a loss of 5%; a loss of $33\frac{1}{3}\%$; a loss of $66\frac{2}{3}\%$; a loss of $6\frac{2}{3}\%$.

(7) A suit which cost $60, was sold for $80. What sum was gained? The gain is what per cent of the cost price?

(8) A cabinet cost $80, and sold for $100. What amount was gained? The gain is what per cent of the cost price?

(9) Find by inspection the gain per cent on the cost price:

Cost	*Selling Price*	*Cost*	*Selling Price*
a. $15	$45	*f.* $150	$160
b. 30	90	*g.* 145	190
c. 10	18	*h.* 180	200
d. 18	27	*i.* 120	160
e. 7.50	10	*j.* 125	175

(10) Raw silk was bought for $3.84 per pound and sold for $5.12. Find the per cent profit based on the cost price. What was the

per cent of profit made by buying and selling 150 pounds at the same rate?

ILLUSTRATIVE EXAMPLE (1): By selling furs for \$840, a dealer gained $66\frac{2}{3}\%$ on the cost price. If he had gained 40% on the cost price, what would be the purchase price?

Solution: \$840 = the selling price = $166\frac{2}{3}\%$ or $\frac{5}{3}$.

a. \$336 = the gross profit = $66\frac{2}{3}\%$ or $\frac{2}{3}$.
\$504 = the purchase price = 100% or $\frac{3}{3}$.

b. \$840 = the selling price = 140% or $\frac{7}{5}$.
\$240 = the gross profit = 40% or $\frac{2}{5}$.
\$600 = the purchase price = 100% or $\frac{5}{5}$.

ILLUSTRATIVE EXAMPLE (2): By selling a rug for \$3200, a dealer lost $16\frac{2}{3}\%$ on the cost price. What was it bought for?

Solution: \$3200 = Selling price ($100\% - 16\frac{2}{3}\% = 83\frac{1}{3}\%$) = $\frac{5}{6}$ cost.
\$3840 = Cost.
\$ 640 = Loss.

(11) Goods which cost \$1.00, are sold for \$1.50. What is the per cent profit based on the cost price?

(12) A set of books cost a dealer \$25; in order to sell them the dealer lost \$5.00. What was the per cent loss?

(13) A dealer buys stockings for \$4 a dozen; he sells them for 50¢ per pr. What is his per cent profit?

(14) At what price must a dealer sell merchandise bought for \$165, to make a profit of $33\frac{1}{3}\%$?

(15) What is the per cent of reduction in each item of the following sale quotations? What is the loss to the store, in money, by the mark-down on:

a. "Miscellaneous Items"?

b. "Embroideries and Laces"?

c. "Art Furniture"?

d. What is the total amount of money loss to the store by the mark-down on the three lots of merchandise quoted?

Miscellaneous Items

COMPRISING LINENS FROM ITALY, SPAIN, JAPAN, CHINA, AND MADEIRA

1 Mosaic embroidery and Neapolitan lace square, 30×30 inches. Was $42.00. Now.....$30.00
1 Italian fil-tiré square, 34×34 inches. Was $25.00. Now.$20.00
1 Florentine and Mosaic square, 36×36 inches. Was $35.00. Now.....$28.00
1 Italian embroidered luncheon set, round, 25 pieces. Was $75.00. Now.....$50.00
1 Filet and cut-work dinner cloth, round, 90 inch. Was $175.00. Now.....$95.00
1 Point d'Espagne cloth, round, 54 inch. Was $150.00. Now.....$80.00
1 Flat Venetian cloth, round, 54 inch. Was $110.00. Now..$65.00
10 Madeira scarfs, 20×63 inches. Were $6.00. Now.....$5.00
16 Madeira scarfs, 20×72 inches. Were $9.00. Now.....$7.00
1 Venise scarf, 23×42 inches, 17th Century Italian. Was $35.00. Now.....$25.00
1 Filet scarf, 37×58 inches, 17th Century Italian. Was $70.00. Now.....$55.00
1 Venise scarf, 31×52 inches, 17th Century Italian. Was $85.00. Now.....$62.00
1 Filet square, 24×24 inches, 17th Century Italian. Was $21.00. Now.....$15.00
1 Filet and Venise square, 28×28 inches, 17th Century Italian. Was $38.00. Now.....$28.00

Finest Embroideries and Real Laces from Belgium and Italy Are Combined in the Following Pieces

1 Square, 29×29 inches. Was $188.00. Now.....$140.00
1 Square, 34×34 inches. Was $230.00. Now.....$180.00
1 Square, 36×36 inches. Was $262.00. Now.....$210.00
1 Scarf, 18×54 inches. Was $115.00. Now.....$90.00
1 Scarf, 22×64 inches. Was $135.00. Now.....$100.00
1 Scarf, 20×62 inches. Was $170.00. Now.....$135.00
1 Scarf, 20×56 inches. Was $210.00. Now.....$165.00
1 Luncheon set, round, 25 pieces. Was $125.00. Now....$100.00

ART FURNITURE

ODD PIECES—MARKED TO CLOSE OUT

ALL FURNITURE SOLD "AS IS"

	Original Value	Price
1 Reception Side Chair	$85.00	$25.00
1 Hand Carved Walnut Chair	85.00	30.00
1 Inlay Mahogany Side Chair	40.00	6.75
3 Chippendale Muffin Stands	41.50	10.50
1 Dutch Inlaid Chair	45.00	16.50
1 Mahogany Side Chair	120.00	50.00
1 Oak Gothic Arm Chair	35.00	17.50
1 Oak Arm Chair	40.00	12.50
1 Chippendale Tea Table (as is)	42.00	11.50
1 Oak Arm Chair	18.50	5.25
1 Adam China Cabinet	137.50	45.00
1 Mahogany Barrel Desk	450.00	225.00
1 Enamel Carved Chair	60.00	18.50
2 Table Desks	65.00	27.50
1 Satinwood Arm Chair	60.00	30.00
1 Queen Anne Chair	45.00	22.50
1 Adam Chair	40.00	22.50
4 Adam Dining Room Chairs	27.50	12.50
1 Louis XVI Arm Chair	250.00	70.00
1 Louis XV Gold Finish Sofa	195.00	95.00
1 Ribbon Back Side Chair	65.00	35.00
1 Hepplewhite Arm Chair	54.00	30.00
1 Wing Chair, carved legs	110.00	50.00

90. Two bases for figuring profit. Gross profit percentage may be figured in either of the two following ways:

(1) On the cost-price.

(2) On the selling-price.

1. Merchants and buyers usually figure profits on the selling price only. The following work will show what a great difference there is between profit figured on cost, and profit figured on selling price.

(1) Example: When an article that cost \$6, is sold for \$9, the gross profit (\$3) represents:

a. Fifty per cent based on cost-price of merchandise.

b. Thirty-three and one-third per cent based on the selling price of the merchandise.

(2) When gross profit on merchandise is equivalent to $33\frac{1}{3}\%$ of its selling price, that profit is equivalent to 50% of the cost price of the merchandise; a gross profit of 20%, based on selling price, is equivalent to 25% based on cost price.

2. Answer the following questions, figuring the profits or loss first on the cost price, second on the selling price, and note the difference:

(1) A pencil bought for 5 cents sold for 4 cents. What is the per cent of loss?

(2) A watch bought for \$25 is sold for \$75. What is the per cent of profit?

(3) Goods bought for \$750, are sold at a profit of 50%. What is the selling price?

(4) Find by inspection the per cent of gain (1) based on the cost, (2) based on the selling price, of the following:

	Cost	Selling Price		Cost	Selling Price
a.	\$50.00	\$60.00	*f.*	\$125.00	\$150.00
b.	15.00	30.00	*g.*	200.00	300.00
c.	10.00	15.00	*h.*	150.00	200.00
d.	75.00	100.00	*i.*	300.00	350.00
e.	45.00	75.00	*j.*	850.00	1000.00

3. The permanent relation between the most commonly used gross profit ratios based on cost, and the corresponding ratios based on selling price may be easily learned from the following schedule for determining the per cent on cost to which certain percentages on selling price are equivalent.

Gross Profits on Selling-Price		*Gross Profits on Cost-Price*
$\frac{1}{3}$	equals	$\frac{1}{2}$
$\frac{1}{4}$	"	$\frac{1}{3}$
$\frac{1}{5}$	"	$\frac{1}{4}$
$\frac{1}{6}$	"	$\frac{1}{5}$
$\frac{1}{7}$	"	$\frac{1}{6}$
$\frac{1}{8}$	"	$\frac{1}{7}$
$\frac{1}{9}$	"	$\frac{1}{8}$
$\frac{1}{10}$	"	$\frac{1}{9}$
$\frac{2}{5}$	"	$\frac{2}{3}$
$\frac{2}{7}$	"	$\frac{2}{5}$
$\frac{3}{8}$	"	$\frac{3}{5}$
$\frac{4}{9}$	"	$\frac{4}{5}$

The solution of the following example illustrates how simple the formula is upon which comparisons like those in the foregoing schedule are based.

ILLUSTRATIVE EXAMPLE (1): If the gross profit is equal to 25 percent on the selling price, what percentage is that gross profit on cost?

Solution:

a. Write the percentage on selling price to which the gross profit is equivalent, as a common fraction, that is, $\frac{25}{100}$.

b. From the fraction's denominator subtract its numerator, and use the remainder as a denominator in a new fraction.

In this case:

From the denominator	100
Deduct the numerator	25
Getting new denominator	75

c. As a numerator for the new fraction use the same numerator that served in the old fraction, that is, 25.

Thus, to help determine the equivalent percentage on *cost price* we have a new fraction: $\frac{25}{75}$.

d. Change the new fraction to a decimal fraction by dividing the numerator, 25, by the new denominator, 75.

The resulting quotient, $.33\frac{1}{3}$, is the percentage on *cost* to which a gross profit of 25 per cent on *selling price* is always equivalent.

ILLUSTRATIVE EXAMPLE (2): Suppose a merchant finds that in one of his departments on sales totaling $116,665 the gross profit was $37,030.33. On dividing the $37,030 by the $116,665 he learns that the gross profit was equal to almost .318 (or not quite 32 per cent) on the *selling price* of the goods which the department sold.

He can determine to what percentage on *cost* that gross profit is equivalent by either of the two following methods:

First Method:

a. Deduct from the *selling price* ($116,665) the gross profit ($37,030).

b. The remainder is the goods' *cost* ($79,635).

c. Divide the gross profit ($37,030) by the cost ($79,635).

Second Method:

a. Write the percentage on *selling price* as a common fraction, that is, $\frac{318}{1000}$.

b. From the denominator (1000) deduct the numerator (318) and get as a remainder 682.

c. With 682 as the denominator and with 318 as a numerator form the new fraction, that is, $\frac{318}{682}$.

d. Change the new fraction to a decimal by dividing the numerator by the new denominator; $\frac{318}{682}$ equals .466.

e. Hence, the gross profit which equalled almost .318, or about 32 per cent on the *selling price* of the goods, equals about .466 or about 47 per cent on their *cost.*

4. To test the accuracy of this short cut method: Divide the gross profit (in this case, $37,030) by the cost ($79,635). The gross profit is about .465 on the cost price. This is close enough.

ILLUSTRATIVE EXAMPLE: If the gross profit on cost is 25 per cent, find the corresponding percentage of gross profit on the selling price.

a. Write the percentage on cost to which the gross profit is equivalent, as a common fraction, that is, $\frac{25}{100}$.

b. To the fraction's denominator add its numerator, using the sum thus obtained as a denominator in a new fraction:

In this case:

To the denominator.....100
Add the numerator..... 25
Getting new denominator.....125

c. As a numerator for the new fraction use the same numerator that served in the old fraction, that is, 25.

To help determine the equivalent percentage on selling price, we have a new fraction: $\frac{25}{125}$.

d. Change the new fraction to a decimal fraction by dividing the numerator (25) by the new denominator (125).

e. The resulting quotient .20 shows that a gross profit of 25 per cent on cost is equivalent to 20 per cent on selling price.

f. The points above given are applicable to gross profit percentages only.

5. In figuring profits, a merchant must consider the following items:

(1) The gross selling price, which is the total amount received for the goods sold.

(2) The gross trading profit, which is the difference between the selling price and the cost price.

(3) The net profit, which is the difference between the expenses and losses of the business and the gross profit, or the profit after all expenses are deducted.

(4) The net selling price, which is the difference between the gross selling price and the sum of all the expenses of the sale paid by the seller.

6. The per cent of gross trading profit is found by dividing the gross profit by the net sales.

(1) When goods are sold at less than their cost, a loss is incurred. The per cent of loss is found by dividing the loss by the net sales.

(2) In reckoning the cost of goods purchased it is customary to add to the wholesale price all the expenses incurred in getting the goods on the shelves ready to sell, and the selling expenses.

91. Figuring cost. In figuring cost, a merchant must consider the following items:

1. Prime, or net, cost.

This includes the first cost, or the amount actually paid for merchandise.

2. Second, or manufacturer's, cost.

This includes the manufacturer's cost, if the merchant is the manufacturer, the wear and tear on machinery, wages of factory hands, lighting, heating, etc.

3. Third cost, or selling cost.

This includes the salaries and expense of sales people, freight and handling of goods, advertising, etc.

4. Fourth, or overhead charges.

This includes rent and insurance, lighting, heating, taxes, office equipment, etc.

5. The total cost is the prime cost, plus all the expenses of the purchase, and is the gross cost.

Problems

(1) The inventory of a stock at the beginning of the year was \$2800. The purchases during the year were \$17,600. At the close of the year the inventory showed a stock of \$5400 unsold. What was the cost of the goods sold? The sales for the year were \$22,500. What was the gross profit? What was the per cent of gross profit?

Solution:

Expenses or Costs for Year		*Income or Receipts for Year*	
Inventory at first of year, Mdse.	=\$ 2,800.	Sales of Mdse.	=\$22,500.
Purchases during the year, Mdse.	=\$17,600.	Net cost of sales	=\$15,000.
Total amount of expenditure, Mdse.	=\$20,400.	Profit on Mdse.	=\$ 7,500.
Inventory at close of year, Mdse.	=\$ 5,400.	\$7,500 ÷ \$15,000 = .50 = 50% = Per cent of profit.	
Net cost of sales, Mdse.	=\$15,000.		

(2) *a.* Complete total cost of purchases in the following table.

Inventory Jan. 1, 1919		Merchandise		Buying Expenses		Total Cost of Merchandise	
6258	75	31359	50	615	20		
8765	29	36824	25	724	15		
3640	50	8756	40	378	24		
5625	50	16786	90	1108	50		
4072	75	22128	50	369	25		

b. Using figures in *a* complete the table:

Inventory Dec. 31, 1919		Cost of Mdse. Sold		Sales		Profit or Loss	Per cent of Profit or Loss
6875	50			29275	50		
8864	25			58065	25		
4275	75			9824	75		
4825	50			16756	84		
3256	75			17650	00		

92. Depreciation losses. The decline in the value of property, or expense in business caused by the loss in value, is depreciation.

1. Estimating the amount of depreciation in property may be done in several ways, but the methods most commonly used are as follows:

(1) A fixed rate, reckoned each year, on the original value of the property.

(2) A decreasing rate, reckoned each year, on the original value of the property.

(3) A fixed rate reckoned on a decreasing value.

The first method is the simplest but would ultimately represent property as of no value.

ILLUSTRATIVE EXAMPLE: A house is bought for \$10,000; when the house has been occupied for ten years its value may be \$4600, which shows a decrease in value, or depreciation, of \$5400; this is a yearly loss or depreciation of \$540. \$540 is $5\frac{2}{5}\%$ of \$10,000; therefore $5\frac{2}{5}\%$ of the original value of the house is charged each year as depreciation.

2. Reckoning a decreasing rate on the original value is based on the principle that the largest amount of depreciation should be charged the first year, because during this year greater depreciation occurs than during any later year. Then the depreciation of each year after is gradually reduced. Illustration: A sewing machine may only be used a short time and must be sold as "a second-hand machine." The owner incurs much greater proportional loss from its use the first year than he does for the second year.

3. The method of estimating the depreciation from a fixed rate reckoned on a decreasing value is as follows:

The original value of a piece of property is $2500, and the rate of depreciation is 10% yearly. What is the property worth at the end of three years? What is the annual amount of depreciation? What is the decreased value each year?

$2500.00 = Original Value.
.10 = Rate of Depreciation.

$2500 — $250.00 Depreciation first year.
−250

$2250 = Decreased value, beginning of second year.
.10

$225 = Depreciation second year.

$2250
−225

$2025 = Decreased value, beginning of third year.

$2025.00
−202.50

$1822.50 = Decreased value, at end of third year or beginning of fourth year.

Problems

(1) Find by first method the amount charged off annually for depreciation from the following data:

Property	*Value*	*Depreciation Rate*
Building	$75,000	5%
Machinery	8400	$6\frac{1}{4}\%$
Fixtures	3200	10%
Tools	2400	$8\frac{1}{3}\%$

(2) Compute by third method the annual depreciation at 6% and the decreased value each year, of five years' depreciation on property costing $12,600.

(3) An automobile which cost $3600 was sold after five years for $900. What per cent should be charged annually for depreciation, and what was the amount of the annual depreciation?

93. Buying and selling expenses. Expenses in trading include:

(1) The buying expenses.

(2) The selling expenses.

1. The buying expenses include all the costs of buying merchandise, and are added to the original price of goods to fix the total cost.

2. The prime cost of merchandise is the original, or purchase price; the total cost of merchandise is the prime cost, plus the buying expenses.

3. All expenses connected with the sale of merchandise, (advertising, rent, heat, and light of building, salaries, depreciation, etc.) are selling expenses. The per cent of selling expenses is found by dividing the selling expenses by the gross sales.

ILLUSTRATIVE EXAMPLE: A merchant bought merchandise during the year amounting to $28,500.00. This was the prime cost. The cost of placing this merchandise in the department stores (freight, custom duties, employees, etc.) was $1965. The total cost of the merchandise purchased is:

$28,500.00 Prime Cost.
1,965.00 Buying Expenses.
$30,465.00 Total Cost.

Problems

(1) The prime cost of merchandise is $48,250; the buying expenses are $1930. What is the total cost? What per cent of the prime cost are the buying expenses?

(2) Note paper is quoted at the following prices: $3.75 a pound less 2%, or $4 a pound less 6%. Which is the better price and how much better? How much will be saved by buying 19 pounds at the cheaper quotations?

(3) A merchant's gross sales during one year are \$156,218; the selling expenses for the same time are \$37,492.32. What per cent of the gross sales are the selling expenses?

Solution: Selling expenses, \$37,492.32 ÷ \$156,218. Gross sales = 24% of selling expenses.

(4) Some of the usual and necessary expenses connected with the carrying on of a department store, or of a department in a department store, are quoted below. Find what per cent each item of expense is of the gross sales.

Department Store: Gross Sales =	\$45,000.00
Rent	1200.00
Salaries	6500.00
Heat and Light	450.00
Advertising	275.00
Insurance and Taxes	480.00
Delivery	960.00
Supplies	175.00
General Expense	650.00
Depreciation	285.00
Debts	165.00

a. What are the total selling expenses in the above list?

b. The total selling expenses are what per cent of the sales?

c. The cost of the goods sold is \$16,820. What is the gross profit? What is the net profit?

d. What is the average or standard per cent of selling cost on gross sales?

e. Which of these per cents are larger than the average? Which are smaller?

f. What does a merchant learn about his business by a study of these comparisons of per cent expenses?

g. Which item of expense shows the greatest loss to the business?

h. Losses that may be prevented or checked are sometimes called "leaks" in business. Do you see any "leaks" in the study of the above per cent comparisons?

i. If the gross sales were \$60,000.00 and the selling expenses were \$15,000.00, what was the per cent of selling expenses?

j. If the selling expenses were reduced to \$12,000.00 without reducing the gross sales, what would be the per cent of selling expenses?

(5) Copy and complete the following table showing the profit or loss in six departments:

Stock Jan. 1, 1919		Purchased during 1919		Total Cost 1919		Stock Dec. 31, 1919		Cost of Mdse. Sold		Sales 1919	
16,128	50	152326	65			18256	28			185224	48
24,235	68	175289	45			27352	65			198264	75
29,256	74	181243	58			28217	43			201315	68
18,435	62	205368	75			19246	85			156248	37
25,278	58	142643	25			21143	76			167281	45
32,564	75	151342	18			27325	68			168432	17

Gross Profits	%	Expenses		Net Profits		%	Net Loss		%
		25286	75						
		23207	81						
		24658	17						
		28275	43						
		15624	08						
		16275	15						

Per cent of Gross Profit = Amount of Gross Profit divided by the sales.

Per cent of Net Profit = Amount of Net Profit divided by the sales.

Per cent of Net Losses = Amount of Net Loss divided by the sales.

94. Reducing selling cost to increase profits. The investment of a firm's money, or its capital, is the merchandise or stock. This investment is profitable when the merchandise is sold at the price originally set by the buyer within a reasonable amount of time.

1. The method used most frequently to increase gross sales is to sell at a small profit and to make frequent "turnovers." A mer-

chandise "turnover" and its advantages to the merchant may be explained as follows: A dealer hires two salesmen to sell \$100 worth of stock in two different parts of a store. Salesman No. 1 sells out his stock in two days, and on the third day begins with a fresh line, valued as before, at \$100. He is again sold out in two days, and stocks up with another \$100 worth of goods which he disposes of in the same amount of time. The merchandise was sold at a $33\frac{1}{3}$ per cent gross profit to the dealer; the weekly rental for the floor space was \$10. The salesman was paid \$2 a day. The statement of salesman No. 1's business reads as follows:

Receipts from sales \$300 plus $33\frac{1}{3}\%$		\$400
Expenses:		
Investment in Stock	\$300.00	
Wages	12.00	
Rent	10.00	322
	Net Profit	\$78

Salesman No. 2 with the same amount of stock at the beginning, sold out only twice in the same amount of time: Expenses were the same as in salesman No. 1's case. Salesman No. 2's account reads:

Receipts from sales \$200 plus $33\frac{1}{3}\%$		$\$266.66\frac{2}{3}$
Expenses:		
Investment in Stock	\$200	
Wages	12	
Rent	10	222.00
	Net Profit	$\$44.66\frac{2}{3}$

Salesman No. 1 returns to the dealer a profit of \$78, because of three turnovers; salesman No. 2 returns to the dealer a profit of $\$44.66\frac{2}{3}$, the result of two turnovers; this shows that the oftener the "turnovers" the larger the profits. If the dealer should increase salesman No. 1's salary to \$15 or to \$20 a week, salesman No. 1 would still be making a larger profit for him than salesman No. 2.

If a merchant's stock is valued at \$50,000 at cost prices (which include buying expenses), and the selling expenses are \$25,000, he must get \$75,000 before he begins to make any profit; therefore he must mark up his goods 50%. If he wishes to make a 10% profit on the cost, he must mark his goods at 60% above cost; so the goods which cost \$1.00 must sell for \$1.60. In this single turnover, a 60% mark up will net a profit of \$5,000.

What would be the profit on a 30% mark up (selling merchandise costing \$1.00 for \$1.30) in two turnovers without increasing the selling expenses? Each turnover of a \$50,000 stock means gross sales of \$65,000; two turnovers make the gross sales \$130,000, and the gross profits \$30,000. Deducting the selling expenses of \$25,000 a profit of \$5000 remains.

Problems

(1) What is the net profit on stock which cost \$20,000 in four turnovers if there is a mark up of 25%, selling expenses being \$12,000?

(2) If a mark up of 15% results in four turnovers of stock costing \$10,000 with selling expenses of \$5000, what is the net profit?

(3) What is the profit or loss in each of the following?

Average Inventory	Selling Expenses	Turnovers	Mark Up
1. \$ 6000	\$1296	1	25%
2. 12,600	2150	2	20%
3. 24,240	4275	3	15%
4. 28,800	4650	4	10%
5. 36,500	8275	5	$12\frac{1}{2}$%

(4) How much profit is made on \$25,000 invested in stock, when selling expenses amount to \$7400 by:

a. 2 turnovers with a mark up of $12\frac{1}{2}$%?

b. 4 turnovers with a mark up of 8%?

c. 3 turnovers with a mark up of 5%?

2. The method of increasing profits by increasing the selling price leads to a decrease in demand, hence is used less frequently than the "turnover" method just explained.

95. Figuring profit or loss on each sale. Most business houses keep a tabulated account not only of the profit or loss of the entire business but also of the profit or loss of each department and each sale.

1. A book called the cost book, which contains the following items, is prepared:

(1) The wholesale cost
(2) A share of the buying expenses
(3) A share of the selling expenses

} of each piece of merchandise for sale in the business.

2. The profit or loss on each sale is made out by the profit clerk who has a copy of the cost book, and a duplicate of each invoice of merchandise sold.

(1) The profit or loss is obtained by finding the difference between the cost of the merchandise recorded in the cost book, and the selling price, recorded on the invoice.

(2) The invoice gives the wholesale cost.

3. The per cent of buying expenses to be added is found as follows:

Divide the total buying expenses by the total cost of merchandise purchased: The quotient thus obtained is the per cent of buying expenses which is to be added to the wholesale cost of each piece of merchandise.

ILLUSTRATIVE EXAMPLE: The expenses of a department store are as follows:

Direct Buying Expenses (Salaries, etc.) $9500
Share of Overhead Buying Expenses (rent, labor, etc.) 5500
Cost of purchases for one year $250,000

Solution: $9500 = Direct Buying Expenses.
5500 = Share of Overhead Buying Expenses.
$15,000 = Total Buying Expenses.
$15,000 ÷ $250,000 = 6%.

4. Adding the Direct Selling Expenses and a share of the Overhead Selling Expenses gives the total Selling Expenses. The per cent of Selling Expenses included in the cost is found by dividing the Total Selling Expenses by the Cost of Merchandise Sold.

ILLUSTRATIVE EXAMPLE: What per cent must be added to the cost of merchandise to cover the selling expenses for the following data:

Direct Selling Expenses (Salaries, etc.) $7725.00
Share of Overhead Selling Expenses 5400.00
Total Selling Expenses $13,125.00
Cost of Merchandise sold $87,500.00

$13,125 ÷ $87,500 = 15% = per cent to be added to the cost of each piece of merchandise, to cover selling expenses.

5. The per cent found by adding the per cent of buying expenses and the per cent of selling expenses, is added to the cost of each piece of merchandise. Business firms usually make use of the per cents which the business of the previous years shows to be approximately correct.

(1) Copy and complete the following table:

Deptmt.	Direct Buying Expenses	Share of Overhead Buying Expenses	Total Buying Expenses	Purchases 1919	Per cent of Buying Expenses
A	3560 00	1720 00	5280 00	66000 00	8%
B	4360 00	3620 00		133000 00	
C	2398 00	1576 00		19870 00	
D	854 00	687 00		22032 00	
E	1575 00	796 00		29649 00	
F	1896 00	634 00		42178 00	

Inventory Jan. 1, 1919	Purchases 1919	Total Cost of Mdse. 1919	Inventory Dec. 31, 1919	Cost of Goods Sold 1919
114000 00	66000 00	180000 00	75000 00	105000 00
54000 00			46000 00	
7860 00			7360 00	
15108 00			15486 00	
16400 00			5326 00	
51987 00			2332 00	

Direct Selling Expenses	Share of Overhead Selling Expenses	Total Selling Expenses	Per cent of Total Expenses	Total Per cent
7370 00	8380 00	15750 00	15%	23%
6260 00	3610 00			
740 00	482 20			
1728 00	1587 00			
1438 00	1412 00			
5056 00	3208 00			

(2) Find the profits or losses on the following sales, from the cost book prices and amounts of invoice records given below:

Cost Book Price	*Invoice Price*	*Profit or Loss*
$18.75	$24.35	
124.50	175.00	
36.75	42.18	
98.00	115.00	
45.00	40.00	

(3) What is the per cent profit or loss figured on selling price?

Figuring profits and buying and selling expenses, may be illustrated by the following example.

ILLUSTRATIVE EXAMPLE (1): The prime cost of an article is $2.00; the buying and selling expenses, 25%; profit required, 10%. Find the selling price, and check the work.

Solution: 25% + 10% = 35% = per cent to cover both buying and selling expenses and the profit. 100% — 35% = 65% = per cent represented by prime cost. That is, $2.00 = 65% of selling price. Therefore $2.00 ÷ .65 or $3.08 will be the proper selling price.

Check: Marked price is $3.08. 10% of $3.08 = $.308 = profit. 25% of $3.08 = $.77 = cost of buying and selling. $.308 + .77 = $1.078 or $1.08 = amount of expenses and profit. $3.08 — $1.08 = $2.00, the prime cost.

ILLUSTRATIVE EXAMPLE (2): What cost should be entered on a cost book of an article whose prime cost is $16.50, with 25% to be added to cover both buying and selling expenses?

Solution:

$16.50 = Prime cost.
Add. 4.125 = 25% of prime cost.
20.625 = Total cost of buying and selling.

(4) What cost should be entered in a cost book for the following purchases?

Prime Cost	*Per Cent to Cover Buying and Selling Expenses*	*Total Cost to Buy and Sell*
$ 24.30	$16\frac{2}{3}$	?
18.27	$33\frac{1}{3}$	?
32.40	$12\frac{1}{2}$	?
56.64	$37\frac{1}{2}$	?
28.80	20	?
35.75	25	?
45.60	15	?
64.50	30	?
56.88	24	?
132.84	18	?

CHAPTER XV

MARKING THE PRICE OF GOODS

96. Marking goods. When the cost price is marked on goods a certain per cent is added to the prime cost to cover the buying expenses. The amount of the per cent to be added is determined by the average expenses of several previous years. For example, the expenses of a certain department were as follows:

Year	*Prime Cost*	*Buying Expenses*
1917	$26,250	$1065.00
1918	$24,175	$1430.50
1919	$42,450	$2148.25
	$92,875	$4643.75

$4643.75 ÷ $92,875 = 5% = average per cent of increase.

Illustrative Example. Merchandise which costs $15 must be marked at what price to cover buying expenses?

Solution:

$15 = Prime cost.
× .05 = Per cent of buying expenses.
$.75 = Buying expenses.
+ $15.00 = Prime cost.
$15.75 = Total marked cost.

1. The method of marking the cost and the selling price of goods varies not only with the different stores, but also with the merchandise. Some merchants use some word or words, consisting of ten different letters representing the ten digits; this word is called a Key. Thus, if the key is "profitable," the corresponding letters and figures are:

P-r-o-f-i-t-a-b-l-e
1-2-3-4-5-6-7-8-9-0

2. When a figure occurs more than once in succession, a repeater is used to avoid giving a clue to the key. Thus, if the cost of an article is \$1.25 and the selling price is \$1.99, using the key "profitable," with M for the repeater, the mark would be $\frac{\text{pri}}{\text{plm}}$. In the following exercises, use this key.

(1) Interpret the following, and find the per cent of increase for each:

a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*	*i.*	*j.*
pie	fbe	pb	rf	rem	ra	bai	rmf	tri	lab
pli	tfe	rb	tf	oem	fi	peme	blt	aie	pble

(2) In the following, fill in the selling price at a "mark up" of 20%.

a. ot	*b.* pir	*c.* pof	*d.* pmabe	*e.* tre
?	?	?	?	?

(3) Fill in the cost mark in each of the following, the given selling price being $33\frac{1}{3}\%$ above cost.

?	?	?	?	?
a. rtb	*b.* bfp	*c.* oatr	*d.* aefm	*e.* pmtrb

3. In marking goods bought by the dozen and sold by the piece, the cost must be divided by 12; this work is simplified by making use of the table of decimal equivalents—thus:

$\frac{1}{12} = .08\frac{1}{3}$
$\frac{5}{12} = .41\frac{2}{3}$
$\frac{7}{12} = .58\frac{1}{3}$
$\frac{11}{12} = .91\frac{2}{3}$

ILLUSTRATIVE EXAMPLE (1): What is the cost of one skirt at the rate of \$81 a dozen?

Solution: $\frac{1}{12}$ of \$81 = \$$6\frac{3}{4}$ = \$6.75.

ILLUSTRATIVE EXAMPLE (2): What is the cost of 5 waists at the rate of \$100 a dozen?

Solution: $\frac{5}{12} = .41\frac{2}{3}$; \$100 × $.41\frac{2}{3}$ = \$$41\frac{2}{3}$ = \$41.66$\frac{2}{3}$.

4. Key words are often used to mark the selling price as well as the cost, thus:

(1) Cost key, "republican". "w" is the repeater.

1234567890

(2) Selling key, "importance" "x" is the repeater.

1234567890

Cost, $52.36
Selling price, $58.72
Mark, bepl
rnam

(3) Mark the cost and the selling price of the following, devising a cost key, and selling key, of your own:

a. Cost $5.60 marked to gain 25%
b. Cost $14.40 marked to gain $33\frac{1}{3}$%
c. Cost $54.56 marked to gain $62\frac{1}{2}$%

(4) Some key words used are contained in the following list:

Now be quick	Purchasing	Republican
Money talks	What prices	Market sign
Impersonal	Backslider	Buckingham

ORAL EXERCISE

(1) What is the cost per article, when the cost per dozen is:

a. $65?	*e.* $7.30 ?	*i.* $14.14?
b. $38?	*f.* $18.24?	*j.* $ 9.63?
c. $47?	*g.* $54.46?	*k.* $23.00?
d. $15?	*h.* $17.25?	*l.* $19.27?

(2) Mark the following priced articles to sell at an $87\frac{1}{2}$% increase on the prime costs given below:

a. 48¢	*c.* 72¢	*e.* $1.04	*g.* $6.72	*i.* $360
b. 64¢	*d.* 96¢	*f.* $2.56	*h.* $6.08	*j.* $448

Illustrative Example: How shall a hardware dealer mark a pair of skates that cost $2.40 so that he may give a discount of 8% from the marked price, and still make a profit of 20%?

Solution: 120% of $2.40 equals $2.88, selling price.

Since the selling price is 8% less than the marked price, $2.88 is 92% of the marked price.

Then the marked price = \$2.88 ÷ .92 = \$3.13. Hence the complete mark using "m" as the repeater, and "profitable" as the key should be $\frac{\text{rfe}}{\text{opo}}$

(3) Mark the articles that cost as follows, so that there will be a profit of 25% after giving the indicated discount:

a. 64¢ discount, 40%
b. 24¢ discount, $3\frac{1}{3}$%
c. 64¢ discount, $33\frac{1}{3}$%
d. \$6.80 discount, 30%
e. \$3.36 discount, $6\frac{1}{4}$%
f. \$4.20 discount, $8\frac{1}{3}$%

(4) In marking goods, if 5% of the cost of each article be added as buying expenses, what would be the marked cost of each of the following articles?

a. Prime cost, \$28.50
b. Prime cost, \$14.75
c. Prime cost, \$13.69
d. Prime cost, \$24.52
e. Prime cost, \$ 7.25
f. Prime cost, \$42.39

(5) Find out the per cent of buying expenses to be added to the wholesale cost of goods purchased in 1920 from the following data:

Year	*Purchases*	*Buying Expenses*
1916	\$12,378.00	\$728.75
1917	\$15,824.75	\$629.50
1918	\$16,724.98	\$825.25
1919	\$13,528.50	\$615.35

(6) Complete the following invoice, and find out the marking price of each bookcase to make a gain of $33\frac{1}{3}$%.

Boston, Mass., June 5, 1919.

George A. Long.
Worcester, Mass.
Bought of Paine Furniture Company
Terms: Net Cash

16	Book cases @ \$38.50 Less 10% and 5%.				

(7) The prime cost of the following articles has been increased the amount of per cent indicated, to include both the buying expenses and the selling costs: mark the tags for each article like the first one below, writing the cost above the line, and the selling price below.

Article	Wholesale Cost	Per cent Increase	Tag	Article	Wholesale Cost	Per cent Increase	Tag
Coat	$55.00	18%	55.00 / 64.90	Dress	$13.76	$12\frac{1}{2}\%$	
Evening wrap	85.00	32%		Dress	29.50	30%	
Hat	15.00	$33\frac{1}{3}\%$		Dress	19.60	25%	
Cape	19.50	$66\frac{2}{3}\%$		Dress	25.00	20%	
Neck-piece	84.00	$62\frac{1}{2}\%$		Dress	18.50	15%	
Suit	45.00	20%		Gown	50.00	10%	
Suit	48.00	$16\frac{2}{3}\%$		Gown	95.00	20%	
Suit	125.00	30%		Gown	72.00	$8\frac{1}{3}\%$	

(8) A merchant buys merchandise at \$36 less $16\frac{2}{3}\%$; he sells to make a gross profit of $33\frac{1}{3}\%$. What is his marking price if he allows 20% discount?

Solution:
$\frac{1}{6}$ of \$36 = \$6 = discount.
\$36 − \$6 = \$30 = net cost to merchant.
$\frac{1}{3}$ of \$30 = \$10 = increase.
\$30 + \$10 = \$40 = dealer's net selling price.
\$40 ÷ 80% ($\frac{4}{5}$) = \$50 = jobber's marked price.

Explanation: Discount is $16\frac{2}{3}\%$ or $\frac{1}{6}$ of \$36, or \$6. \$36—\$6 = \$30, the net cost of the merchandise. $33\frac{1}{3}\%$ profit is equal to $\frac{1}{3}$ of \$30 or \$10, which added to the net cost equals \$40, the net selling price.

A 20% discount being allowed, the merchant sells for 80% of his marked price, which is \$50, the list price.

(9) A manufacturer buys merchandise at \$25 less 10%. At what price must it be marked to sell at a discount of 25%, and still make a $33\frac{1}{3}\%$ profit?

(10) Find the net amount of the following pieces of merchandise, and figure out the selling price mark for each, if sold at an increase of $33\frac{1}{3}\%$. The stock sold was bought at the following prices.

3 fur coats @ \$275, less 20%
5 fur coats @ \$450, less $12\frac{1}{2}\%$
12 fur coats @ \$625, less 10%

CHAPTER XVI

COMMISSIONS

1. Give as many reasons as you can why an experienced person in any business is more valuable than an inexperienced one to the business, the customer, to all others employed in the business. Is this true of all other employments?

2. How is store merchandise obtained?

3. Why is it an advantage to a business house to buy and sell through an agent?

4. Do you know what merchandise in stores is bought or sold through agents? Where are these agents located?

5. What is the amount called that agents receive for their services in buying and selling?

97. Commission. Commission agents, merchants, or brokers receive their goods (called a consignment), see that they are properly cared for, make the sale, return the proceeds to the owner, less the expense of handling and the charge for services rendered. These services in buying and selling for others, are often paid for by a fixed per cent of the value of the article sold; the percentage allowed a commission merchant is called his *commission*. The agent or person to whom the merchandise is sent is the *consignee*.

1. The sum left after the commission and other expenses have been deducted, is called the *net proceeds*. The commission is reckoned at a certain rate per cent upon the value of the sales and purchases.

2. Finding the commission.

Illustrative Example: A commission agent sold 60 bales of cotton at $50 a bale: he received 20% commission. How much money did he receive?

Solution: 60 bales @ \$50 = \$300, selling price of cotton. 20% of \$300 = \$60.00, amount of commission.

3. To find the rate of commission divide the commission by the gross proceeds.

EXERCISE

(1) Find the commission and the net proceeds of the following:

Merchandise.	*Quantity*	*Price*	*Rate of Commission*
a. Suits	500	\$18	3%
b. Rugs	250	\$24	$4\frac{1}{2}$%
c. Blankets	600	\$6.50	$1\frac{1}{4}$%
d. Coats	800	\$90	$4\frac{1}{4}$%
e. Skirts	800	\$12.50	2%
f. Books	450	\$1.50	$1\frac{3}{4}$%
g. Pr. of Stockings	175 doz.	\$1.50	$2\frac{1}{2}$%
h. Shoes	400 pr.	\$6.75	3%
i. Rubbers	65 pr.	\$0.75	$1\frac{1}{2}$%
j. Waists	250	\$8.25	$2\frac{2}{3}$%

(2) A commission merchant made sales as follows:

Monday, \$350	Thursday, \$2500
Tuesday, \$2700	Friday, \$3650
Wednesday, \$1700	Saturday, \$3450

Find his commission for each day at $2\frac{1}{2}$%.

(3) An agent charged 5% for buying broadcloth for \$4000. How much did he receive?

(4) An agent sold a consignment of wool for \$24,000. At 4% commission how much money did he receive?

(5) A traveling salesman sold \$40,000 worth of merchandise at a commission of $5\frac{1}{2}$%. How much money did he receive?

(6) What will an agent receive for selling 12 tickets at \$62.50 each—commission 6%?

(7) A dealer sent 150 cases of shoes to an agent who sold them at \$24.60 a case, paying \$77 for freight charges; 10¢ a case, cartage; 4¢ a case, storage. Find the net proceeds and the agent's commission at $2\frac{3}{4}\%$.

(8) An agent sold 6250 pairs of rubbers at \$0.85 a pair. Find the rate of commission if the proceeds of the sale were \$5000.

(9) An agent was paid \$26.25 for selling 350 books at \$2.50 apiece. What was the rate of commission?

(10) When \$249.60 commission is paid for selling \$7680 worth of merchandise, what is the rate?

(11) What is the rate of commission if a dealer pays an agent \$82.80 for collecting a claim of \$1440?

4. To find the amount of investment.

Illustrative Example: If an agent's rate of commission is 4%, what is the value of the goods he must sell in order to earn \$100?

Solution: .04)\$100.00

\$ 2500 = Amount.

To find the amount of investment, divide the commission by the rate per cent of the commission.

Problems

(1) What amount of goods must be purchased in order that an agent may receive a commission of \$525 on a $3\frac{1}{2}\%$ commission?

(2) A commission of $2\frac{1}{2}\%$ was paid to an agent for selling woolen goods. He received \$612.50. What was the amount of his sales?

(3) A charge of \$28.20 was made by an agent at the rate of 5%. What amount of money did he collect from the purchaser?

5. To find investment and commission.

Illustrative Example: If \$3150 is sent to an agent to purchase silk, how much will he invest, his rate of commission being 5%?

Explanation: Since \$3150 includes 100% of the amount invested, plus 5% commission, then \$3150 is 105% of the cost of the silk. Therefore, the silk cost \$3150 ÷ 1.05 or \$3000.

\$1.00 = investment.
.05 = commission
\$1.05 = actual cost of each dollar invested.

\$1.05)\$3150
\$3000 = sum invested.

To find the investment when the commission is included in the remittance, divide the remittance by 1 plus the rate of commission.

REVIEW

(1) An agent receives \$25,008.40 to invest in wool. His commission is 3%. How much money did he invest in wool?

(2) An agent receives \$6679.70 to invest in cotton. After deducting his commission of $1\frac{1}{2}$%, what amount of money does he invest in cotton?

(3) The salespeople in a Cleveland store received a fixed salary of \$8 a week and a commission of 2% on their weekly sales. What would be the total earnings of A, whose sales amounted to \$125? Of B, whose sales amounted to \$225? Of C, whose sales amounted to \$380? Of D, whose sales amounted to \$410?

(4) A salesman receives a salary of \$15 a week and a commission of $1\frac{3}{4}$% on all goods sold. If he sells \$25,000 worth of goods in the year, what does he earn in a year of 50 weeks?

(5) Find out the rates of commission in your business, and work out problems concerning them.

(6) How many bbls. of flour at \$15.50 per bbl. may be purchased for \$790.50, if the agent charges 2% commission?

(7) If an agent charges 12% commission, and keeps \$24 as his fee, how much was collected?

(8) How many pounds of coffee at $16\frac{2}{3}$¢ a pound can be bought for \$1020, if this amount includes the agent's commission of 2%?

(9) The salesmen in the department store of A. Davis & Co. receive the following commissions:

a. On goods sold in Department I, 10%.
b. On goods sold in Department II, $12\frac{1}{2}$%.
c. On goods sold in Department III, $16\frac{2}{3}$%.

d. On goods sold in Department IV, 8%
e. On goods sold in Department V, 15%.

(10) The table below shows the sales made by these five salesmen:

Salesmen	Dept. I	Dept. II	Dept. III	Dept. IV	Dept. V
J. A. Horne.....	258.16	306.28	528.62	456.42	612.40
A. T. Burr.....	372.28	460.72	560.78	612.78	278.85
M. E. Brooks....	512.16	504.20	327.42	495.30	298.67
F. L. Lang.....	432.08	648.75	396.50	643.72	521.50
N. C. Draper....	642.19	532.12	428.60	317.40	298.90

a. Find the amount of commission paid to each salesman.
b. Find the total commission for each department.
c. Find the total commissions paid to all salesmen.
d. Find the total sales of each salesman.
e. Find the total sales in each department.
f. Find the total sales for all departments.

(11) A book agent receives a salary of $15 a week and a commission of 5% on all the books he sells. During one week he sold 50 books at $2 each, and 3 dozen books at $3 each. What did he receive for his week's work?

CHAPTER XVII

BILLS, ACCOUNTS, AND INVENTORIES

Do you keep an account of the money earned and the money paid out during the week? What are the advantages of keeping such accounts? Why is "Keeping Accounts" a very important part of every business?

98. Accounts. Every person who earns and spends money is doing business for himself, and the simple form of cash account is the record which shows to him just how his money affairs stand. The simplest form of business record is the *cash account.* A cash account is of no value unless it is correct in every item; it not only shows the cost of living, but also how to save money; everybody should keep one.

(1) Harold Fraser helps to pay for his education by keeping hens. His receipts and expenditures for March, 1921, are as follows:

Income.

Mar. 1. Balance on hand $5.60
Mar. 8. Received for $11\frac{2}{3}$ doz. eggs @ .69 per doz.
Mar. 10. Received for 2 fowls sold, ($8\frac{1}{2}$ lb. @ .50 a lb.)
Mar. 15. Received for $10\frac{1}{2}$ doz. eggs @ .68 per doz.

Expense.

Mar. 1. Cost of pullets @ $2.50
Mar. 1. Cost of food 1.75
Mar. 8. Cost of shells 60

What are the total receipts? The total payments? The balance of cash on hand Mar. 29, 1921?

(2) The following is an account of money received and paid out during one week.

Cash Account of W. A. Strong

1920			Rec.	Pd.
June	5	Cash on hand	$ 2.25	
"	"	Wages	9.50	
"	"	Board		$ 4.50
"	6	Laundry		.75
"	"	Waist, hat		3.73
"	7	Stamps, Stationery		.50
"	8	Carfares, Telephone		.35
"	9	Recreation (theatre)		.75
		Balance		1.17
			$11.75	$11.75
June	13	Cash on hand	$ 1.17	

(3) What are the total receipts in the above cash account? The total payment? The balance cash on hand?

1. The right side of the cash account is the credit side (cr.) or statement of money paid out; the left side is the debit side (dr.) or statement of the money received. Finding the difference between the two amounts is called "Balancing the Account." The amount more or in excess, on the debit side, is the balance of cash on hand. In starting a new account the balance is carried over to the debit side, and entered as though it were cash just received.

(1) Small business houses use the following convenient method of keeping a record of cash received and paid.

a. Find the daily cash balances.

b. Add the balance of the preceding day, the receipts from cash sales and receipts on account; from this sum, subtract the different amounts listed under "cash paid," thus: \$356.68+\$356.72+\$126.78—\$67.32—\$78.85—\$5.60=?

c. Find the totals for each column.

Date	Cash Received				Cash Paid						Daily Cash Balance
	Cash Sales		On Account		Purchases		On Account		Expenses		
											\$356.68
Apr. 1	\$356	72	\$126	78	\$67	32	\$ 78	85	\$5	60	
2	297	25	87	42	56	48	49	16	9	10	
3	368	29	115	39	125	13	132	45	24	36	
4	275	32	76	45	143	89	37	48	8	65	
5	485	88	209	30	96	15	59	95	37	20	
6	396	30	98	67	29	60	47	75	19	05	

d. Copy the following and fill in the balances.

Robert Hall's Account with Citizen's National Bank

Balance June 1.			\$375.00
Deposited June 1.			65.00
			?
	Check	\$42.80	
		?	
			?
Deposited, June 10.			\$150.00
	Check	\$46.00	
		?	
		?	?

		Cash Account		
1920			Dr.	Cr.
Feb.	2	Cash on hand	$ 6.35	
"	"	Board and laundry		$ 6.50
"	3	Tickets, stamps, tel.		.87
"	5	Week's wages	11.50	
"	7	Board and laundry		6.50
"	9	Clothing		12.25
"	10	Week's wages	11.50	
"	14	Board and laundry		6.75
"	17	Books, amusements		2.15
"	19	Week's wages	11.50	
"	21	Board and laundry		6.50
"	25	Week's wages	11.50	
		Balance		
Mar.	1	Cash on hand		

EXERCISE

(1) Find the amount of money received and paid out during the month of February in the above cash account.

(2) Find the balance on hand.

(3) What does Dr. stand for? Cr?

(4) What is an account?

(5) Rule cash-account forms for problems *a*, *b*, and *c*. Enter the data given and balance each account.

a. Cash on hand, May 1, \$1.75; received week's salary, May 1, \$13.25; paid for tickets, books, papers, May 1, \$1.18; paid for board, May 1, \$7.00; May 2, paid for a pair of shoes, \$2.50, and 25¢ for shoe blacking; May 3, paid for laundry, 75¢; May 5, paid 17¢ for stamps and 50¢ for theatre tickets.

b. Cash on hand, Sept. 1, \$2.50; received week's salary, \$15.00; paid for board, Sept. 1, \$8.75; for laundry, \$.96; Sept. 3, paid 50¢ for carfares, 8¢ for stamps, 15¢ for stationery, 25¢ for charity; Sept. 5, found 25¢, paid 15¢ for candy, \$1.25 for underwear; Sept. 6, paid 98¢ for a hat.

c. Cash on hand, Aug. 1, \$5.75; received wages, \$13.75; received Aug. 1 for extra work, 75¢; paid for board, Aug. 1, \$6.75, for laundry, 95¢; Aug. 2, paid \$2.25 for care of teeth; Aug. 3, received 75¢ for extra work; Aug. 4, paid 98¢ for a hat; Aug. 5, paid 76¢ for carfares, papers, stamps.

(6) Make an account for a boy who as treasurer of a club, received and paid out the following sums: Received, subscriptions of members, \$21.50; from sale of tickets for entertainments, \$16.75; paid out, rent of room, \$8.75; printing and advertising, \$6.95; music and other expenses, \$5.00.

(7) Can you suggest a convenient form in which a person can keep an account of personal receipts and expenditures? Make up such an account, containing at least six receipt items, and six expenditures.

(8) A merchant's receipts for the successive days of the week beginning with Monday were as follows: \$136.95; \$125.98; \$350.16; \$92.25; \$105.24; \$372.12. His expenses on Wednesday were \$112.63; on Saturday \$441.25. Make out his weekly statement.

(9) Keep a cash account for a week and show it to your teacher.

99. Statements and bills. Merchandise is not always paid for when purchased, but a record or account of such purchases is kept, and payments made as in the account below.

1. A *statement* is the record of an account up to a certain date.

Boston, Mass., Oct. 5, 1915.

Mr. John Black,
Boston, Mass.
To R. H. White Co.

			Dr.	Cr.
Sept.	3	Balance Due	$ 3.24	
"	5	1 Suit	23.00	
"	8	1 Hat	5.00	
"	9	Cash		$10.00
"	15	6 Shirts $.75	4.50	
"	18	3 Collars $.33$\frac{1}{3}$	1.00	
"	30	Cash to balance		

(1) Study carefully the above account.

a. Who is the creditor? Who is the debtor? Why?

b. What is the amount of the purchases? Of the credits?

c. What is the balance? How is it found?

(2) Make out a statement of the following items, similar to the above:

Mrs. J. R. Day purchased from the Jordan, Marsh Co. during the month of May last, the following articles of merchandise which were charged to her account:

Balance due from previous month, $8.75; May 1, bought

7 yd. serge at $1.25; May 5, bought 1 suit, $15.75; May 12, bought a book for $3.75; May 15, bought 2 pairs of shoes @ $3.75 a pair; May 18, bought $\frac{1}{2}$ doz. pairs of stockings at $37\frac{1}{2}$¢ a pair; May 25, returned 1 pair of shoes at $3.75.

(3) What are the advantages in keeping an account in any business?

a. To the employer?

b. To the employee?

c. To the customer?

(4) What are some of the disadvantages?

2. A detailed statement of the amount, kind, and price of the goods bought or sold, together with the names of the parties to the transaction, terms of credit, conditions of sale or purchase, discount allowed, etc., is called either a bill or an invoice. The term "invoice" is now often used interchangeably with "bill." The following bill is one of the simplest forms and is the one most used in a great many lines of business.

Los Angeles, Cal., Feb 1, 1920.

Morehouse, Martens Co.,
Columbus, Ohio.

Bought of A. Hamburger & Sons Co.

Terms, Cash — Telephone: Main 269.

Jan.	8	54 yd. Ingrain Carpeting	@	$ .94	$ 50	76	$		
		38 yd. Brussels "	@	2.95	112	10			
	10	12 Rugs	@	19.75	237	00			
	15	250 yd. Straw Matting	@	.98	245	00		644	86

Received payment,
A. Hamburger & Sons Co.
(Per James Smith)

(1) In the above bill who bought the goods? Who sold the goods?

(2) Name the debtor and the creditor in this bill.

(3) What were the articles bought? When was the bill made out?

(4) The items should contain (*a*) the quantity of the article purchased, (*b*) its price and (*c*) its cost. (Entering cost on bill is sometimes called "filling.") State the quantity, price, and cost of each item of this bill.

(5) What was the total amount of the bill? (This is sometimes called footing.)

3. When the bill is paid, the words "Received payment" or "Paid" are written at the foot, followed by the creditor's name. This is called receipting the bill.

State the name of the person who receipted the bill for the firm.

100. Price and cost. In a sale of a given quantity of material, the price is the amount paid for one unit. The cost is the amount paid for the entire quantity.

Rule a bill-form similar to that on page 265; use your own name for the purchaser and the name of some firm in your town who deals in the articles purchased, for that of the seller. Use the date of to-day, fill, foot, and receipt the bill.

$8\frac{1}{2}$ yd. Velvet @	\$2.75	per yd.
$12\frac{3}{8}$ yd. Cretonne @	$1.87\frac{1}{2}$	per yd.
$6\frac{2}{3}$ yd. Tricotine @	3.75	per yd.
$9\frac{3}{4}$ yd. Silk @	3.50	per yd.
$7\frac{5}{8}$ yd. Baronet Satin @	2.64	per yd.
$4\frac{1}{2}$ yd. Tricolette @	2.98	per yd.
5 yd. Lace @	.79	per yd.

WHOLESALE DRYGOODS

Chicago, Ill., June 12, 1920.

Messrs. O. T. Johnson Co.,
Galesburg, Ill.

Bought of Marshall, Field & Co.,
Franklin St. and Fifth Av.

Terms, 60 days.

10	pc. Crêpe de Chine			$				
	38 41 40 41 40 40 42 38 42 41	403	98¢	394	94			
8	Chiffon Taffeta							
	39 40 39 38 38 39 40 39	312	$1.25	390	00			
6	Lining Satin							
	42 40 39 41 39 40	241	$ .37½	90	37½			
12	Tailors' Serge 40 41 38 41 39 40 42 40 40 42 38 38	479	$1.13	541	27			
8	Nainsook							
	39 42 38 40 41 40 39 41	320	$ .25	80	00	$1496	58½	

EXERCISE

(1) Name the debtor and creditor in the above bill.

(*a*) This bill is not receipted. Why?

(*b*) The series of numbers: 38, 41, etc., represent the

number of yards in each of the different pieces of cloth. The total number of yards should be found by horizontal addition.

(*c*) Explain the difference between the cost and price of different articles.

(2) Make out in proper form, fill, foot, and receipt the following bills:

a. Mrs. A. F. Ring bought of Denver Drygoods Co., Denver, Col., Jan. 9, 1919:

25 yd. Carpet @ $1.75
14 yd. Carpet @ 0.75
8 Rugs @ 3.75
24 Stair Pads @ .37½

b. On June 4, 1920, Mrs. John May bought of Riker, Kumler Co., Dayton, Ohio:

24 yd. Muslin @ $ .37½
12 yd. Linen @ 1.25
8 yd. Silk @ 1.75
10 yd. Satin @ 2.50
5 yd. Serge @ 1.98

c. B. Altman & Co., New York, bought of Tiffany & Co., May 3, 1920:

12 Gold Watches @ $85.50 each
25 sets of Silver Knives @ $48.00 a set
40 sets of Silver Spoons @ $20.50 a set
10 Silver Trays @ $19.75

d. Houghton & Dutton Co. bought of Jordan, Marsh Co:

2 dozen Men's Caps @ $.98 each
5 dozen pr. Men's Hose @ $0.62½ each
½ dozen Silk Vests @ $1.25 each
5 dozen Collars @ $1.50 a dozen
50 Ties @ $.25 each

(3) Make out the bill which Mrs. W. S. Dean, 27 Marlow St., Milwaukee, should receive January 1 for the following articles charged December 21 to her account with the Boston Store, Milwaukee. A doll for 85¢; 2 boxes of blocks @ 35¢; doll carriage for $1.35; toy motor car for 75¢; "Alice in Wonderland," 48¢; "Little Women," 98¢; Shakespeare's Works (16 volumes) @ 42¢ each; $4\frac{3}{4}$ yd. ribbon @ 38¢ a yd; 12 yd. baby ribbon @ $2\frac{1}{2}$¢ a yd; $1\frac{1}{2}$ lb. nuts @ 20¢ a lb; $2\frac{1}{2}$ lb. raisins @ 28¢ a lb.; 3 lb. candy @ 85¢ a lb.

(4) Make out and receipt bill for the following, supplying dates, names, and addresses: 4 shot guns @ $13.75; 10 shell bags @ 50¢; 4 shot gun covers @ $1.25; 12 pocket knives @ 48¢; 3 pairs Indian clubs @ 17¢ each; 4 baseballs @ $1.25; 16 yd. silk @ $1.87; 2 pairs gloves @ $.75; 12 yd. flannel @ $62\frac{1}{2}$¢; 5 dozen handkerchiefs @ $2.50 a dozen; 3 tablecloths @ $2.75 each; 2 hats @ $6.75 each.

(5) What is the meaning of the following abbreviations used in connection with bills and accounts?

@, cr., dr., pay't, No. or #, $\frac{a}{c}$, bal., per, rec'd., terms, 30 days.

(6) State as many different ways of receipting bills as you can.

(7) Collect as many different kinds of bills, statements, accounts, etc., as you can, and point out the differences in them.

(8) How do most business people pay their bills?

101. Checks. Do you know what a check is, as used in connection with banks in paying bills? Get a blank check from some bank and make a copy of it.

1. What are the advantages of paying by check? The disadvantages?

A bill for $16.75 was received by Mr. E. A. Horne

from A. J. Brown, who sent the following check in payment of the same:

BOSTON, May 3, 1919. No. 25

OLD COLONY TRUST COMPANY 5–129

PAY TO THE ORDER OF

Albert J. Brown $ 16.75

Sixteen and $\frac{75}{100}$ DOLLARS

Edward A. Horne.

2. What are the essential parts of a check? Where is the money deposited with which this check is to be paid?

3. If you receive a check from someone, you must indorse it preferably at time of cashing; that means that your name must be written on the back exactly as it appears on the face of the check.

4. Who made out this check? To whom is it payable? Whose name should appear on the back of the check when it is cashed?

5. How are the sums of money written? Why written twice? What is the last item on the check?

6. If your name were incorrectly written on the face of a check, how would you indorse it? How could you cash a check received by you, made out, and indorsed by someone else?

(1) Make out a check on the Federal Trust Co. for $37.50, payable to Mary E. Baker, over your own signature.

(2) Bring to the class a number of cancelled checks and trace them from the time they were issued until they were filed as receipts by the drawer.

(3) Show why a cancelled check is a kind of receipt for the payment of money.

(4) Mr. A. W. Penn has received in his mail the following bill, which he wishes to pay by check. Make out the check. Copy the bill and inclose both in an envelope to send to Bowman & Co., Harrisburg, Pa.

New York, May 8, 1920.

Mr. A. W. Penn,
125 East Broadway,
Bought of Bowman & Co.,
Harrisburg, Pa.,
Drygoods Merchant.
Terms, Cash:
125 suits @ $12.75 = $1593.75.

WILLIAM G. WEBBER CO.

ESSEX AND WASHINGTON STREETS

SALEM, MASS.

A MONTHLY SETTLEMENT OF ALL ACCOUNTS IS REQUIRED

ITEMIZED STATEMENT OF YOUR ACCOUNT FOR THE MONTH OF OCTOBER, 1918

SOLD TO Mrs. Frances Brainard
Symsfield,
Mass.

YOUR CHECK IS A RECEIPT. DETACH AND RETURN THIS STUB IF YOU DO NOT WISH ANOTHER RECEIPT.

7. Why are bill-heads similar to the one above, an advantage to a customer? To a dealer? How does this bill-head differ from the usual form?

102. Inventories. An inventory is an itemized account of the property possessed by an individual, corporation, or firm.

1. An inventory is made at regular intervals during the year to find out the value of the stock on hand.

2. Inventories are usually taken by two or more clerks working together; the number of items of each kind is counted, and the cost price marked on the goods is read aloud by one, while the second clerk records the facts. The value of each item is extended in the office where the inventory is sent, and the total value of the "stock on hand" is made.

(1) A simple form for periodic inventories may be used similar to the following:

Quantity	Name of Item	Cost		Cost Extension	
250 yd.	Brussels Carpet	$1	75	$437	50
325 yd.	Tapestry Carpet	1	25	406	25
275 yd.	Ingrain Carpet	0	98	269	50

(2) Rule an inventory similar to the one above, enter the following items, find the cost of each item, and the total cost.

12 pc. cotton, 56 yd. each—42¢ per yd.
18 pc. flannel, 35 yd. each—98¢ per yd.
12 pc. gingham, 44 yd. each—75¢ per yd.
6 pc. muslin, 36 yd. each—45¢ per yd.
10 pc. scrim, 42 yd. each—68¢ per yd.

(3) From the prices and quantities listed in the following table make inventories similar to the form in (1) combining prices and quantities in columns numbered to correspond.

Price List				Inventory		
Article	1.	2.	3.	1.	2.	3.
Hosiery	$9.98	$2.25	$ 3.75	125 pr.	265 pr.	475 pr.
Union Suits	5.98	4.50	3.65	150	225	445
Skirts	4.75	6.98	12.49	598	450	1075
Camisoles	0.98	1.49	2.69	1056	872	615
Waists	4.75	6.49	7.50	560	476	365
Books	1.25	3.50	2.19	298	175	110
Umbrellas	0.98	1.49	2.98	365	478	265
Parasols	1.25	3.98	4.75	275	198	256
Hats	2.98	4.25	5.50	165	138	149
Caps	0.25	0.69	1.79	278	325	264

3. See inventory page 272. Different styles of goods are given "Lot Numbers" by either the manufacturer or the merchant, and this number is entered in the inventory in place of a description. The "size" column shows the different sizes of the articles in stock. Each size may be entered on a separate line, or all sizes may be entered on the same line. Thus, $\frac{2}{6}$, $\frac{3}{4}$, means 2 No. 6's, 3 No. 4's.

4. Goods purchased or sold during the taking of inventory may be added or subtracted by using the "Add" or "Deduct" columns, so that the value of stock at the end of the inventory may be obtained.

5. The cost extension is found by multiplying the cost per item by the number of items, plus the number of items in the "Add" column, or minus those in the "Deduct" column.

6. Depreciation of the value of goods is entered in the "%" depreciation column. If an article has become shop-worn, or faded, and is worth only one-third of its value, a $33\frac{1}{3}\%$ depreciation is entered in the "%" depreciation column, and then $33\frac{1}{3}\%$ of the cost extension of that item is entered in the "Loss Assumed" column; the total of this column is subtracted from the total of the "Cost Extension."

ORAL PROBLEMS

(1) Goods are worth $\frac{1}{8}$ of their original cost. What is the per cent depreciation?

(2) What does this per cent depreciation mean?

(3) What is the per cent depreciation when goods are worth $\frac{1}{3}$ of their original cost?

(4) What is the per cent depreciation when goods are worth $\frac{1}{6}$ of their original cost?

(5) Waists bought for $11.40 can now be purchased for $3.80. What per cent depreciation should be entered in the "per cent depreciation" column? What would be the amount of the loss on 18 of these waists?

FORM OF INVENTORY

Add	Deduct	Lot No.	Size	Name and Quantity	Cost Price	Cost Extension	% Dep.	Loss Assumed
		A 26	$9\frac{1}{2}$	12 pr. Tan Hose	1 98	23 76		
	2	B 13	10	15 pr. Black Hose	2 75	35 75		
3		C 8	$\frac{3}{9}$ $\frac{5}{10}$	8 pr. White Hose	79	8 69	10%	.86

(6) Rule an inventory form similar to the one above and enter the following items. Find the value of each item listed and the total value of the stock.

Lot no. 18A; Size $6\frac{1}{2}$, 24 pr. Tan Dogskin Gloves, $4.50 per pr.
Lot no. 10B; Size 6 , 50 pr. Black Kid Gloves, $2.75 per pr.
Lot no. 5C; Size $5\frac{1}{2}$, 36 pr. Gray Mocha Gloves, $3.75 per pr.
Lot no. 2D; Size 7 , 18 pr. White Silk Gloves, $1.50 per pr.
Lot no. 3E; Size $7\frac{1}{2}$, 16 pr. Tan Fabric Gloves, $1.25 per pr.

Lot no. 3E can now be purchased for $1 per pair. The following sales were made during stock taking:

1 pair Lot no. 10B
1 pair Lot no. 5C
1 pair Lot no. 2D

7. A perpetual inventory or stock record is kept in a book, one page being used for each item carried in stock;

both purchases and sales are recorded at cost price. Below is a form of perpetual inventory.

PERPETUAL INVENTORY

NO. B 20 SMALL WARES											
Date		Purchases				Sales			Balance		
1918		No.	Cost	Value		No.	Value		No.	Value	
May	7	40	12	4	80				40	4	80
"	9					35	4	20	5		60
"	11	60	12	7	20					7	80
"	20					17	2	04		5	76

8. The stock record or perpetual inventory shows the amount of stock kept in warehouses or in store rooms, and from it the amount of stock on hand can be estimated without delay. When the balance on hand has decreased to the "danger point" which is fixed for each item, a new supply is bought.

(1) Rule a page for a stock record similar to the above model and enter the following data, making a perpetual inventory of the stock of the New England Supply Co.

Purchases

March 5, 16 rugs @ $12.75
April 9, 24 rugs @ $12.75
May 10, 18 rugs @ $12.75

Sales

March 10, 3 rugs; Mar. 11, 5 rugs.
April 12, 3 rugs; April 15, 6 rugs.
May 13, 9 rugs; May 18, 12 rugs.
May 23, 4 rugs; May 29, 5 rugs.

(2) Why is making up inventories or "Taking Stock" a very important part of a business at the time when books are to be closed?

(3) Find out from local firms all you can relative to the making of inventories; get all the information merchants will give you as to the manner and time of taking inventories, and the value of them to the business.

(4) When should an article be inventoried at less than cost? Merchants often have a "pre-stock-taking" sale. Why is this done?

(5) The inventory reveals many important facts. What are they? What will be the effect on the profit and loss of the business if the inventory is valued too high?

(6) What will be the effect on the profit and loss of the business if the inventory is valued too low?

CHAPTER XVIII

PAYROLLS, EXPRESS AND FREIGHTAGE, POSTAL SERVICE

Payroll 1231 For the week ending May 6, 1919

No.	Name	No. of hours' work each day						Total No. Hours	Wages Per Hour	Total Wages		Remarks
		M	T	W	T	F	S					
1.	M. E. Smith	7	7	7	7	7	7	42	25¢	10	50	
2.	A. C. Fox	8½	8½	9	8½	7½	8	50	37½¢	18	75	
3.	J. E. Kane	9	9	9½	9	9	9½	55	50¢	?		
4.	M. A. Marr	8	7½	8	8½	8	8	?	35¢	?		
5.	S. A. Dorr	8	8	8	8	8	8	?	35¢	?		
6.	R. E. Finn	9	9	9	8½	9	8½	?	45¢	?		
7.	C. S. O'Hare	7	7	7½	7½	7	7	?	25¢	?		
8.	D. C. Bell	8	8	8	8	8	8	?	40¢	?		
9.	E. H. Carr	9	9	9	9	9	9	?	50¢	?		
								?		?		

103. Payrolls. Copy the above payroll and fill out the blank spaces as shown in Nos. 1 and 2. Some firms use checks in paying off employees, but large business concerns find it more convenient and satisfactory to pay in cash by

the envelope system. To do this it is necessary first to find the amount of money needed, and then the bills and fractional currency that are required to pay each employee. A blank called a change memorandum, showing the method of finding just the denomination wanted for the payroll is used, similar to the one shown below.

(1) Fill out the change memorandum given below for the payroll given above. Make totals, and check results.

No.	BILLS					COINS					
	$20	$10	$5	$2	$1	50¢	25¢	10¢	5¢	1¢	
1.		1				1					
2.		1	1	1	1	1	1				
3.											
4.											
5.											
6.											
7.											
8.											
9.											

1. When the amount of the payroll, the necessary bills, and the fractional currency, have been ascertained, a check payable to the order of payroll is made out and attached to a payroll memorandum, similar to the form on page 277, and both sent to the bank where the necessary amount of each denomination is secured. The payroll memorandum should foot in the same way as the payroll book.

2. In a large payroll, the experienced and efficient bookkeeper can frequently make an accurate estimate of the kind of change required by first scanning the payroll to find the number of pennies required, then the number of nickels, etc.

(1) Make out a payroll memorandum for the preceding payroll, using the form on p. 277 as a model.

(2) Using the model payroll 1231, page 275, find the amount of it at the following wages per hour: 1, 20¢; 2, 26¢; 3, 28¢; 4, 29¢; 5, 38¢; 6, 37½¢; 7, 66⅔¢; 8, 25¢; 9, 62½¢.

(3) Make a change memorandum for problem (2).

(4) Make a payroll memorandum for problem (2).

(5) Estimate the bills and change required for payroll following and check totals.

First National Bank,
Newark, New Jersey.
Payroll Memorandum.
L. Bamberger & Co.
require the following

Pennies	12	12
Nickels	3	15
Dimes	7	70
Quarters	6	1 50
Halves	7	3 50
Dollars	11	11 00
2's	7	14 00
5's	6	30 00
10's	9	90 00
20's	4	80 00
		230 97

(6) Copy the following payroll and fill in amount due each employee and the totals.

	Name	Total number of hours	Wages per hour	Total Amount	Denomination								
					$10	5	2	1	.50	.25	.10	.05	.01
1	H. A. Smith	36½	$.35	?									
2	A. F. Gray	48	.37½	?									
3	R. M. Winters	42¾	.50	?									
4	E. A. Collins	40	.33⅓	?									
5	W. M. Hart	36	.45	?									
6	F. R. Cole	47¼	.36	?									
Total number of each bill and coins.													

Name	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Per Hour Rate	Per Week Total Wages	
Mary Ring	7	8	8	7	8	8	$.25		
Anna Small	8	8	8	8	8	8	.35		
Frances Ivy	9	9	8	9	8	8	.20		
Rose Gray	7	8	8	7	8	8	.30		
Helen Long	6	6	6	7	7	7	.40		
Agnes Small	8	7	7	8	8	8	.25		
Ray Fisher	9	8	8	9	8	9	.35		
Anna Black	8	8	9	9	8	9	.30		

(7) Calculate total wages, make payroll memorandum, and change memorandum for above problem.

104. Expressage. The tables given below show the express rates since Oct. 13, 1920, and suggest the manner in which charges are made for given amounts to certain distances; the country is divided into blocks, each block covering a certain designated territory, and fixed rates are made for certain weights and distances.

1. First-class matter includes all merchandise; second-class matter applies to food and drink. The difference in charges for first-class matter and second-class matter for large packages and long distances, is considerable, but for small packages, the difference, if any, is very slight.

105. Freightage. Bulky goods (furniture, etc.) are generally sent by freight and are subject to different rates, according to quantity and character. All freight is divided into six classes: pianos, musical instruments, furniture, statuary,

etc., are first-class matter, and are charged the highest rates; furniture encased is subject to a double first-class rate. Carloads of baled hay, etc., are generally considered fifth-class matter, while carload lots of brick, building blocks, etc., are considered sixth-class matter, on which the lowest charges are made.

Shipments weighing less than 100 lb. are charged as 100 lb. between most points.

	Between Boston and Chicago		*Between Boston and Philadelphia*		*Between St. Louis and Boston*	
	1st class	*2d class*	*1st class*	*2d class*	*1st class*	*2d class*
5 lb.	$.52	$.51	$.43	$.40	$.53	$.53
6	.54	.51	.45	.40	.57	.53
7	.58	.51	.47	.40	.60	.53
8	.60	.51	.48	.40	.64	.53
9	.66	.51	.52	.40	.68	.53
10	.68	.51	.53	.40	.71	.53
15	.84	.63	.60	.45	.89	.67
20	1.00	.76	.71	.53	1.18	.81
25	1.17	.88	.79	.59	1.26	.95
100	3.67		2.14		4.02	

	Between Boston and New York		*Between Boston and San Francisco*		*Between Boston and Pittsburg*	
	1st class	*2d class*	*1st class*	*2d class*	*1st class*	*2d class*
5 lb.	$.40	$.35	$1.07	$1.07	$.47	$.44
6	.42	.35	1.21	1.21	.50	.44
7	.43	.35	1.36	1.34	.53	.44
8	.44	.35	1.50	1.34	.54	.44
9	.45	.35	1.65	1.34	.57	.44
10	.47	.35	1.79	1.34	.59	.44
15	.53	.40	2.65	1.88	.72	.54
20	.59	.44	3.23	2.42	.84	.63
25	.66	.50	3.94	2.96	.97	.73
100	1.59		14.75		2.84	

WRITTEN EXERCISE

(1) Find the total cost of sending the following packages:

a 16 lb first class, from Boston to New York.

b. 25 lb. second class, from Boston to Pittsburg.

c. 10 lb. first class, from Boston to San Francisco.

(2) Find the total cost of sending the following packages:

a. 21 lb. first class, from Chicago to Boston.

b. 18 lb. second class, from Boston to Pittsburg.

c. 10 lb. second class, from Boston to St. Louis.

d. 5 lb. first class, from Boston to Philadelphia.

e. 8 lb. first class, from Boston to New York.

New York, New Haven & Hartford R. R.
Local Freight Tariff Between
Boston, Mass.,
and the following stations.

Stations	Rate per 100 lb. Classes						Miles
	1	2	3	4	5	6	
Andover, Conn	28	24	20	15	11	9	94
Bridgewater, Mass	15	13	11	8	6	5	27
Clinton, Conn	32	27	22	18	13	11	132
Dedham, Mass	12	10	8	7	5	4	$8\frac{1}{2}$
Hyannis, Mass	25	21	18	14	10	8	78
New Rochelle, N. Y	35	30	25	19	14	12	195
No. Cohasset, Mass	13	11	9	7	5	4	19
Waterbury, Conn	33	28	23	18	13	11	149
Duxbury, Mass	17	14	12	9	7	6	38
Plymouth, Mass	17	14	12	9	7	6	37
New Bedford, Mass	20	17	14	11	8	7	52
Lowell, Mass	22	19	15	12	9	7	61
Mt. Vernon, N. Y	35	30	25	19	14	12	209

(3) Using the table, find the amount of freight on 15,000 lb., third-class matter, from Boston to Plymouth.

(4) Using the table, find the amount of freight on 25,360 lb. first-class matter, and 320 lb. second-class matter, from Boston to New Rochelle, N. Y.

(5) Using the above table, find the amount of freight on 165 lb. first-class matter, 180 lb. second-class matter, 1500 lb. third-class matter, 2400 lb. fifth-class matter, and 3600 lb. sixth-class matter from Boston to Andover, Conn.; to Bridgewater, Mass.; to Clinton, Conn.; to Dedham, Mass.; to New Rochelle, N. Y.

106. United States Postal Service. All mailable matter for transmission by the U. S. Mails, within the U. S. or to Mexico, Cuba, Hawaii, Porto Rico, Philippine Islands, and Canada, is divided into four classes: first, second, third, fourth.

1. First-class matter includes letters, and all mail which is sealed. Type-written and written matter is first-class, sealed or unsealed. Rate: 2 cents an ounce or fraction thereof. Post-cards and postal cards, 1¢ each. Second-class matter includes printed newspapers and periodicals, and the rate is 1¢ for each 4 oz. When sent by publishers or news agents, the rate is 1¢ a lb.

2. Third-class matter includes books, circulars, pamphlets, proof sheets, and engravings; the rate is 1¢ for each 2 oz. and the weight is limited to 4 lb. All postal matter of first, second, or third class may be registered at the rate of 10¢ for each package in addition to the regular rate of postage. The rates on special delivery letters are 10¢ a letter in addition to the regular postage.

3. Foreign rates of postage: letters, 5¢ an ounce for the first ounce, 3¢ for each additional ounce; postal cards, 2¢ each; newspapers, etc., 1¢ for 2 oz. Germany and Great Britain have a letter rate of 2¢ an ounce.

4. Fourth-class matter includes all domestic parcel post mail. For parcel post purposes the United States is divided into eight postal zones—the rate of postage depending upon the zone where the parcel is to be delivered. Regular postage stamps are used on parcel post packages. The rate for par-

cels weighing 4 oz. or less is 1¢ for each ounce, regardless of distance. The rates for parcels weighing more than 4 oz. (pound rate) are shown in the following table. The weight limit for zones 1 and 2 is 50 lb.; for all other zones, 20 lb.

Weight in pounds	Local	Zones							
		1st Up to 50 miles	2d 50 to 150 miles	3d 150 to 300 miles	4th 300 to 600 miles	5th 600 to 1,000 miles	6th 1,000 to 1,400 miles	7th 1,400 to 1,800 miles	8th Over 1,800 miles
1	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2	.06	.06	.06	.08	.11	.14	.17	.21	.24
3	.06	.07	.07	.10	.15	.20	.25	.31	.36
4	.07	.08	.08	.12	.19	.26	.33	.41	.48
5	.07	.09	.09	.14	.23	.32	.41	.51	.60
6	.08	.10	.10	.16	.27	.38	.49	.61	.72
7	.08	.11	.11	.18	.31	.44	.57	.71	.84
8	.09	.12	.12	.20	.35	.50	.65	.81	.96
9	.09	.13	.13	.22	.39	.56	.73	.91	1.08
10	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
11	.10	.15	.15	.26	.47	.68	.89	1.11	1.32
12	.11	.16	.16	.28	.51	.74	.97	1.21	1.44
13	.11	.17	.17	.30	.55	.80	1.05	1.31	1.56
14	.12	.18	.18	.32	.59	.86	1.13	1.41	1.68

Weight in pounds	Local	Zones							
		1st Up to 50 miles	2d 50 to 150 miles	3d 150 to 300 miles	4th 300 to 600 miles	5th 600 to 1,000 miles	6th 1,000 to 1,400 miles	7th 1,400 to 1,800 miles	8th Over 1,800 miles
15	.12	.19	.19	.34	.63	.92	1.21	1.51	1.80
16	.13	.20	.20	.36	.67	.98	1.29	1.61	1.92
17	.13	.21	.21	.38	.71	1.04	1.37	1.71	2.04
18	.14	.22	.22	.40	.75	1.10	1.45	1.81	2.16
19	.14	.23	.23	.42	.79	1.16	1.53	1.91	2.28
20	.15	.24	.24	.44	.83	1.22	1.61	2.01	2.40
21	.15	.25	.25						
22	.16	.26	.26						
23	.16	.27	.27						
24	.17	.28	.28						
25	.17	.29	.29						
26	.18	.30	.30						
27	.18	.31	.31						
28	.19	.32	.32						
29	.19	.33	.33						
30	.20	.34	.34						
31	.20	.35	.35						

Weight in pounds	Local	Zones							
		1st Up to 50 miles	2d 50 to 150 miles	3d 150 to 300 miles	4th 300 to 600 miles	5th 600 to 1,000 miles	6th 1,000 to 1,400 miles	7th 1,400 to 1,800 miles	8th Over 1,800 miles
32	.21	.36	.36						
33	.21	.37	.37						
34	.22	.38	.38						
35	.22	.39	.39						
36	.23	.40	.40						
37	.23	.41	.41						
38	.24	.42	.42						
39	.24	.43	.43						
40	.25	.44	.44						
41	.25	.45	.45						
42	.26	.46	.46						
43	.26	.47	.47						
44	.27	.48	.48						
45	.27	.49	.49						
46	.28	.50	.50						
47	.28	.51	.51						
48	.29	.52	.52						
49	.29	.53	.53						
50	.30	.54	.54						

EXERCISE

(1) What is the usual letter postage? What does a letter weigh that requires a 4¢ postage? A 10¢ postage?

(2) How much would you pay in stamps for a letter weighing 9 oz.? $2\frac{3}{4}$ oz.? For newspapers weighing 15 oz.? 18 oz.?

(3) Find the cost of mailing 84 advertisement booklets, if each booklet weighs $6\frac{1}{2}$ oz.

(4) Find the cost of postage on the following articles, assuming that the wrappings are included in the weights given:

a. Eight books weighing $11\frac{1}{2}$ oz. each.

b. Six magazines weighing $4\frac{1}{2}$ oz. each and a letter weighing $1\frac{1}{4}$ oz.

c. 18 letters weighing $2\frac{3}{4}$ oz. each and 3 newspapers weighing $4\frac{1}{2}$ oz. each.

d. Two books weighing 1 lb. 2 oz. each, two registered letters weighing $\frac{1}{2}$ oz. each, and 1 doz. newspapers weighing 7 oz. each.

e. A letter weighing $\frac{3}{4}$ oz. sent by special delivery, 15 letters weighing less than $\frac{1}{2}$ oz. each, and 3 books weighing 1 lb. 1 oz. each.

(5) Which is the cheaper—to send a manuscript weighing 3 lb. 6 oz. by express at a cost of 45¢, or by mail? To send one weighing 2 lb. 4 oz. if the expressage on it is 45¢ also?

(6) How much is the postage on a special delivery sealed package weighing 8 oz.? On 1500 lb. of newspapers sent by the publishers? On 3600 pounds of newspapers sent by the publishers?

(7) How much is the postage on a boy's sweater weighing 12 oz., to be sent to a post office in the third zone?

(8) What is the expense of sending a package weighing 19 lb. by parcel post to a city 350 miles distant? Of sending

a 37 lb. package 135 miles? Of sending a 12 lb. package 275 miles?

(9) What is the advantage of parcel post delivery to the farmer? To the consumer? To the manufacturer? To the merchant? To all tradespeople?

107. Postal money orders. An order issued by the postmaster in one place directing a postmaster in another place to pay a stated sum of money to a specified person, or his order, is a postal money order. Such an order is issued upon deposit of the amount of money to be sent, the fee for sending, and a written application stating the following facts:

1. The amount of the order.
2. Name and address of payee.
3. Name and address of sender.

FEES FOR MONEY ORDERS DRAWN ON DOMESTIC FORMS

Payable in the United States (which includes Guam, Hawaii, Porto Rico, and Tutuila, Samoa); or payable in Bermuda, British Guiana, British Honduras, Canada, Canal Zone (Isthmus of Panama), Cuba, Newfoundland, at the United States Postal Agency at Shanghai (China), in the Philippine Islands, or the following islands in the West Indies: Antigua, Bahamas, Barbados, Dominica, Grenada, Jamaica, Martinique, Montserrat, Nevis, St. Kitts, St. Lucia, St. Vincent, Trinidad, and Tobago and Virgin Islands.

For Orders From	$ 0.01	to $ 2.50	..... 3 cents.
From	2.51	to 5.00	..... 5 cents.
From	5.01	to 10.00	..... 8 cents.
From	10.01	to 20.00	.....10 cents.
From	20.01	to 30.00	.....12 cents.
From	30.01	to 40.00	.....15 cents.
From	40.01	to 50.00	.....18 cents.
From	50.01	to 60.00	.....20 cents.
From	60.01	to 75.00	.....25 cents.

NOTE.—The maximum amount for which a single Money Order may be issued is $100. When a larger sum is to be sent additional orders must be obtained. Any number of Orders may be drawn on any Money Order office on any one day.

Problems

(1) If you wished to send a money order for $35.50 to James A. Long, 125 Central St., Duluth, Minn., what would you write in each blank space in the following application?

APPLICATION FOR DOMESTIC MONEY ORDER

Spaces below to be filled in by purchaser, or, if necessary, by another person for him

Amount

..... Dollars .....Cents

Pay to Order of .....

(Name of person or firm for whom order is intended)

.....

Whose Address is No.....Street

Post Office .....

State.....

Sent by.....

(Name of Sender)

.....

Address of sender No.....Street.

Purchaser must send order and coupon to payee

(2) What must be paid to the postmaster in addition to the amount you wish to send to Mr. Long?

3. The order issued by the postmaster consists of two parts; (1) a receipt for the sender and (2) the order proper which is mailed to the payee. To whom will Mr. Long present the order for payment? What amount will be paid?

(4) Make out an application for a money order which you wish to send to Gay & Davis Co., 125 South St., Albany, N. Y., for $23.69. What will the order cost?

(5) Find the cost of postal money order for the following amounts:

a. $18.60	*e.* $66.80
b. $24.85	*f.* $48.00
c. $30.70	*g.* $32.40
d. $42.75	*h.* $71.00

(6) Mr. Smith sends a money order to pay for the following articles:

1 dozen handkerchiefs @ $.37½ each.
½ dozen pairs of stockings @ $2.25 per pair.
2 pair gloves @ $2.75 per pair.
½ dozen neckties @ $0.98 each.
1 pair shoes @ $9.00
1 hat @ $6.75
Find the cost of the money order.

(7) Mr. Harris bought six money orders for the following amounts:

$8.75; $7.10; $24.16; $13.95; $11.05; $7.65. Find the total cost.

CHAPTER XIX

MATHEMATICS OF THE TRADES

108. Millinery.

1. Hat facings, plain and gathered. The following allowances are usually made for fullness for gathered hat facings:

Malines	— 4 or 5	times	the	length	of	the	edge	wire
Chiffon	— 3 or 4	"	"	"	"	"	"	"
Silk	— 2 or 3	"	"	"	"	"	"	"
Velvet	— $1\frac{3}{4}$	"	"	"	"	"	"	"

Problems

(1) How many widths of 27-inch wide malines are required to face a hat having an edge wire 49 inches long? How many yds. of malines are needed if the brim is 5 inches wide?

Solution: *a.* 49 in. = length of edge wire.
$\times 5$ (times for fullness)
$27\overline{)245}$ inches of malines.
$9\frac{2}{7}$ or 10 widths

For 245 inches as many widths are needed as 27 inches, the width of the maline, is contained times in 245 inches, or 10 widths.

Solution: *b.* 5 inches = width of brim
1 inch for turnings
6 inches of maline needed for 1 width.
$\times 10$
60 inches of maline, or $1\frac{2}{3}$ yds. for 10 widths.

(2) How many widths of maline 32 inches wide are needed for the facing of a hat having a 42-inch edge wire? How many yards of maline are needed if the brim is 6 inches wide?

(3) When chiffon is 33 inches wide, into how many 6 inch widths can $1\frac{1}{2}$ yards chiffon be cut? How many yards of hat facings can be made from these widths?

(4) A piece of velvet $1\frac{3}{4}$ yards long and 20 in. wide is to be used for a hat facing. The edge wire of the hat brim is 38″ long

and the brim 4 inches wide. How much of the velvet will it take and how much will be left?

(5) How many yards of 27-inch silk will be needed for a hat having a 44-inch edge wire and a brim $5\frac{1}{2}$ inches wide?

(6) Wire hat frames that cost \$5.40 a dozen were sold at 60¢ each. What was the gain on $1\frac{1}{2}$ dozen frames? [1]

(7) A milliner bought 2 pieces of hat lining, one containing $42\frac{3}{4}$ yd., the other $64\frac{1}{2}$ yd. She sold $13\frac{3}{4}$ yd., $12\frac{5}{8}$ yd., $17\frac{1}{2}$ yd., and $9\frac{3}{8}$ yd. How many yards remained?

(8) A dealer bought $58\frac{1}{2}$ pieces of net, each piece containing 12 yards, paying 5¢ a yard wholesale. She sold the lot for the retail price of 15¢ a yard. What was the total gain? What was the gain per cent?

(9) It takes $1\frac{1}{2}$ yd. crinoline to cover a hat frame. How many similar frames can be covered from a piece of crinoline containing 42 yards?

(10) A milliner bought $3\frac{1}{2}$ dozen pieces of hat braid for \$10.50. She sold 28 pieces at 42¢ each, and the remainder at 50¢ each. Did she gain or lose? What per cent?

(11) Violets which cost \$11.40 a dozen bunches were sold at \$1.35 a bunch. What was the gain on $1\frac{1}{2}$ dozen bunches?

(12) A milliner buys 15 bolts of ribbon, 10 yards to a bolt, for \$12.75. Five per cent was not salable because it was damaged. The good ribbon was sold for $37\frac{1}{2}$¢ a yard. What was the gain?

(13) A dealer bought 24 yards of velvet at $\frac{3}{4}$ of a dollar, and sold it at $\frac{7}{8}$ of a dollar a yard. Find the gain per cent.

(14) The hat braid used on a hat cost \$4.50, an ostrich plume \$4.75, and making \$2.75. For how much must the hat be sold to gain 90 per cent?

(15) A milliner bought $1\frac{3}{4}$ dozen Panama hats at \$48.00 a dozen and sold $\frac{2}{3}$ of them at \$5.00 each. The others were closed out at the end of the season at a 40 per cent mark down. Did she gain or lose? What per cent?

(16) A hat was marked \$32.00, which was 60% above cost. How much can a milliner reduce the price of this hat, and still make 25%?

(17) A milliner copied a \$75 pattern hat in Paris for \$25, and paid 50% duty in Boston, on the copy. What was the net cost of the hat? What was the gain per cent?

[1] Figure per cent of gain or loss on cost price, in the problems of this section.

(18) She made 12 copies of this hat at a cost of \$25 each and sold them for \$48 each. What was the entire gain if the pattern was sold at the close of the season for \$25?

(19) The week's expenses for a millinery establishment are \$250.00. The sales for the week amount to \$381.00, but one customer owing \$49.00, another owing \$73.00, and another owing \$44.00, failed to pay their bills promptly. What per cent of the week's expenses must be taken from the working capital?

(20) The net profits for one week during the busy season were 75% of the receipts. What were the receipts for the week if \$72.48 was the week's expenses?

(21) A milliner who did \$26,400 worth of business one year paid 20% of the amount for rent and light, 50% for salaries, 5% for packing and delivery of hats, and lost 3% in bad debts. How much was paid for each item of expense? What were the total expenses for the year? What were the profits?

(22) She used 20% of the profits for her own expenses and $\frac{3}{4}$ of the remainder for new stock for the next year. The rest she saved and put in the bank to start the next year's business. How much money did she put in the bank?

109. Receipts and bills. The following forms are commonly used for receipts:

a.

Cleveland, Ohio.
June 12, 1919.

Received of Mary E. Smith, twenty-five dollars (\$25.00) in full to date.

Hall & Company
per H.

b.

Worcester, Mass.
Dec. 1, 1919.

Received of John E. Clexton, ninety-eight dollars and $\frac{75}{100}$ (\$98.75) on account, balance due, \$20.

F. A. Small.

EXERCISE

(1) Make out and complete a bill for a hat which you made and sold to a customer June 10, 1919, using the following items:

36 yd. Braid	@ $0.55
$1\frac{3}{4}$ yd. Satin	@ $2.50
Ostrich Feather	$18.00
Making of Hat	$5.50
Lining and Findings	$1.00

(2) Copy, fill and foot the following bill for a hat which you have made for a customer. Date of bill: March 9, 1920.

Frame.....	$1.50
3 yd. Lace .....@	$0.75
5 Roses.....@	$1.25
$\frac{3}{4}$ yd. Velvet.....@	$2.75
2 bunches Foliage...@	0.62\frac{1}{2}$
Lining and Findings	$1.50
Making.....	$3.00

(3) What are the important parts of a bill? Why is the date of a bill important?

(4) What is meant by "fill" a bill? By "foot" a bill?

(5) Has a receipted bill any value? Give reasons for your answer.

(6) Write a receipt for the bill in Ex. 1.

(7) Write a receipt for the bill in Ex. 2.

(8) What are the important parts of a receipt?

(9) Explain the expressions "in full" and "on account."

(10) Can they both be used in the same receipt? Why?

(11) Why are initials sometimes placed under the signature?

(12) Name different papers which serve as receipts.

(13) What is the value of receipts?

(14) Make out and receipt a bill for a hat, using the following items:

36 yd. Braid @	$ .29
$\frac{7}{8}$ yd. Velvet @	3.20
Goura Feather @	18.00
Making of Hat	3.50

(15) Fill, foot, and receipt the following dressmaker's bill:

8 yd. Lawn @	$ .37$\frac{1}{2}$
18 yd. Lace @	.62$\frac{1}{2}$
Findings	2.75
Making	15.00

(16) Date, fill, and foot the bill for the following transaction:

A. M. Wing & Co., of New York, sold to E. F. James & Co., of Boston, discount 5%:

16 doz. Waists @	$ 45.00 a doz.
8 doz. Skirts @	25.80 a doz.
12 doz. Coats @	101.40 a doz.

(17) John H. Prince owed Henry C. Abbott $470.25 and paid $200.00 today. Write the receipt for the person who paid the money.

(18) Geo. W. Cowan paid $156.00 on a bill for $325.00 which he owed Plimpton, Fisk & Co. Write the receipt for the person who paid the money.

(19) John J. King & Co. received $250.00 as part payment of a bill for $875.00 held against Fred C. Wood. Write the receipt for this payment.

Hat Problems

(1) *a.* A hat factory employs 25 women to sew on crowns and put in linings at $1.50 per dozen hats. What are the average earnings per woman when the weekly output is 4800 hats?

b. In the same factory 3 men varnish these hats at 15 cents per dozen. What are the weekly earnings of the varnishers?

(2) (*a*) Find the cost of the labor of preparing for the market 5 dozen Milan hats which have been put through the following processes:

a. Sewing the braid into hat shape	$0.45 per hat
b. Sizing	.96 per doz. hats
c. Hat blocking	3.00 per 100 hats
d. Steaming	.80 per 100 hats
e. Hydraulic pressing	2.40 per 100 hats
f. Wiring by machine	.15 per doz. hats
g. Varnishing	.12½ per doz. hats
h. Sewing on crowns and lining	1.50 per doz. hats

(*b*) The Milan braid for these hats cost 75¢ a piece, and it takes 2 pieces for each hat. What is the cost of the materials for 1 hat, including 2½¢ for wire? For the 5 dozen hats?

(3) What is the total amount paid for packing and casing these hats for the market at $1.60 per dozen?

(4) These hats are sold for $48 a dozen, with 3 per cent discount for cash. What does the manufacturer receive for 1 dozen hats? For 5 dozen hats? What is his profit on the 5 dozen hats?

(5) What is the net profit to the manufacturer, if 25% must be deducted for the running expenses of his factory? How many dozen hats must he sell to make a profit of $2000?

(6) The retail dealer who buys these hats at $48 a dozen, with 3% cash discount, sells them at $5.50 each. What is his profit? What is his gain per cent?

110. Dressmaking. 1. Estimating amount of material required for hems, tucks, ruffles, etc.

(1) Turn the hem upon a piece of material so that it will measure 1 inch when finished. Unfold the cloth and measure the amount of material which is taken up by the hem. How much material did you allow for the first turning?

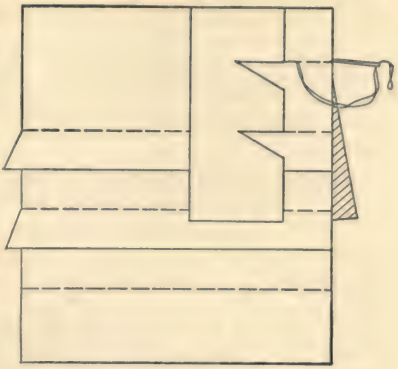
NOTE: In the following problems, allow $\frac{1}{4}$ of an inch for the first turning, except for very narrow hems, on which $\frac{1}{8}$ of an inch is turned. Cut a gauge for a 1-inch hem.

(2) In the same way cut a gauge for a $\frac{3}{4}$-inch hem. How

much material is required? Cut a gauge for a $\frac{5}{8}$-inch hem. What is the amount of material required? How much material is taken up by an inch and a quarter hem? By a $2\frac{1}{2}$-inch hem? By a $\frac{5}{16}$-inch hem? By a $\frac{3}{8}$-inch hem?

(3) Using a half-inch gauge, make a half-inch tuck. How much material is taken up by two tucks? By three tucks? By six tucks?

(4) Cut a quarter-inch gauge and make one quarter-inch tuck. Measure the amount of material which is taken up by one tuck. By three tucks. By five tucks.



Measuring for Tucks from Stitching to Fold

(5) Cut the gauges for quarter-inch tucks with quarter-inch spaces showing between the tucks. How much material is taken up by three quarter-inch tucks with quarter-inch spaces showing between the tucks?

(6) How much material is taken up by a $1\frac{1}{2}$ inch hem and three half-inch tucks?

(7) How much material is taken up by a $1\frac{3}{4}$ inch hem?

(8) The material for a ruffle is eight inches deep. If it is to have a $1\frac{1}{4}$-inch hem and 2 half-inch tucks, how wide will it be when finished?

ILLUSTRATIVE EXAMPLE (1): A piece of muslin containing eight quarter-inch tucks and a three-quarter-inch hem is five inches wide. How wide was it before it was tucked and hemmed?

Solution: 5 inches = width finished
4 inches = amount taken up by eight quarter-inch tucks ($8 \times \frac{1}{2}$ in.)

1 in.—amount taken by $\frac{3}{4}$ inch hem. $\frac{3}{4}+\frac{1}{4}$ (turning)
5 inches
4 inches
1 inch
10 inches = width before it was tucked and hemmed.

Illustrative Example: (2) The muslin for the front of a shirtwaist is 36 in. wide. In it there are to be made 32 sixteenth inch tucks. How wide should the muslin be after the tucks have been made?

Solution: 36 in. = width of muslin
32 in. $\times \frac{1}{8}$ = 4 inches taken up by the tucks.
32 in. = width of muslin after being tucked.

2. Tucking problems.

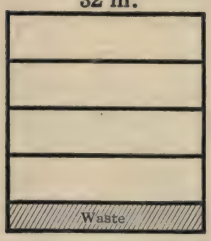
(1) The back of a camisole was 14 inches across when finished. It had in it 4 groups of tucks, each group taking up $1\frac{1}{8}$ inches of material. How wide was the piece before it was tucked?

(2) A piece of muslin 34 inches wide was 18 inches wide after tucking. How many quarter-inch tucks were made in it?

(3) How many half-inch tucks must be put in a skirt that is five inches too long, to make it right? How long should it be when finished if it is 39 inches long now?

(4) How many inches of goods are taken up by 10 groups of tucks, each group containing three sixteenth-inch tucks and one quarter-inch tuck? How wide was the material before it was tucked if it was $13\frac{3}{4}$ inches wide when finished?

(5) How deep a ruffle can be made from a strip of Georgette crêpe 16 inches deep, if a $2\frac{1}{4}$ inch hem is put on the bottom and five half-inch tucks above it?



(6) How many quarter-inch tucks can be made in a piece of material $22\frac{1}{2}$ inches long if there are quarter-inch spaces between the tucks and a quarter of an inch space showing at the top and bottom of the piece?

Illustrative Example (1): How many widths 8 inches deep can be cut from a yard of goods 32 inches wide? How many yards of ruffling can be made from these widths?

Solution: 1 yd. = 36 in.

$8)\overline{36}$

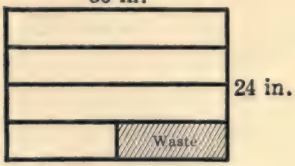
$4(\frac{1}{2})$ widths, the half width cannot be used for this ruffle.

32 in. = length of 1 width

$\times 4$

$36)\overline{128}$ in. = length of 4 widths

$3\frac{5}{9}$ yd. of ruffling.



ILLUSTRATIVE EXAMPLE (2): How many yards of material 36 inches wide are needed for $3\frac{1}{2}$ yd. of ruffling which is to be cut 6″ deep?

Solution: 36″ = length of 1 width.

For $3\frac{1}{2}$ yd. of ruffling as many widths are needed as 1 is contained times in $3\frac{1}{2}$ $1)\overline{3\frac{1}{2}}$

$3\frac{1}{2}$ widths. In order to get $3\frac{1}{2}$ widths 4 widths must be bought.

6 inches = depth of 1 width

$\times 4$

24 in. or $\frac{2}{3}$ yd. = depth of 4 widths;

$\frac{2}{3}$ of a yd. needed for the ruffle.

A Tucked Skirt

3. Ruffles. The amount of fullness allowed for a ruffle is $1\frac{1}{2}$ times the width of the garment for which it is made.

(1) How many yds. of ruffling are needed for the bottoms of 2 dozen aprons, one yd. wide?

(2) How many pieces of 36 in. lawn are needed for 3 yds. of ruffling? For $8\frac{1}{2}$ yds. of ruffling? If these pieces are sewed together, how many yards of ruffling will there be?

(3) A petticoat measures $2\frac{1}{3}$ yds. around the bottom. How many yards of ruffling are needed for it? If the ruffle is 8 inches deep before making and is cut from 36 in.

wide goods, how many yds. of material are needed for the ruffle?

(4) How many widths of ruffling can be cut from 6 yds. of muslin, if the ruffle is 4 in. deep, finished, and has a $\frac{3}{4}$-in. hem, and four $\frac{1}{8}$-in. tucks?

(5) The profits of a dressmaker for ten months (one year) were \$3350, but she lost during the two months' dull season \$462.00. What was the gain per cent for the year if her receipts were \$11,552?

4. Factory sewing problems.

(1) During a six weeks' busy season in a Boston suit factory 7 men operatives made skirts at 50c each. If the weekly output was 504 skirts, what did each man earn a week? The following 6 weeks was a dull season and the output was 301 skirts weekly. What did each man earn a week? What were the earnings of each man for the 12 weeks?

(2) In the same factory 12 women are paid 25¢ each for finishing coats during the 6 weeks' busy season and the output is 600 coats. What are the weekly earnings of each woman? During the 6 weeks' slack season the output is 275 coats. What are the weekly earnings of each woman? What are the earnings of each woman for the 12 weeks?

(3) A man is paid 75¢ per 100 for making machine buttonholes. What does he receive for the buttonholes on 600 coats, each coat having 5 buttonholes? What do his earnings average an hour if it takes 4 days of 6 hours each to make this number of buttonholes?

(4) What is the total cost to a manufacturer of the following suit if 50% of the actual cost is added for cost of manufacturing and selling:

$6\frac{3}{4}$ yd. material, @	\$1.75
Cutting	.65
Operating	2.25
Finishing	.50
Pressing	.45
Buttonholes	.25
Findings	.35
Buttons	.41

(5) 150 of these suits are sold to retail dealers at $30.00 each. What per cent profit does the manufacturer make?

(6) What are the weekly earnings of a woman who makes 1½ dozen infants' dresses a day and receives from the manufacturer $1.35 a dozen? [1]

(7) What is the weekly output in a shirt-waist factory if the operators average 2 dozen waists daily and 75 operators are employed?

(8) What is the average weekly wage of 4 buttonhole makers who are paid 12½¢ a hundred for making machine buttonholes in these waists when there are 6 buttonholes to the waist?

(9) What are the daily earnings of a girl who makes machine buttonholes on 35 dozen waists in 1 day, each waist having 5 buttonholes and the pay being 8c for each hundred buttonholes. What are her weekly earnings?

(10) The finishers sew buttons on those waists at the rate of 1¢ for 1 dozen buttons. How many dozen waists, 5 buttons to a waist, must a finisher average to earn $8.75 a week?

(11) A factory having a weekly output of 6912 shirtwaists requires how many expert pressers averaging 12 dozen shirtwaists a day? Each worker is paid 25¢ a dozen waists. What are the weekly earnings of each presser?

(12) A worker receives 25¢ per hundred yards for strip tucking. If she makes 62,400 yd. of tucking, how much is she paid? How many yards must she tuck to earn $468?

5. Earnings and savings.

(1) A girl taking the Salesmanship Course in High School earned $2 a day on Saturday of each week, and 50¢ a day for school days after 2:30 P. M. What did she earn a week?

(2) What are the total earnings of a High School girl taking the Salesmanship Course, who has had the following number of days' practice work in the department stores of her own city, during the school year, from Sept. 8, 1918, to Sept. 8, 1919?

Sept. 4 wks. $4.50 a week.....(after school and Saturdays)
Oct. 4 wks. $4.50 a week.....(" " " ")
Nov. 3 wks. $4.50 a week.....(" " " ")
Dec. 4 wks. $12.00 a week.....(time allowed from school)
Dec. 1 wk. $12.00 a week.....(vacation)

[1] Allow 6 working days a week, unless otherwise stated.

Jan.	4 wks. $4.50 a week	(after school and Saturdays)
Feb.	4 wks. $4.50 a week	(" " " ")
Mar.	4 wks. $4.50 a week	(" " " ")
Apr.	3 wks. $4.50 a week	(" " " ")
Apr.	1 wk. $12.00 a week	(vacation)
May	4 wks. $4.50 a week	(after school and Saturdays)
June	4 wks. $4.50 a week	(" " " ")
July	4 wks. $12.00 a week	(vacation)
Aug.	4 wks. $12.00 a week	(vacation)

(3) A girl earning $8.50 a week spent $7.65 for a hat. What per cent of her week's wages did she spend? What is your opinion of this purchase based upon her week's earnings?

(4) If an operator wastes $\frac{1}{3}$ of an hour each day how many hours will she waste in 4 weeks of 6 days each? If she is paid $1.25 a day and works 8 hours a day, how much has her employer lost by her waste of time? Can you suggest any way in which she could make up lost time?

(5) An operator made 5 dozen shirtwaists one week, and 6 of them were "seconds." What per cent of her week's work was below the standard?

(6) A girl is paid 25¢ for 108 yd. of strip tucking. How many hundred yards must she tuck a day to earn $15.00 a week?

(7) A skilled hat maker earning $18.00 a week for 43 weeks a year, was offered 120% more per week in another factory. She found that the second factory kept its employers only 16 weeks in the year. Which was the better paying position? Why?

(8) A girl earned $9.00 a week during her first 16 weeks in trade and put $.50 in the savings bank each week. The next 32 weeks she received an increase of $33\frac{1}{3}\%$ in her weekly earnings, which enabled her to save 20% weekly. How much had she in the bank at the end of the 48 weeks? What per cent of her earnings had she saved?

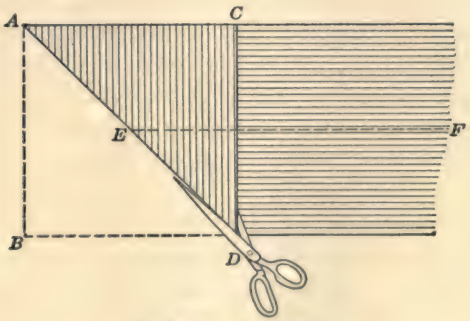
(9) A girl earning $9.50 a week, spends 16¢ a day for carfare, 25¢ a day for luncheon, pays $3.50 weekly toward the

family support, and puts 25¢ in the bank each week. With the remainder she buys her own clothes, pays her incidental expenses, and has a small sum left for recreation. How much was used for family support and how much had she for clothing, incidentals and recreation for the year if she worked 50 weeks?

(10) Make an itemized account of your own expenses for one year.

(11) A trade school graduate received $8.00 for her first week's wages. Three months later she received a 25 per cent increase. What was her last wage? 50 cents saved per week was what per cent of her increase?

(12) A girl who had been trained in designing and costume-sketching began to work at $9.00 a week. In two years afterwards she was earning 125 per cent more. What was her last wage?



Cutting a True Bias

6. Cutting a true bias. To make a true bias, the cloth is laid on the work table and the straight end "A-B" is folded back so that the edge lies along the selvedge of the goods "A-C." When the material is cut along the line "A-D" a true bias edge is made on the material and on the piece cut away. The figure above shows also the method of measuring

a length of material having one bias and one straight end. Its length is determined by measuring from a point at the middle of the bias edge "A-D" designated by "E" to the middle of the straight edge "F." From paper cut 2 pieces of true bias $1\frac{1}{2}$ inch wide.

(1) Find out the difference between "on the bias" and "through the bias." Illustrate.

(2) From the same material cut the same number of 2-inch wide straight pieces (from selvedge to selvedge).

(3) If the bias pieces were sewed together and the straight pieces were sewed together, how would the two compare in length? How much longer, approximately, is one bias width than one straight width of the same material?

(4) Which ruffle requires the greater number of widths, the straight ruffle or the bias ruffle?

(5) If a bias width is approximately $\frac{1}{3}$ longer than a straight width of the same material, how many bias widths will be required for a ruffle which takes 5 straight widths? Which requires the greater amount of fullness, a straight ruffle or a bias ruffle? Why?

7. Dressmakers' supplies.

Counting Table

2 = 1 pair
20 = 1 score
12 = 1 dozen
12 doz. = 1 gross
12 gross = 1 great gross

(1) Find the cost of $2\frac{1}{2}$ gross spools of cotton @ $62\frac{1}{2}¢$ per dozen spools.

(2) What is the cost of one yd. of soutache braid at $4.32 per dozen pieces (12 yd. to a piece)? If a dressmaker sells the braid, making $1\frac{1}{2}$ cents on every yard, what is her gain on the lot?

(3) Cards of hooks and eyes that cost a dressmaker $7.20

a gross are sold at 12 cents a card. She makes $2.52. How many cards does she sell?

(4) A modiste bought $38\frac{1}{2}$ yds. of lining at $33\frac{1}{3}¢$ a yd. and received 5 per cent discount. She sold her lining at 50¢ a yd. What was her per cent gain?

(5) What is the gain in button molds that cost 60¢ per gross, when the selling price is 15¢ a dozen? What is the gain on $4\frac{1}{2}$ gross?

(6) How many shirtwaists, each requiring $2\frac{7}{8}$ yd. of cloth can be made from 46 yds. of material? If $34.50, less 5% discount, was paid for the goods, what was the cost of the material for one shirtwaist?

8. Making estimates for garments.

(1) How many middy blouses can be made from a 33 yd. piece of goods if $2\frac{3}{4}$ yds. are required for one?

(2) What is the cost of trimming for the lot at $1\frac{1}{3}¢$ per yd., if one blouse requires 3 yds. of braid?

(3) What is the total cost if to the cost of the materials is added 25% for the making? (Material, 50 cents per yard.)

(4) How many skirts can be made from 68 yds. of material if each skirt requires 4 yds.? What is the total cost, if the material cost 49¢ per yd., and the labor on each skirt 48¢?

(5) A plain apron made of 36″ gingham is $28\frac{1}{2}$ in. long, finished. How much gingham is used for one apron (without the band) having a hem $2\frac{1}{4}$ in. deep?

(6) The band $2\frac{1}{2}$ inches wide for this apron is 36 in. long. How much gingham must be bought for the band? What is the cost of one apron, the gingham being $62\frac{1}{2}¢$ a yard ?

(7) How many strips $3\frac{1}{4}$″ wide and 36″ long can be cut from a yd. of 39″ wide material? Would the strips be cut to better advantage lengthwise or crosswise?

(8) How many yards of 27-in. goods will be required for a ruffle for a skirt measuring $2\frac{1}{2}$ yd. around the bottom, if

the ruffle is cut 8 inches wide, allowing once and a half the width of the skirt for fullness?

(9) What will it cost for sewing aprons for 24 girls if each apron requires $\frac{5}{8}$ yd. for a bib, $\frac{7}{8}$ yd. for sleeves, and $1\frac{1}{2}$ yd. more to complete it? The cloth is $37\frac{1}{2}¢$ per yd.

(10) What is the cost of 24 skating bags containing $\frac{5}{8}$ yd. each at $33\frac{1}{3}¢$ yer yard?

(11) How many yards of sheeting $2\frac{1}{4}$ yd. wide must be bought to make $1\frac{1}{2}$ dozen sheets, if each sheet measures $2\frac{1}{2}$ yd. when finished, with a one-inch hem at one end and a $2\frac{3}{4}$-inch hem at the other, and 3 inches allowed on the whole, for shrinkage?

(12) Which is the more expensive dress material to purchase:

a. 12 yd. of poplin, 27 in. wide at 30¢ a yd., or 9 yd. of poplin 36 in. wide at 35¢ a yd.?

b. 8 yd. goods 27 in. wide at 70¢ a yd., or 4 yd. material of the same quality 54 in. wide at $1.30 per yd.? Why?

(13) How many jabots requiring $\frac{5}{8}$ yd. each, can be made from $11\frac{1}{4}$ yds. of lace?

(14) If you cut 18 in. from a remnant of lace $1\frac{1}{8}$ yd. long, what part of a yard remains?

(15) How many coats, each requiring $2\frac{7}{8}$ yd. cloth, can be cut from a piece of cloth measuring $28\frac{3}{4}$ yd.?

(16) What is the saving in each yard of cloth if you buy a $6\frac{1}{4}$ yd. remnant for $1.50, the regular price of the goods being 30¢ a yd.?

(17) A girl's sailor collar measures 39″. How many yards of soutache braid will be required for five rows around it?

(18) A tailor has a piece of cloth containing $9\frac{1}{8}$ yd. If he cuts from it a suit of clothes, using $2\frac{5}{8}$ yd. for the coat, $\frac{8}{9}$ yd. for the vest, and $2\frac{1}{4}$ yd. for the trousers, how much cloth will he have left?

(19) I want to shorten a skirt $6\frac{1}{2}$ inches. How many tucks one-quarter inch, will I have to make?

Dressmakers' Problems

(1) A modiste bought 28 yd. silk at 98¢ a yard and received a 6% discount. She sold it to a customer for 25% more than she paid for it. What did she gain on the sale?

(2) Silk was offered at a special sale for 98¢ a yd. A dressmaker bought 35 yd. and received a dressmaker's discount of 5%. She sold $15\frac{1}{2}$ yd. to one customer at $1.16 a yd., and $17\frac{1}{2}$ yd. to another customer at $1.08 a yd. The remainder she was unable to sell. Did she gain or lose? What per cent?

(3) A dressmaker bought 75 yd. of white lining for $33.75. She sold 40% of it at a gain of 25% and the remainder at a loss of 5%. Did she gain or lose? What per cent?

(4) A dressmaker had $62.40 left after paying her assistants $62\frac{1}{2}$% of the week's receipts. How much did she pay her assistants?

(5) What is discount?

(6) Why do retail merchants give dressmakers a special discount?

(7) Name other classes of people who receive discounts from department stores.

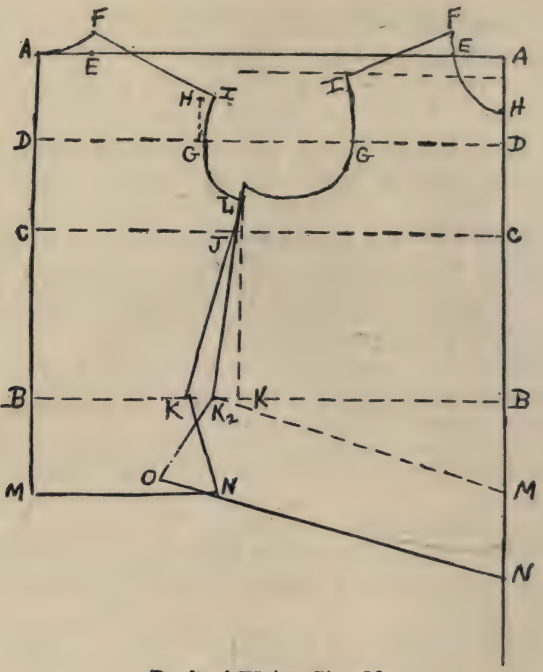
(8) Explain how both the purchaser and the dealer profit by a discount.

(9) A modiste pays her helper $2.50 each for making shirtwaists; the materials for each waist cost $2.25; $33\frac{1}{3}$% of the cost is added for overhead expenses. The waists sell for $7.65 each. What is her profit on each waist? What is the gain per cent?

(10) What is meant by "overhead expenses" in example (9)?

(11) A dressmaker bought 2400 yd. of gingham at $33\frac{1}{3}$¢ per yd. and received 10% discount on the amount of her purchase. She sold the goods for 50¢ per yard. What was her total profit?

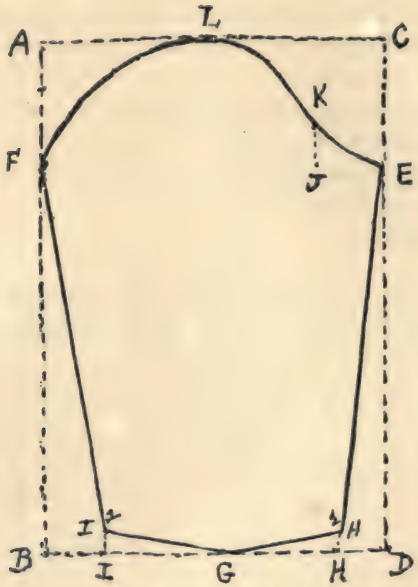
(12) A modiste purchased $33\frac{3}{8}$ yd. of organdie muslin at 75¢ per yd. and received a 10% discount on her bill. She sold $12\frac{1}{2}$ yd. to one customer, $9\frac{3}{4}$ yd. to another, and $10\frac{1}{8}$ yd. to a third, receiving $1.00 a yard in each case. How many yards did she sell? What did she receive for the muslin sold? How many yards had she left? What did she pay for the muslin? What did she gain?



Draft of Waist, Size 36

Scale $\frac{1}{4}'' = 1''$.

Measurements for Back	*Measurements for Front*
A–M = 20″ = length	H–N = $20\frac{1}{2}''$ = length of front
A–B = 16″	D–G = $6\frac{3}{4}''$ = chest
A–C = 9″	C–J = $12\frac{3}{4}''$ = bust
A–D = 4″	B–K = $14\frac{1}{2}''$
D–G = 7″	H–P = $1\frac{1}{4}''$
C–J = $7\frac{1}{4}''$ = width of bust	E–A = $2\frac{1}{4}''$
B–K = 6″	K–L = 9″ (under arm)
E–F = $\frac{3}{8}''$	I–I = 16″ (arm-size)
K–N = 4″	AF–FH = $12\frac{1}{2}''$ (neck)
K–L = 9″	A–H = $1\frac{5}{8}''$
O–K = 4″	
B–M = 4″	



Sleeve Draft

Measurements

Scale = 1″	A–F = 5″
Length of sleeve, 18″	F–B = 18″
Width of sleeve, $13\frac{1}{2}$″	B–I = $2\frac{1}{2}$″
Arm-size, $15\frac{3}{4}$″	I–G = $4\frac{1}{4}$″
Under arm, 9″	F–E = $13\frac{1}{2}$″
	J–E = $1\frac{1}{4}$″
	J–K = $\frac{3}{4}$″

9. Drafts.

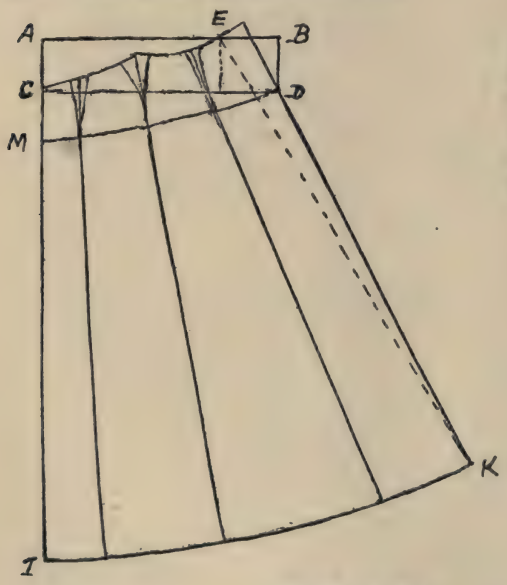
(1) Sketch the style of waist you desire, using the draft, page 306. Estimate the amount of material required, if the shirtwaist were made of silk, 27″ wide. Estimate the amount of material required if the material were a 50-inch wide challie.

(2) Draft the pattern page 306 reducing the scale $\frac{1}{8}$″.

(3) A dressmaker paid the following prices for the materials of a gown. What was the cost?

$8\frac{1}{2}$ yd. of satin at \$4.25 a yard.
$6\frac{1}{4}$ yd. of trimming at \$6.50 a yard.
5 ornaments at \$2.50 each.
$4\frac{3}{4}$ yd. of chiffon at \$3.50 a yard.
3 yd. lining at \$1.50 a yard.
\$3.75 for findings.

(4) The dressmaker added 10% to the cost of the materials and \$50.00 for making. What was the total amount of the bill for the gown, to the customer?



A –B = 17″
C –D = 17″
M–D = 18″
E –K = 31″
A –E = $13\frac{1}{2}$″
A –1 = 30″
1 –K = 45″

Draft for Skirt
Length of skirt, 36″ finished

(5) How many yards of 27″ goods will this skirt require, allowing 4″ for hem, and $1\frac{1}{2}$″ for seams. What would the material cost at $1.75 a yard?

(6) How many yards of 50″ goods will the above pattern call for? Cost of the material at $3.25 per yard?

(7) If each measurement were increased 1″ estimate the amount of 27″ goods the skirt would require.

Textile Problems

(1) The average bale of cotton weighs 500 lbs. The United States raises $\frac{4}{5}$ of the world's annual crop of $7\frac{1}{2}$ billion pounds. How many bales are raised annually in the United States? What is the crop worth in money to the United States if the average bale sells for $56.00? For $75.00? For $85.00?

(2) The average bale of wool weighs 250 lbs. How many fleeces does it take to make a bale when the average fleece weighs $6\frac{3}{4}$ lb.? If wool sells for 38c per lb., what is the value of 1 bale?

(3) The total wool production in the United States in 1907 was 298,294,750 lbs. What was the value of the wool production of that year at 38c per lb.?

(4) In 1904 the United States imported 16,037,411 lb. of raw silk valued at $55,873,440. What was the average value per lb.?

Receipts of wool in pounds from and including Jan. 1, 1919, compared with the same period in 1918, are as follows:

	1919	1918
Domestic	28,144,450	26,649,963
Foreign	57,498,606	102,857,355
Totals	85,643,056	129,507,318

(5) Answer the following questions from the above statement:

a. What is the per cent of increase in pounds of domestic wool from Jan. 1, 1918, to Jan. 1, 1919?

b. What is the per cent of decrease in pounds of foreign wool from Jan. 1, 1918, to Jan. 1, 1919?

c. What is the total per cent decrease of wool production from Jan. 1, 1918, to Jan. 1, 1919?

(6) What are the present average quotations on wool per pound? Per bale?

(7) State as many reasons as you can for the decrease in amount of production.

(8) The materials used for a cretonne-covered waste-basket were as follows:

$1\frac{3}{4}$ yd. cretonne	\$.75 per yard
4 sheets folding card paper at	.03$\frac{1}{4}$ per sheet
4 yds. ribbon at	.37$\frac{1}{2}$ per yard
$2\frac{1}{4}$ yd. kid paper lining	.12$\frac{1}{2}$ per yard
Needles, thread, etc	.25
Making	.75

What was the cost of the basket? For how much must the basket be sold to gain $37\frac{1}{2}\%$?

(9) A girl is paid \$8 for making 100 boxes. What does she receive for 1 box? The cost of material for one box is 42 cents. What is the net cost of 1 box? Of 100 boxes? What is the selling price of each of these boxes if they are sold at a profit of $12\frac{1}{2}$¢ each? What is the total profit on the 100 boxes?

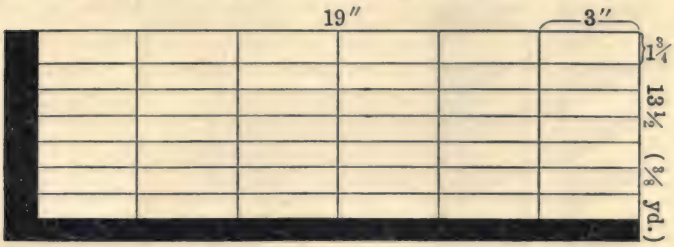
(10) An employee wastes $\frac{1}{4}$ of an hour each day. How much time will she waste in 4 weeks of 6 working days each? She receives \$2 a day, working 8 hours each day. How much has her employer lost by her waste of time? If there are 250 girls in the same industry, wasting the same amount of time a day how many hours of time are lost? What is the employer's loss if these girls are paid at the rate of 25 cents an hour?

(11) The United States cotton crop in 1916 was . . 12,975,569 bales
The world's crop in 1916 was . . . . . 17,990,099 bales.

What per cent of the world's crop was the crop of the United States in 1916? The price of cotton advanced from 13 cents per pound to 27 cents per pound from 1916 to 1917. What was the per cent of increase in price per pound in one year? What was the value of 1 bale (500 lbs.) in 1917? What was the value of the cotton crop to the United States in 1916?

(12) In 1902, fifty-three per cent of the flax-seed crop of the U. S. amounting to 29,284,880 bu. was raised in North Dakota. How many bu. did North Dakota raise? A bushel of flax-seed makes $2\frac{1}{2}$ gallons of oil. How many gallons of oil could be manufactured from the North Dakota crop?

111. Samples. The width of samples of cotton, wool, silk or linen, must be cut on the width of the material, and the length of the sample on the length of the material.



Sample Cutting

ILLUSTRATIVE EXAMPLE: How many rectangular samples $1\frac{3}{4}''$ wide by 3″ long can be cut from a piece of material $\frac{3}{8}$ of a yard wide by 19 inches long?

SOLUTION: $19 \div 3 = 6(\frac{1}{3})$ on the length.
The fraction can not be used. $\frac{3}{8}$ of a yard $= 13\frac{1}{2}''$.
$13\frac{1}{2} \div 1\frac{3}{4} = 7(\frac{5}{7})$ on the width. The fraction can not be used.
$7 \times 6 = 42$ samples can be cut from this amount of material.

(1) How many rectangular samples 3″ long by $1\frac{1}{4}''$ wide can be cut from 1 yd. of 36 in. wide gingham?

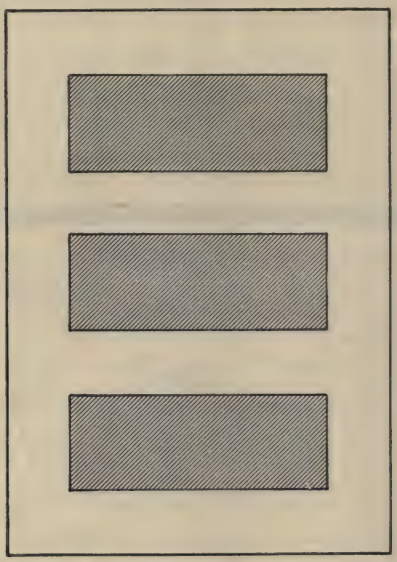
(2) How many yards of 32 in. goods are required for 480 rectangular samples $2\frac{1}{8}''$ long by $1\frac{1}{2}''$ wide?

(3) How many rectangular samples $2\frac{1}{8}''$ long by $1\frac{1}{2}''$ wide can be cut from a half yard of 30-in. crêpe?

1. Making sample cards.

Samples should be so mounted that the spaces in the rows must be in the right proportion to the size of the samples, the size of the card, and the width of the margins; the worker must divide the number of inches left for margins and spaces into such parts as will give the card an attractive appearance. Side margins and spaces between the rows of samples should be the same width. The spaces between the samples in the rows should be narrower than the spaces between the rows. The top margin should be wider than the bottom margin.

ILLUSTRATIVE EXAMPLE: Find the width of the margins and spaces for a sample card 5 inches by 8 inches having 3 rectangular samples $1\frac{1}{3}$ in. wide by 3 in. long.



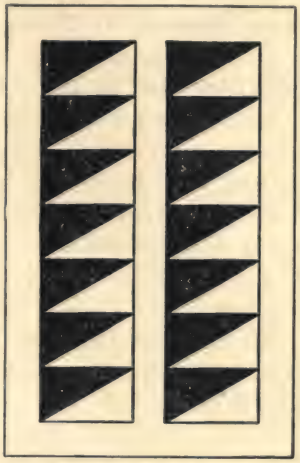
Mounting of Rectangular Samples

Solution:

5 inches = width of card
3 " = length " sample.
2 " = number of in. left for side margins.
$\frac{1}{2}$ of 2 " = 1 inch, width of each side margin.

8 inches = length of card.
$3 \times 1\frac{1}{3} = 4$ " = width " three samples.
4 inches = amount left for top and bottom margins and spaces.
$\frac{3}{4}$ inch = width of each space
$1\frac{1}{2}$ " = width of top margin
1 " = width of bottom margin.

2. Triangular samples are arranged in rows extending lengthwise of the card with the samples touching each other. The spaces between the rows are the same width as the side margin. Triangular samples are rectangular samples divided on the diagonal.



Mounting of Triangular Samples

EXERCISE

(1) Find the width of the margins and spaces for a sample card $6\frac{1}{2}$ by $9\frac{1}{2}''$, having 4 rectangular samples $2\frac{3}{4}''$ long by $1\frac{1}{2}''$ wide. Illustrate.

(2) How many rectangular samples $2\frac{3}{4}$ long by $1\frac{1}{4}''$ wide can be mounted on a sample card $5\frac{5}{8}''$ by $8\frac{1}{2}''$ with allowance for proper margins and spaces?

(3) Find the width of the margin spaces for a sample card $6\frac{1}{2}''$ by $8\frac{1}{4}''$ having 2 rows of 10 triangular samples $2\frac{1}{8}''$ long by $1\frac{1}{8}''$ wide. Illustrate.

(4) How many triangular samples $2\frac{1}{2}''$ long by $1\frac{1}{2}''$ wide can be mounted on a sample card $5\frac{1}{4}''$ by $7\frac{3}{4}''$ with allowance for margins? How many such samples can be cut from $2\frac{3}{4}$ yds. of 36-inch goods? If the material costs 75¢ per yd., the sample cards 1¢ for 5, and the mounting of the samples 25¢ per 100, what was the total cost of the samples?

(5) How many samples are there in an order for 2 dozen sample books, 360 samples to a book?

(6) A manufacturer pays $4.75 per thousand for mounting lace samples. How many cards, 5 samples on a card, must a girl finish to earn $9.50 a week?

112. Box-factory work.

(1) One piece of pulp board measures 8″ by 8″. How many pieces 2″ by 2″ can be cut from it? Illustrate.

(2) How many pieces 6″ by 2″ can be cut from a piece of pulp board 18″ by 24″? From a piece 12″ by 6″?

(3) A sheet of pulp board measures 26″ by 38″. How many boxes (without covers) can be made from 5 sheets by cutting two sheets into bottoms measuring 5″ by 5″, and 3 sheets into sides measuring $5\frac{1}{8}$″ by $2\frac{1}{4}$″?

(4) A girl working in a box factory earns $12 per week. If she makes 480 boxes each week what is the cost of labor for each box?

(5) Four handkerchief boxes were cut from a sheet of pulp board costing 3 cents. What was the cost of one box? Of 91,250 boxes?

113. Fancy lamp shades.

(1) Lamp shades are covered with silk or other suitable material, which is cut in widths from selvedge to selvedge. Three times the circumference is allowed for side pleats. How many widths of 27-inch-wide silk will be required to cover a shade 54 inches in circumference? Allowing one inch for turnings, how many yards of silk are needed, if the height of the shade is 12 inches?

(2) Box-pleated ruchings used for trimming lamp shades are in strips lengthwise. How many strips 3 in. wide can be cut from silk 36 inches wide? (Pleats touch but do not overlap.)

(3) Fold two $\frac{3}{4}$ inch triple box pleats in a 2 inch wide strip of thin paper. One triple box pleat takes up how many times the width of a pleat?

(4) How many yards of triple box pleating $1\frac{1}{2}$ inches wide can be made from $6\frac{1}{2}$ yd. of 36″ material?

(5) What is the gain per cent on a lamp shade which sold for $19.75 when the cost of material was as follows:

$1\frac{1}{2}$ yd. cretonne	$ .75 per yard
5 yd. trimming	$.62\frac{1}{2}$ per yard
2 yd. silk fringe	$1.25 per yard
1 frame	$1.75
Making	5.00

(6) The materials used for a 26 inch lamp shade with collar were as follows:

$1\frac{1}{8}$ yd. silk, @	$1.25 a yard
$\frac{3}{4}$ yd. filet net, @	2.75 a yard
$2\frac{3}{16}$ yd. of silk fringe, @	1.98 a yard
1 frame	2.65
Making	4.50

a. Find the cost of shade.

b. The shade was sold for $25.00. What was the gain per cent?

114. Food problems.

BREAKFAST

DISHES	*Amounts*	*Protein in Ounces*	*Fuel Value in Calories*	*Market Cost in 1920*
Baked apple, cooked	1 (large)	.022	100	
with Sugar	1 tb.	.000	57	$.06
Rolled oats with	$\frac{1}{2}$ cup (cooked)	.147	100	
Milk, and	$\frac{1}{2}$ cup (whole)	.170	85	.10
Sugar	1 tb.	000	57	
Roll with	1	.106	100	.04
Butter	2 tsp.	.003	61	
Coffee with	1 cup	000	000	
Cream (18.5%) and	2 tb.	.025	55	.07
Sugar	2 tsp.	000	38	

DINNER

DISHES	*Amounts*	*Protein in Ounces*	*Fuel Value in Calories*	*Market Cost in 1920*
Cream soup (cream sauce with 2 tbs. flour, 2 tbs. butter, 1 pt. milk)	$\frac{1}{2}$ cup	.576	300	$.15
Broiled steak	$3\times2\times1\frac{1}{2}$	.635	230	.30
Baked potato	1 large	.092	100	.02
Roll	1	.106	100	
Butter (for roll and potato)	1 tb.	.005	92	.05
Apple Tapioca	$\frac{1}{2}$ cup	.481	512	.15
Coffee or Tea with	1 cup	000	000	
Cream and	2 tb.	.025	55	.07
Sugar	2 tsp.	000	38	

SUPPER

Creamed salmon on	$\frac{1}{2}$ cup			
Toast	1 slice, $4\times4\times\frac{1}{2}$	.574	200	$.30
Bread and	2 slices, $\frac{3}{4}$ in. thick	.183	146	.04
Butter	2 tsp.	.003	61	
Lettuce	$\frac{1}{4}$ head	.024	11	.05
Oil dressing	1 tb.	000	100	.05
Chocolate	1 cup	.247	275	.15
Cake (one egg cake)	1 slice	.134	198	.05
Blackberries with	$\frac{1}{2}$ cup	.078	100	
Cream	4 tb.	.050	110	.20
Sugar	2 tsp.	000	38	

(1) Find the total cost of the three meals for the day in the menus given above.

(2) At that rate per day, what would the food cost for a family of eight for a week?

(3) How many ounces of protein does the quantity of food given for the three meals contain?

(4) To how many calories is this amount of food equal?

(5) If the average man needs 3000 calories a day, how much more or less than the necessary quantity is furnished by this menu?

(6) Which is the better economy—for the purchaser to buy at a sale a lot of small lumpy potatoes, 25% of which would be lost by peeling, for 95¢ a bu., or a lot of smooth potatoes $16\frac{2}{3}\%$ of which would be lost in peeling, for $1.00 a bu.?

(7) Which of these two lunches should be chosen for a person who can eat but little starchy food:

a. Bread, butter, rice croquettes, cheese.
b. Chicken, potatoes, butter, milk.

(8) Find out the present day prices of the food stuffs in menu, and estimate the total cost of the day's food supply.

(9) A family can buy eggs at 2 for 15 cents, or at 72¢ per dozen. How much money is saved by buying the eggs by the dozen?

(10) By co-operative buying eight families in one large apartment house can purchase 12 dozen eggs for $7.20. Each family receives $1\frac{1}{2}$ dozen eggs a week. What is each family's weekly egg bill? What is the amount saved by this co-operative purchasing when eggs are quoted at 72¢ per dozen?

(11) The following market quotations were given at a large co-operative store.

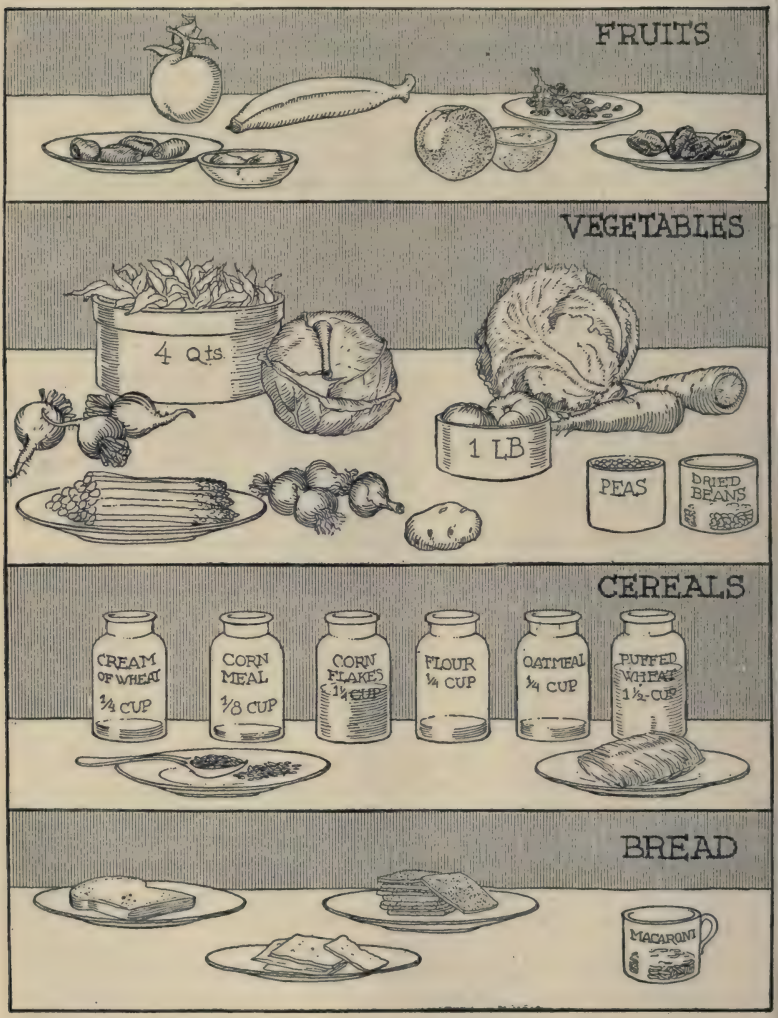
a. Butter, 75¢ per lb., or 1 tub (28 lb.)	$20.00.
b. Flour, 7¢ per lb., or 1 bbl. (196 lb.)	11.76.
c. Sugar, 9¢ per lb., or 1 bbl. (196 lb.)	15.72.
d. Potatoes, 5¢ per qt. or 1 bu.	1.28.

Estimate the amount saved by buying in the larger quantities, in each of the above articles. Could the eight families quoted in example 10, purchase their foods in this way? What are the advantages of co-operative purchasing?

(12) A family buys butter at the rate of 20 cents a quarter of a pound. If the family uses 12 lb. of butter a month, how much would be saved each month by buying it at 72¢ per pound?

(13) A family can buy sugar at the rate of $7\frac{1}{2}$ lb. for 42¢, or 25 pounds for $1.25; which is the better rate? How much is saved by buying 75 lb. at the better rate?

(14) Salt can be bought in 3¢ bags containing 1 lb. or in 5¢ bags containing 4 lb. If a family uses $5\frac{1}{2}$ lb. of salt a month, how much will be saved in a year by buying the 5¢ bags?



100 Calorie Portions of Some Common Foods

(15) Mr. Green's two boys hired a war garden and raised the following vegetables: 2 bu. potatoes; $1\frac{1}{2}$ bu. beans; 40 squash; 2 pk. of peas; 12 doz. corn; and 50 heads of lettuce. Their expenses were \$8.75. How much money did they save by their industry, at the present day price quotations on these vegetables?

(16) A cooking class used the following materials during their lesson:

12 quarts of strawberries @ 25¢.
6 pounds of sugar @ $9\frac{1}{2}$¢;
1 dozen quart jars @ $12\frac{1}{2}$¢
1 dozen jar rubbers @ 1¢.

They sold the preserve for 85¢ per quart. What was the total profit?

(17) Mrs. Green uses $2\frac{1}{2}$ tons of coal for her kitchen range during July and August: Mrs. Smith, her neighbor, uses a gas range, burning 5,625 cu. ft. of gas, during the same period. The gas costs \$1.25 for 1000 cu. ft.

a. What is Mrs. Smith's gas bill for July and August?

b. What is Mrs. Green's coal bill for July and August at the present price quotations?

c. Assuming that the same amount of heat is required in each case which is the cheaper fuel?

115. Increased cost of living in ten years.

Food	Average Cost in 1908		Average Cost in 1918	
	Wholesale	Retail	Wholesale	Retail
Wheat Flour	\$6.40	\$7.50	\$11.50	\$13.50
Butter	.34	.36	.48	.58
Sugar	$.04\frac{1}{2}$	$.05\frac{1}{4}$	$.08\frac{1}{2}$	$.09\frac{1}{2}$
Sirloin Steak	.26	.28	.42	.54
Potatoes	.42 (bu.)	.45 (bu.)	.65	.68
Eggs	.16 (doz.)	.20 (doz.)	.60	.72

(1) How much did the wholesale cost of flour advance in ten years? What per cent did it advance?

(2) How much did the retail cost advance in ten years? What per cent did it advance?

(3) The retail price in 1908 was what per cent higher than the wholesale price? (This is called "The Margin of Profit.")

(4) The retail price in 1918 was what per cent above the wholesale price? Was the margin of profit any greater in 1918 than in 1908?

(5) Give all the reasons you can for this advance in the prices of the above commodities.

(6) Answer the questions, 1, 2, 3, 4, for butter, sugar, steak, potatoes, and eggs.

(7) A child should consume one quart of milk daily; an adult should consume 1 pint daily. What would be the cost of the weekly supply of milk for a family of 5 children and two adults at present prices? The monthly bill?

CHAPTER XX

INTEREST

1. How will the study of ways of saving money or investing money be of service to you? How will this knowledge make you more valuable to your employer? Why should everyone have a bank account? Which is better,—to watch your money go, or to watch it grow?

2. If I hire an automobile for 4 hours at $3 an hour, how much do I pay for the use of it?

3. If I pay $24 a month for rent, what do I pay for the use of the house for 1 year?

4. If the house is worth $3600, what per cent of its value do I pay for the use of it?

5. If I borrow $3600, what do I have to pay for the use of it? Why? People who wish to use buildings, land, horses, etc., which belong to others, hire them and pay for their use. In the same way, people who need money hire it, and pay a certain per cent for the use of it. Money paid for the use of money is called interest, and is reckoned as a certain per cent of the amount borrowed, or principal. When a person says he will lend you a dollar at 6% per year, he means that you must pay him $.06 (6% of $1) for the use of it for one year. If the rate is 5% your interest is $.05, etc.

116. Interest. A commercial year has 360 days; what part of 1 year is 60 days? 6 days? What is the interest on $1 for 1 year at 6%? For 60 days? For 6 days? Since the interest for 1 year or 360 days equals $.06, and the interest of 60 days or $\frac{1}{6}$ of a year equals $.01, and the interest of 6 days equals .001, it is clear that .001 of any principal is equal to the interest for 6 days at 6%. What is the interest on $1 at 6% for 1 day?

(1) How do you find .01 of a number? .001 of a number? State a short way of finding the interest on any principal for 60 days at 6%; for 6 days.

ORAL EXAMPLES

(1) Find the interest on each of the following for 6 days at 6%:

a. \$300	*e.* \$425	*i.* \$218	*m.* \$212
b. \$260	*f.* \$672	*j.* \$537	*n.* \$958
c. \$125	*g.* \$850	*k.* \$241	*o.* \$470
d. \$236	*h.* \$906	*l.* \$198	*p.* \$832

(2) Find the interest on each of the above amounts for 12 days at 6%; for 18 days; for 24 days; for 30 days.

(3) Find the interest on each of the following for 1 day at 6%:

a. \$180	*e.* \$420	*i.* \$480	*m.* \$450
b. \$240	*f.* \$540	*j.* \$210	*n.* \$230
c. \$120	*g.* \$600	*k.* \$390	*o.* \$270
d. \$360	*h.* \$720	*l.* \$570	*p.* \$510

(4) Find the interest on each of the above amounts, for 3 days at 6%; for 9 days; for 2 days.

ILLUSTRATIVE EXAMPLE (1): Find the interest on \$624 for 48 days at 6%.

Solution: Finding .001 of \$624 by pointing off three places to the left, gives \$0.624, the interest for 6 days. $48 \times \$0.624 = \$29.952$ or 6 times the interest for 48 days. Dividing this result by 6, gives \$4.992, the required interest. This work may be shortened by cancellation:

$$48 \times .624 = \$29.952$$
$$\$29.952 \div 6 = \$4.992$$
$$\frac{48 \times \$0.624}{6} = \$4.992$$

ILLUSTRATIVE EXAMPLE (2): Find the interest on \$225 for 1 year, 4 mo., 12 days at 7%.

Solution: 1 yr., 4 mo., 12 da. = 492 days.

$225 \times .001 = .225$.

$492 \times .225 \div 6 = \$18.45$ Interest at 6%

\$18.45 = Interest at 6%

3.075 = " " 1% ($\frac{1}{6}$ of Interest at 6%)

\$21.525 = " " 7%.

(5) At 6% find the interest on each of the following problems (reduce the time to days):

	Principal	*Time*		*Principal*	*Time*
a.	$840	1 yr., 2 mo., 6 da.	*f.*	$425.75	1 yr., 1 mo., 1 da.
b.	$560	1 yr., 8 mo., 12 da.	*g.*	$320.50	1 yr., 2 mo., 6 da.
c.	$230	1 yr., 3 mo., 9 da.	*h.*	$250.25	1 yr., 3 mo., 3 da.
d.	$420	1 yr., 7 mo., 8 da.	*i.*	$175.10	1 yr., 4 mo., 4 da.
e.	$165	1 yr., 9 mo., 10 da.	*j.*	$280.60	1 yr., 5 mo., 5 da.

Problems

Find the interest on:

(1) $350 from May 1, 1910, to Sept. 10, 1914, (find the exact number of days) at 3%; at 2%; at 1%.

(2) A merchant borrowed $[illegible]0 from Jan. 1, 1921, to July 23, 1921, at 9%. How much did the principal pay for its use? What would he have had to pay at 4%? At [illegible]%?

(3) A manufacturer borrowed $2700 from Oct. 15, 1919, to Jan. 1, 1921, at 6%; what did he have to pay for the use of the money? Find what he would have to pay at each of the following rates:

1%, 2%, 3%, 4%, 5%, 7%, 8%, 9%, 10%, 12%

(4) If you loaned $500 at 6% on July 1, 1920, and it was paid back on Dec. 1, 1920, how much interest would it earn for you? What amount of money would you receive?

(5) A student borrowed $250 on Sept. 4, 1919, in order to complete his education. The rate of interest was 5%. What amount of interest was due Sept. 1, 1920?

(6) If your father has $6000 and spends $500 a year, how many years will the money last him? If he puts the $6000 in a bank, which pays a 5% interest, and draws out for expenses $500 at the end of each year, how long will the money last him?

(7) A dry goods dealer borrowed $4725 on the 1st of May. On the 1st of December following, he paid the amount which was $4890.375. What rate of interest did he pay?

(8) A man deposited $800 in a bank paying 6% interest; he kept it there until it was doubled. How long did he keep it there?

117. Exact interest. Business men and the United States Government reckon the exact number of days in a month, and the days are 365ths of a year. This is known as Exact Interest. When banks charge interest on loans they have made, they reckon interest by the commercial year of 360 days; but when banks themselves are paying interest on deposits, they divide the number of days by 365. Can you see the reason for this?

(1) What is the interest on the following amounts?

a.	\$954	67 da.	5%	*c.*	\$1800	4 mo. 4 da.	4%
b.	\$360.72	23 da.	1%	*d.*	\$620.60	97 da.	$2\frac{1}{2}$%

1. The 6% method.

(1) What is the interest on \$1 for 1 yr. at 6%? For 2 yr.? For 8 yr.? For 10 yr.? For 15 yr.?

(2) If the interest on \$1 for 1 yr. at 6% is \$0.06, what is the interest on \$1 for 2 mo. or $\frac{1}{6}$ of a year? For 1 mo.? For 30 days? For 8 mo.? For 10 mo.?

(3) If the interest on \$1 for 1 mo. at 6% is \$.005 (5 mills) what is the interest for 1 day? For 5 days? For 20 days? For 13 days? For 27 days?

(4) From the above it is clear that:

Interest on \$1 for 1 yr. at 6% is \$.06

Interest on \$1 for 1 mo. at 6% is \$.005

Interest on \$1 for 1 day at 6% is \$0.000$\frac{1}{6}$

Problems

(1) Find the interest on \$2460 for 2 yr., 3 mo., 8 days at 5%.
(2) Find the interest on \$15.40 for 3 yr., 4 mo., 5 days at 3%.
(3) Find the interest on \$68.70 for 1 yr., 3 mo., 8 days at 2%.

118. Cancellation method.

(1) Express the per cent as a common fraction; express years and months as days, and divide by 360 for a commercial year, or 365 days for exact interest. Multiply by the principal.

ILLUSTRATIVE EXAMPLE: Find the interest on $2500 for 2 yr., 8 mo., 15 days at 5%.

Solution: $5\% = \frac{5}{100}$ 2 yr., 8 mo., 15 da. $= \frac{975}{360}$ of a year.

$$\frac{\overset{25}{\cancel{\$2500}} \times \overset{1}{\cancel{5}} \times \overset{325}{\cancel{975}}}{\underset{20}{\cancel{100}} \quad \underset{72}{\cancel{360}}} = \frac{8125}{24} = \$338.54, \text{ the required interest.}$$

Do problems 1, 2, and 3 above, by the cancellation method, and compare results.

119. Short methods of computing interest.

1. The following will be found to be excellent rules for finding the interest on any principal for any number of days. When the principal contains cents, point off four places from the right of the result to express the interest in dollars and cents. When the principal contains dollars only, point off two places.

Two per cent.—Multiply the principal by the number of days to run, and divide by 180.

Two and one-half per cent.—Multiply by number of days, and divide by 144.

Three per cent.—Multiply by number of days, and divide by 120.

Three and one-half per cent.—Multiply by number of days, and divide by 102.86.

Four per cent.—Multiply by number of days, and divide by 90.

Five per cent.—Multiply by number of days, and divide by 72.

Six per cent.—Multiply by number of days, and divide by 60.

Seven per cent.—Multiply by number of days, and divide by 51.43.

Eight per cent.—Multiply by number of days, and divide by 45.

Nine per cent.—Multiply by number of days, and divide by 40.

Ten per cent.—Multiply by number of days, and divide by 36.

2. To find the interest at any rate, find the interest at 6%; increase or decrease this by the fractional part by which the required rate is greater or less. Thus if at 7% the interest is $\frac{1}{6}$ more. (See illustrative Example (2) page 322.)

ILLUSTRATIVE EXAMPLE: What is the interest of $200 for 6 da. at 3%?

Solution: 20¢ is the interest for 6 days at 6%. 3% = $\frac{1}{2}$ of 6%. Hence, $\frac{1}{2}$ of 20¢ = 10¢:—the interest for $200 for 6 days at 3%.

(1) State a short method for changing 6% interest to 4% interest; or to any rate, when interest at 1% is known.

(2) 6% interest multiplied by 2, equals interest at what rate? 6% interest divided by 3, equals what rate of interest? Divided by 2, equals what rate of interest? By 6? By 4?

(3) The interest of $1 for 1 yr. or 12 mo. at 6% = ?
The interest of $1 for $\frac{1}{6}$ yr. or 2 mo. (60 da.) = ?
How do you find .01 of a number?

(4) What is the interest on $25 for 60 da. @ 6%? On $250? On $560? On $480?

3. Bankers' sixty-day method.

(1) The above work shows that the interest on any sum at 6% for 60 days is as many cents as dollars loaned. Therefore, moving the decimal point two places to the left in the principal, gives the interest for 60 da. @ 6%.

Problems

(1) What is $\frac{1}{2}$ of 60 days? 60 days plus $\frac{1}{2}$ of itself equals how many days? 60 days minus $\frac{1}{2}$ of itself equals how many days? State a short way of finding the interest at 6% for 90 days; for 30 days.

(2) What is the interest at 6% on \$60 for 60 days? For 90 days? For 30 days? On \$120 for 60 days? For 90 days? For 30 days? On \$500 for 60 days? For 90 days? For 30 days?

(3) What is $\frac{1}{4}$ of 60 days? 60 days plus $\frac{1}{4}$ of itself equals how many days? 60 days minus $\frac{1}{4}$ of itself equals how many days? State a short way of finding the interest at 6% for 75 days; for 45 days.

(4) What is the interest at 6% on \$400 for 75 days? For 45 days? On \$300 for 75 days? For 45 days? On \$700 for 45 days? For 75 days?

(5) What is $\frac{1}{3}$ of 60 days? 60 days plus $\frac{1}{3}$ of itself equals how many days? 60 days minus $\frac{1}{3}$ of itself equals how many days? State a short way of finding the interest for 80 days; for 40 days.

(6) What is the interest on \$200 at 6% for 80 days? For 40 days? On \$160 for 80 days? For 40 days? On \$320 for 40 days? For 80 days?

(7) What is $\frac{1}{6}$ of 60 days? 60 days plus $\frac{1}{6}$ of itself equals how many days? 60 days minus $\frac{1}{6}$ of itself equals how many days? State a short way of finding the interest for 70 days. For 50 days.

(8) What is the interest on \$300 for 70 days at 6%? For 50 days? On \$450 for 70 days? For 50 days? On \$240 for 70 days? For 50 days?

(9) What is $\frac{1}{10}$ of 60 days? How do you find .1 of a number? Removing the decimal three places to the left in the principal, gives the interest for how many days at 6%?

(10) What is the interest on \$250 for 6 days at 6%? On \$160? On \$280? On \$725? On \$420?

(11) State a short way of finding the interest on any amount for 6 days at 6%. For 3 days. What part of 6 days is 2 days? 1 day?

(12) State a short way of finding the interest on any amount at 6% for 2 days. For 4 days. For 1 day.

(13) 6 days plus $\frac{1}{6}$ of itself equals how many days? 6 days minus $\frac{1}{6}$ of itself equals how many days? State a short way of finding the interest at 6% on any amount for 7 days. For 5 days. For 14 days. For 15 days. For 28 days. For 25 days.

(14) 6 days plus $\frac{1}{3}$ of itself equals how many days? State a short way of finding the interest on any amount for 8 days. For 4 days. For 16 days. For 12 days. For 24 days.

(15) 6 days plus $\frac{1}{2}$ of itself equals how many days? State a short way of finding the interest at 6% on any amount for 9 days; for 27 days.

(16) What is the interest on \$320 at 6% for 9 days? For 27 days? On \$600 for 9 days? For 27 days?

ILLUSTRATIVE EXAMPLE: Find the interest on \$2500 at 6% for 75 days.

Solution: 75 days = 60 days + 15 days. *Operation.*

Removing the decimal point in \$2500. two places to the left gives the interest for 60 days. 15 days is $\frac{1}{4}$ of 60 days. $\frac{1}{4}$ of \$25.00 = \$6.25; therefore, the interest of \$2500 for 75 days at 6% = \$31.25.

\$25.00
+6.25
\$31.25

(17) Find the interest on \$675 for 7 days; for 42 days; for 84 days; for 210 days; for 420 days.

(18) Find the interest on \$150.50 from Sept. 1, 1910, to January 11, 1911, at 6%.

(19) Find the interest on \$450.60 from Sept. 5, 1916, to the present date at 6%.

ORAL EXERCISE

(1) Separate the days in the following exercises into convenient parts of 60 days or 6 days—thus:

54 days = 9 times 6 days; 87 days = 60 days + 20 + 6 + 1.

a. 9 days	*c.* 7 days	*e.* 69 days	*g.* 13 days
b. 52 "	*d.* 43 "	*f.* 5 "	*h.* 75 "

(2) Find the interest on:

Principal	*Time*	*Rate*	*Principal*	*Time*	*Rate*
a. \$742	77 da.	3%	*b.* \$576	3 mo. 5 da.	$1\frac{1}{2}$%

120. Compound interest. When interest is added to the principal at the end of each period as it becomes due to become a new principal for the next period, the entire interest is called the compound interest of the original principal.

The amount is the final amount when compound interest is reckoned.

Interest may be compounded quarterly, semi-annually, annually, or at the end of any period agreed upon.

The following solution will show the method of computing compound interest.

ILLUSTRATIVE EXAMPLE: Find the compound interest on \$650 for 3 yr., 3 mo., at 5%, interest compounded annually. It will be seen from the illustration that interest and principal are combined at regular intervals; compound interest is interest upon the principal and unpaid interest after it comes due. Compound interest is usually allowed in most savings banks upon deposits remaining for a full interest term. Money loaned at compound interest at 6%, more than doubles itself in 12 years.

Operation

\$650 = Principal
32.50 = int. for 1st year
\$682.50 = Principal for 2nd year
34.125 = int. for 2nd year
\$716.625 = Principal for 3rd year
35.831 = int. for 3rd year
\$752.456 = Principal for 4th year
9.41 = int. for 3 mo.
\$761.866 = amt. for 3 yr., 3 mo. at 5%
−650.
\$111.866 = compound int. of \$650 for 3 yr., 3 mo. at 5%.

1. When compound interest is payable semi-annually or quarterly, we first find the amount of the given principal for the first interval, and make it the principal for the second interval, etc. When the principal contains years, months and days, we find the amount for the nearest exact interval, and upon this amount compute the interest for the remaining months and days. This amount is then added to the last amount before subtracting the original principal.

The chief use of compound interest is among savings banks, private banking houses, building and loan associations, and insurance companies.

2. Compound interest is usually computed by the use of tables showing the amount of interest and principal when \$1 is loaned at compound interest.

Problems

1. Find the compound interest on \$1500 at 6% for 2 yr., 6 mo.
2. Find the amount and the compound interest on \$2800 at 6% for 3 years, interest compounded semi-annually.
3. If interest is compounded quarterly, what will be the amount and compound interest on \$2000 for 1 yr., 3 mo. at 6%?

The following table shows the compound amount of $1 for intervals of one to ten years:

TABLE OF COMPOUND INTEREST

Years	Amt.	1%	1½%	2%	2½%	3%
1	$1.00	1.0100	1.0150	1.0200	1.0250	1.0300
2		1.0201	1.0302	1.0404	1.0506	1.0609
3		1.0303	1.0456	1.0612	1.0768	1.0927
4		1.0406	1.0613	1.0824	1.1038	1.1255
5		1.0510	1.0772	1.1040	1.1314	1.1592
6		1.0615	1.0934	1.1261	1.1596	1.1940
7		1.0721	1.1098	1.1486	1.1886	1.2298
8		1.0828	1.1264	1.1716	1.2184	1.2667
9		1.0936	1.1433	1.1950	1.2488	1.3047
10		1.1046	1.1605	1.2189	1.2800	1.3439

Years	Amt.	3½%	4%	4½%	5%	6%
1	$1.00	1.0350	1.0400	1.0450	1.0500	1.0600
2		1.0712	1.0816	1.0920	1.1025	1.1236
3		1.1087	1.1248	1.1411	1.1576	1.1910
4		1.1475	1.1698	1.1925	1.2155	1.2624
5		1.1876	1.2166	1.2461	1.2762	1.3382
6		1.2292	1.2653	1.3022	1.3400	1.4185
7		1.2722	1.3159	1.3608	1.4071	1.5036
8		1.3168	1.3685	1.4221	1.4774	1.5938
9		1.3628	1.4233	1.4860	1.5513	1.6894
10		1.4105	1.4802	1.5529	1.6288	1.7908

Example (1): Find the compound interest on $2500.50 for 5 yrs. at 6%, interest compounded annually.

Solution:

$1 compounded annually at 6% for 5 yrs. amounts to $1.3382, as shown by the above table.

$2500.50 × $1.3382 = $3346.1691, amount of $2500.50 compounded annually for 5 yrs. at 6%.

$3346.1691 — $2500.50 = $845.6691, compound interest.

Note: When interest is compounded semi-annually, take $\frac{1}{2}$ the

rate for twice the time. When interest is compounded quarterly, take $\frac{1}{4}$ the rate for 4 times the time.

Example (2): Find the compound interest on \$10,000 at 8% for 2 yr., 3 mos., interest compounded quarterly.

Solution:

$\frac{1}{4}$ of 8% = 2%

4 × 2 years 3 months = 9 years

The amount of \$1.00 compounded at 2% for 9 yrs. = \$1.1950

10000 × \$1.1950 = \$11,950

\$11,950—\$10,000 = \$1,950 compound interest.

Problems

(1) Interest is compounded annually on the following amounts. Find the compound interest on each amount.

1. On \$360 for 2 yr., 10 mo., at 6%
2. " \$510 for 3 yr., 8 mo., " 7%
3. " \$650 for 2 yr., 9 mo., " 5%

(2) A child received a gift of \$200 on his tenth birthday. What would this money amount to, if deposited in a bank paying 4% int. compounded semi-annually, and withdrawn when he was 18 years old?

(3) Draw a diagram showing the yearly increase of this amount of money, for the eight years.

121. Short method of computing compound interest.

1. Rule for calculating interest compounded annually:

n. the number of years

r. the rate of interest

p. the number of dollars of the principal

s. the amount or sum at the end of n years.

$$\text{Formula: } s = p \times (1.00 + r)^n$$

Illustrative Example: On January 1, 1916, Mr. Smith put \$725 in a bank paying 3 per cent interest compounded annually. What did this money amount to by January 1, 1919?

$$s = \$725 \times (1.00 + .03)^3$$

Note: A symbol written above another symbol and on the right of it is called an exponent and denotes how many times the latter is taken as a factor. Thus $(1.00 + .03)^3 = (1.03)^3 = 1.03 \times 1.03 \times 1.03$.

2. Rule for calculating interest compounded semi-annually:
n. the number of half-year periods
r. the rate of interest (annual rate divided by 2)
p. the number of dollars of the principal
s. the amount or sum at the end of n periods
Formula: $s = p \times (1.00 + r)^n$

ILLUSTRATIVE EXAMPLE: An athlete won a prize of $500 in gold. He put the money in a savings bank paying 4 per cent compounded semi-annually. What will his money amount to at the end of 2 years?

$$s = 500 \times (1.00 + .02)^4$$

3. Rule for calculating interest compounded quarterly:
n = the number of quarter year periods
r = the rate of interest (annual rate divided by 4)
p = the number of dollars of the principal
s = the amount or sum at the end of n periods
Formula: $s = p(1.00 + r)^n$

(1) The government bases its compound interest calculations on the average price of a War Savings Stamp, $\$4.17\frac{1}{2}$. Show that $\$4.17\frac{1}{2}$ compounded quarterly for $4\frac{1}{2}$ years amounts to $5.00, the maturity value of the stamp (approximately).

(2) If the stamp is surrendered for cash before the date of maturity it pays a little less than 3 per cent interest. Find the interest on $\$4.17\frac{1}{2}$ at 3 per cent compounded quarterly for $4\frac{1}{2}$ years. How much less is this than the interest on $\$4.17\frac{1}{2}$ compounded quarterly for $4\frac{1}{2}$ years?

122. Thrift measured in dollars and cents.

A few of the wonders of compound interest when applied to saving money:

1. Simple interest is the interest on principal only. If you deposit $100 in a bank that computes interest semi-annually

at 6%, at the end of every six months your money will have earned three dollars. At the end of one year and a half you would have $109. This shows you that the working principal has not increased at all.

(1) What would this same sum earn by putting it out at compound interest at the same rate? In six months it would earn $3, making a total of $103. From this time on, the interest would be added to the principal, and become a part of it, and it would earn money so that instead of having $100 at work you would have $103. For the next six months the interest would be $3.09 making the principal for the following interest period $106.09. The working principal is increased with every interest date by the addition of the interest earned. The dollar that goes to work is the successful dollar because it makes itself a dollar and something, while the careless dollar goes off somewhere and is never seen again.

2. To illustrate how money makes money study the following table of one dime a day saved.

How a Dime a Day Saved and Invested Grows

End of	*Int at 4%*	
1st year		$ 36.50
2nd year	$ 1.46	74.46
3rd year	2.97	113.93
4th year	4.55	154.98
5th year	6.19	197.67
6th year	7.90	242.07
7th year	9.68	288.25
8th year	11.53	336.28
9th year	13.45	386.23
10th year	15.44	438.17

(1) Complete the above table up to the twentieth year. What will the dimes thus saved amount to in 20 years? Draw a plan similar to the above and show "How a nickel a day saved and invested grows," when placed in a bank paying 5%, interest compounded semi-annually. Show what these savings will amount to in 15 years.

(2) If you save 25 cents a day for 1 year, what will you have at the end of a year? Make a table similar to the above showing how your money grows at 5%, compounded annually, saving 25 cents a day.

(3) Consulting the above quotations make a table, and show how money invested in $100 Treasury Savings Certificates, will increase from Dec. 31, 1919, to Jan. 1, 1924, interest compounded quarterly at 4%.

(4) Make a table showing how money invested in $1000 Treasury Savings Certificates will increase, interest compounded quarterly at 4%, from Dec. 31, 1919, to Jan. 1, 1924.

3. Compound interest is the foundation upon which the majority of fortunes are built. It works silently but incessantly, week days, and holidays.

The following table shows the results of systematic savings invested at 4% interest compounded semi-annually for five years. Complete the table to ten years.

How Savings Grow

Monthly Savings	1 year	2 yrs.	3 yrs.	4 yrs.	5 yrs.
$ 1.00	$ 12.24	$ 24.98	$ 38.24	$ 52.03	$ 66.39
2.00	24.48	49.96	76.48	104.06	132.77
3.00	36.73	74.95	114.71	156.10	199.16
4.00	48.97	99.93	152.95	208.13	265.55
5.00	61.21	124.91	191.19	260.16	331.94
10.00	122.42	249.81	382.38	520.32	663.87

Weekly Deposit	Rate	5 yr.	10 yr.	20 yr.
$ .25	4% compounded	$ 73.00	$ 162.00	$ 403.00
.50	twice a year	146.00	324.00	806.00
1.00		293.00	650.00	1614.00
2.00		585.00	1301.00	3228.00

(1) Construct a table like "How Savings Grow" on page 334 and show how savings grow at 5%.

(2) Construct another table showing the results when the payments are made monthly and the interest compounded twice a year.

4. How to double your income.

(1) You are earning $20 a week and receive $1040 a year. Deposit $5 each week in a savings bank at 4% interest compounded semi-annually. When $1000 is accumulated draw it out and invest in some good security drawing 6% interest.

Show that in	5	years	you	will	have	saved.....	$ 1454.03
	10	"	"	"	"	"	..... 3888.99
	15	"	"	"	"	"	..... 5998.57
	20	"	"	"	"	"	..... 9497.13
	25	"	"	"	"	"	..... 14206.13
	28	"	"	"	"	"	..... 17779.11

$17,779.00 at 6% will yield $1,066.92 annually or $20.51 per week. Figure out this result.

(2) A man who earns $30 a week deposits one-fourth of his income in a savings-bank paying 4% interest, compounded semi-annually. In how many years will his savings amount to $26,000?

(3) From the table "How Savings Grow" find the amount of his savings at the end of 5 years; 10 years; 20 years.

ROXBURY CO-OPERATIVE BANK

VALUE OF SHARES

NOVEMBER 2, 1919. (OCTOBER PAID)

Profits at the Rate of $5\frac{1}{8}$ Per Cent Per Annum

Number of Series	Date of Issue	Amount Dues paid per Share	Profits per Share	Total Value per Share	Withdrawal Value per Share
37	Nov. 1907	$144.00	$53.50	$197.50	$197.50
38	May 1908	138.00	48.72	186.72	186.72
39	Nov. 1908	132.00	44.22	176.22	176.22
40	May 1909	126.00	39.97	165.97	165.97
41	Nov. 1909	120.00	35.97	155.97	155.97
42	May 1910	114.00	32.21	146.21	146.21
43	Nov. 1910	108.00	28.69	136.69	136.69
44	May 1911	102.00	25.40	127.40	127.40
45	Nov. 1911	96.00	22.35	118.35	116.11
46	May 1912	90.00	19.51	109.51	107.56
47	Nov. 1912	84.00	16.88	100.88	99.20
48	May 1913	78.00	14.46	92.46	91.01
49	Nov. 1913	72.00	12.25	84.25	83.02
50	May 1914	66.00	10.23	76.23	75.21
51	Nov. 1914	60.00	8.41	68.41	67.57
52	May 1915	54.00	6.75	60.75	60.09
53	Nov. 1915	48.00	5.30	53.30	52.24
54	May 1916	42.00	4.04	46.04	45.22
55	Nov. 1916	36.00	2.97	38.97	38.36
56	May 1917	30.00	2.06	32.06	31.63
57	Nov. 1917	24.00	1.32	25.32	25.06
58	May 1918	18.00	.74	18.74	18.60
59	Nov. 1918	12.00	.34	12.34	12.27
60	May 1919	6.00	.09	6.09	6.07

(1) Is there a co-operative bank in your town?

(2) How does a co-operative bank differ from a regular Savings Bank?

(3) What are the advantages of depositing savings in a co-operative bank?

(4) What is the rate of interest paid in the Roxbury Co-operative Bank?

(5) Mr. Hall owns the following shares in the Roxbury Co-operative Bank:

2 shares of the issue of May, 1908
5 shares of the issue of Nov., 1910
3 shares of the issue of Nov., 1915
5 shares of the issue of May, 1917

a. What are these shares worth Nov. 2, 1919? (Consult the table).

b. What are the profits on all the shares?

c. What is the amount of dues paid?

d. If he were obliged to withdraw the fifteen shares what would he receive?

e. How much would he lose by withdrawing all his shares at this time?

f. Which shares could most profitably be withdrawn if he were obliged to have $600 to use in his business?

g. What would be his amount of dues paid, Nov. 1, 1919, if he had twice as many shares in each of the foregoing issues?

h. What would be the profit on his shares?

i. What would be the total value?

j. What would be the withdrawal value?

THIS IS THE WAY PENNIES GROW

Saved each week	4 per cent per annum compounded semi-annually				
	1 year	2 years	3 years	4 years	5 years
$0.01	$ 0.53	$ 1.08	$ 1.66	$ 2.25	$ 2.88
.10	5.30	10.82	16.56	22.54	26.75
.25	13.26	27.06	41.41	56.34	71.68

(6) Extend the above table to 10 years' time and make the amounts "Saved each week" \$0.50, \$0.75, \$1.00; show how these amounts of money increase from 1 to 10 years.

(7) Find out the actual amount of earnings of your money in the 10 years, on each amount saved each week.

CHAPTER XXI

GOVERNMENT SECURITIES

1. Why was saving money and materials necessary during the war?

2. How did the United States Government raise money to carry on the war, and at the same time provide the best way to invest small sums?

3. Why are Government Bonds and War Savings Stamps excellent investments?

4. Why is investing in Government securities showing good citizenship? How does it show thrift?

5. If the proportion of thrifty people in your community were increased, what effect would it have on the town in which you live?

6. What can you do to increase the number of thrifty people in your town?

7. What are the best ways to invest small savings?

8. How does money spending affect one's character?

9. What is the price of a Thrift Stamp? What is its purpose?

10. How many Thrift Stamps can be bought for $1.00? $4.75? $7.25? $.50?

The value of a stamp for each month is shown in the following table:

Month	1918	1919	1920	1921	1922
January	$4.12	$4.24	$4.36	$4.48	$4.60
February	4.13	4.25	4.37	4.49	4.61
March	4.14	4.26	4.38	4.50	4.62
April	4.15	4.27	4.39	4.51	4.63
May	4.16	4.28	4.40	4.52	4.64
June	4.17	4.29	4.41	4.53	4.65
July	4.18	4.30	4.42	4.54	4.66
August	4.19	4.31	4.43	4.55	4.67
September	4.20	4.32	4.44	4.56	4.68
October	4.21	4.33	4.45	4.57	4.69
November	4.22	4.34	4.46	4.58	4.70
December	4.23	4.35	4.47	4.59	4.71

Jan. 1, 1923 ..... $5.00

All the roads leading to the City of Success enter by one gate, "Thrift."—S. W. Strauss.

123. United States War Savings. The war savings plan is one by which you can lend small savings to your government at 4% interest, compounded quarterly. This is done by purchasing Thrift Stamps and War Savings Stamps. Thrift Stamps cost 25 cents to be applied in payment for a War Savings Stamp. Thrift Stamps do not earn interest: their purpose is to help people accumulate in small sums the amount necessary to pay for a War Savings Stamp. Sixteen Thrift Stamps will represent a value of $4, and can be exchanged for a War Savings Stamp by paying the difference between the $4, and the price of a War Savings Stamp the month in which the exchange is made.

Mr. Frank A. Vanderlip, former president of the National City Bank of New York, says of the War Savings plan:

"The War Savings plan offers the safest, most convenient, and most profitable method of accumulating savings that has ever been presented to any people. A War Saving Stamp is really a United States Government little baby bond, backed up by the entire resources of the United States."

He has also written a new money table. Here it is:

10 mills a cent
25 cents a thrift stamp
16 thrift stamps a war savings stamp
20 war savings stamps a war savings certificate.

War Savings Stamp Problems

(1) A man bought 200 War Savings Stamps at $4.12 each, Jan. 1, 1919. How much money will he gain when the stamps mature in January, 1924?

(2) There were 23,857,000 pupils in all the schools and colleges in the United States in 1916. If each of them buys a Thrift Stamp at $4.15 each, how much money will they have invested?

(3) Jan., 1918, a class of 54 children each bought a War Savings Stamp at \$4.19. How much money did they lend to Uncle Sam?

(4) John buys 76 War Savings Stamps in April, 1919, when stamps cost \$4.15 each. If he should hold them until Jan., 1924, how much money will he gain?

(5) A florist invested his profits of \$224.64 in War Savings Stamps at \$4.16 a piece. How many War Savings Stamps did he buy?

(6) The population of the United States in 1918 was 105,000,000. If 75% of these people bought a War Savings Stamp at \$4.19 apiece, in January, how much money did they lend the Government?

(7) Mr. Black invested 4% of his salary in War Savings Stamps during 1919. He bought 25 stamps at an average price of \$4.16. What was his salary?

(8) A War Savings Stamp in January, 1919, cost \$4.12. In January, 1924, the Government will pay you \$5 for the stamp. What will be the rate of simple interest you receive?

(9) Dr. Gray bought an automobile for \$1200. He received a discount of 5% for cash payment. He put the money saved into War Savings Stamps at \$4.14 each. How many War Savings Stamps did he buy?

(10) If two members in each of the 20 million families in the United States invested in a W. S. S. at \$4.19 apiece each month of the year 1918, how many War Savings Stamps would be purchased? How much money would be loaned to the United States in this way?

BUYING W. S. S. SYSTEMATICALLY

Stamps (1919)	*Average Cost*	*Maturity Value*
1 W. S. S. a month, or \$0.96 a wk.	\$ 50.16	\$ 60.00
2 W. S. S. a month, or \$1.93 a wk.	100.32	120.00
3 W. S. S. a month, or \$2.89 a wk.	?	?
1 W. S. S. a week, or \$4.18 a month	?	?
2 W. S. S. a week, or \$8.36 a month	?	?
3 W. S. S. a week, or \$? a month	?	?
4 W. S. S. a week, or \$? a month	?	?

(11) Complete the above table figuring each stamp at average cost of \$4.18.

Problems

(1) The population of the United States in 1918 was 105,000,000. If 25% of these people bought a War Savings Stamp apiece in February, at $4.21, how much money did they lend the government?

(2) If your salary is $1800 a year and you put 25% of that amount in War Savings Stamps at an average price of $4.15, how many War Savings Stamps do you buy?

(3) Each one of a club of twenty members bought a War Savings Stamp when the price of stamps was $4.18. How much money did the whole club lend to the government?

(4) In 1918, Mr. Green bought twenty War Savings Stamps which cost $83.80. In April he was obliged to redeem his War Savings Stamps to pay a bill. The stamps were worth $4.27 apiece that month. How much did he receive for his twenty stamps? How much money did he gain on the original price?

(5) Seventy-five per cent of a class containing 40 pupils have bought Thrift Stamps to the amount of $60. What is the average amount of money lent to the government by each of those who are saving?

(6) In January, 1919, Mr. Smith bought 250 War Savings Stamps at $4.12 each. How much money will he gain when the stamps mature in January, 1924?

(7) Mary Smith bought a War Savings Stamp on Jan. 1, 1919, for $4.12. The government will redeem this stamp on Jan. 1, 1924, for $5.00. Alice Cook invested $4.12 at 4 per cent, simple interest, for five years. Which is the better investment?

THE BOND MARKET

LIBERTY BONDS

Dec. 26, 1919.	Open	High	Low	Close 3 P.M.	Close Dec. 25.
$3\frac{1}{2}$s 1947	99.54	99.86	99.54	99.72	99.52
1st 4s 1942	93.20	93.20	93.20	93.20	93.20
2d 4s 1942	91.66	91.90	91.66	91.80	91.60
1st $4\frac{1}{4}$s '47	93.40	93.50	93.40	93.40	93.36
2d $4\frac{1}{4}$s '42	91.68	91.94	91.62	91.70	91.64
3d $4\frac{1}{4}$s '28	94.50	94.90	94.50	93.74	94.36
4th $4\frac{1}{4}$s '38	91.70	91.90	91.64	91.86	91.68
Victory $4\frac{3}{4}$s	99.32	99.32	99.06	99.06	99.28
Victory $3\frac{3}{4}$s	99.32	99.32	99.10	99.20	99.36

Quoted in dollars and cents per $100 bond.

(8) The above quotations in Liberty Bonds were given on Dec. 26, 1919. What profit will a person receive who buys ten 3½s at the lowest price quoted, and keeps them until maturity?

(9) What profit will a person receive who buys the following amount of bonds at the above "open" quotations, and keeps them until maturity? (Par value $100.)

Ten 2d 4s 1942
Fifteen 1st 4s 1942
Ten 1st 4¼s 1947
Twenty Victory 4¾s 1924

(10) Mr. Smith owns the following amount of Liberty Bonds:

Two $100 3½s 1947
One $50 1st 4s 1942
Three $100 3d 4¼s 1928

a. What are they worth to-day? *b.* What would Mr. Smith have by selling them now, instead of allowing them to mature? *c.* What interest has he received to date on these bonds (amount of his coupons)?

GOVERNMENT BONDS

Jan. 12, 1920	Open	High	Low	Latest
Lib. 3½s 1932–47	99.60	99.92	99.50	99.52
1st 4s 1932–47	92.96	92.96	92.38	92.88
2d 4s 1927–42	91.90	91.90	91.72	91.74
1st 4¼s 1932–47	93.18	93.24	93.16	93.16
2d 4¼s 1927–42	92.36	92.36	92.24	92.26
3d 4¼s 1928	94.42	94.42	94.	94.06
4th 4¼s 1933–38	92.60	92.60	92.50	92.56
Victory 3¾s 1922–23	99.	99.	98.94	99.
Victory 4¾s 1922–23	98.96	98.96	98.90	98.98

(11) Mr. Crane buys the following amount of Liberty Bonds at the above "Low Quotations":

50 Lib. 1st 4¼s ..... '47
25 Lib. 2d 4¼s ..... '42
25 Lib. 3d 4¼s ..... '28

a. What is the purchase price? *b.* What is the redemption price? *c.* What gain is made by the increase in the redemption price over the purchase price? *d.* What are the present day quotations on

these bonds? *e*. Would he gain or lose by selling them at to-day's quotations? How much? *f*. What are the dates of maturity of the bonds purchased? *g*. What will the yearly interest be on each of the bonds purchased?

(12) Mr. Green owns the following United States Bonds. What is his annual income?

Eight $100 Lib.	$3\frac{1}{2}$s	1947
Six $1000 1st	$4\frac{1}{4}$s	1947
Four $500 2d	4s	1942
Five $500 3d	$4\frac{1}{4}$s	1928

(13) What are the present quotations on these bonds? What will be the value of each bond at maturity?

(14) If the eight $100 $3\frac{1}{2}$s were issued June 15, 1917, what amount would the first coupon, dated Dec. 15, 1917, state was to be paid to bearer? When would the next coupon be due? When can these coupons be cashed?

CHAPTER XXII

PROMISSORY NOTES

1. What advantage to a business are cash customers? Charge customers? Why do some business houses refuse to take charge customers?

2. Is it always a convenience for a merchant in business to pay cash for goods? How can a merchant carry on his business if he has not a large supply of cash?

3. What are banks? How are banks able to make loans?

4. What means do banks employ to prevent a borrower from forgetting the obligation of repaying the amount forwarded?

5. Why is lending money, and receiving money for it, a help to the lender? What institutions make this their principal business?

6. What are some of the dangers connected with borrowing money?

On Aug., 15, 1915, Daniel Cobb borrowed from Henry Smith fifty dollars, to pay a bill. He gave Henry Smith a paper which reads as follows:

A Promissory Note

$50.00	Boston, Mass., Aug. 15, 1915.
'Sixty days after date I promise to pay to	
.....	
Henry Smith	or order
Fifty..... $\frac{00}{100}$	Dollars
At First National Bank	
with interest at 5% for value received.	
	Daniel E. Cobb.

124. Promissory notes. This paper is a promissory note or a note. In this note Daniel Cobb makes a contract, whereby he agrees to pay back the money, with the specified interest, at the appointed time.

1. The necessary items of this promissory note are as follows:

(1) Where the transaction took place: Boston, Mass.
(2) When it took place (date): Aug. 15, 1915.
(3) Name of promisor or maker: Daniel Cobb.
(4) (Why) For value received: "For value received."
(5) Promises to pay: "I promise to pay."
(6) When—time stated: sixty days after date.
(7) To whose order: to the order of Henry Smith.
(8) Amount of money stated: fifty dollars, "$50."
(9) Amount of interest: with interest at 5%.

2. All interest-bearing notes must contain those various items although they may vary in form.

3. The person who promises to pay is the maker, the promisor, or the payor. Who is the maker in the above note?

4. The person to whom the promise is made is the payee or the promisee. Who is the payee in the above note?

5. The person who holds or owns the note—either the payee or some person to whom the payee has sold the note is the holder. Who is the holder of the above note?

6. The sum of money named in the note is the face or principal of the note. What is the face of the note given above?

7. The face of the note is written twice—once in figures, and once in words—to avoid mistakes.

8. The date on which a note matures, or becomes due is the date of maturity. What is the date of maturity of the above note?

EXERCISE

(1) Compute the amount due at maturity on the following notes:

\$450.00 Buffalo, New York, Jan. 1, 1920.

Sixty days after date, I promise to pay to the order of John A. Long, Four Hundred Fifty Dollars, with interest at 6%. Value received.

Henry Thompson.

\$560.00 Los Angeles, Cal., Dec. 31, 1919.

Thirty days after date I promise to pay to the order of Carl A. Poor, Five Hundred Sixty Dollars, with interest at 4%. Value received.

Thomas Carroll.

(2) A promissory note for \$840 dated Jan. 13, 1920, and due in two months, was not paid until June 24, 1920. If the note did not bear interest, what was the amount paid with 6% interest?

(3) On Jan. 18, 1919, I borrowed \$680 with interest at 4% per annum. For how much must I draw my check to settle my indebtedness, on Aug. 3, 1919?

(4) A real estate agent received the following offers for a piece of property: *a.* \$15,000 cash, or, \$5,000 cash and a 6 months' note for \$10,500. Which was the better offer, and how much better, if the rate of discount was 6%? (See §§ 125 and 126.)

(5) Write out a 90-day note for \$1500, dated to-day, payable to James Carr or order, at some bank in your city. Discount the note at 6%, and find the proceeds.

(6) Write a promissory note for $1000 due in 60 days, supplying date and names. If this note is discounted, who endorses it?

(7) Write out a note with the following data:

Face—$800
Date—Jan. 1, 1920
Time—90 days
Discount—Feb. 15, 1920
Payee—John Smith
Maker—James Gould
Payable at the Third National Bank

What are the proceeds of the note?

(8) The following note is discounted May 1, 1919, @ 5%. Find the proceeds.

$860.00 Buffalo, New York, April 2, 1919.

Sixty days after date, I promise to pay to James Hunter..... or order Eight Hundred Sixty.....Dollars
Payable at the First National Bank.
For value received.
Due June 3, 1919.
Discounted May 1, 1919. John A. Dore.

$2500.00 Detroit, Michigan, May 3, 1920.

Three months after date, I promise to pay to the order of Frederick E. Tyrrell, Twenty-Five Hundred Dollars, with interest at 5%. Value received. Louis R. Wall.

(9) Find the interest on the above note.

(10) Blank forms, like the following, are generally used in making out notes.

(11) Compute the amount due at maturity on the note.

$a		b	c 19
d	After date	e	Promise to Pay to the
Order of			f
	g		Dollars
With interest at ..... h ..... %			
Value Received			
			i

a. Tell what is to be written in each of the spaces indicated by the letters.

b. Copy the blank and fill in the spaces, using the following data.

Your own town
Date of to-day
Your own name (maker of note)
To the order of a friend of yours.
Amount of money, $200
Interest at 6%.

(12) On May 1, 1918, William H. Crowley of Plymouth, Mass., bought lumber to the value of $640 from the Eastern Lumber Company. Mr. Crowley gave a written promise to pay in 3 months, with interest at 4%. Write the promise.

What amount must Mr. Crowley pay when the note becomes due? On what date must the bill be settled?

(13) On June 1, 1919, John A. Elwell of Butte, Montana, bought from the Oriental Tea Store $275 worth of tea. One hundred dollars was paid in cash, and a thirty-days' note was given for the balance at 6%. Write the note. When

should the note be paid? Who is the payee? How much was required to settle the note?

(14) Make out notes supplying the date and place of writing, from the following data. Find the date of maturity, and the amount of money due on that date.

	Face	*Time*	*Maker*	*Payee*	*Rate*
a.	$ 450	2 mos.	Fred A. Turner	Harry Warner	4%
b.	540	3 mos.	Samuel Tuttle	A. E. Washburn	6%
c.	690	30 da.	George Ring	Henry Smith	3%
d.	1200	4 mos.	Albert Upton	Charles Loring	5%
e.	1500	60 da.	Louis Wall	Max Vogel	8%
f.	1375	5 mos.	Edward Walsh	Joseph Knight	7%
g.	1450	90 da.	Walter Ward	Peter Sargent	4%
h.	2500	8 mos.	Henry Pool	Alvin White	5%

$560.00 Boston, Mass., Sept. 1, 1921.

On Demand after date, I promise to pay to the order of Carl Woods, Five Hundred Sixty Dollars, with interest at 5%. Value received.

Samuel E. Wheeler.

(15) The above note is a demand note, and is due when payment is demanded. Find out the amount due if payment is demanded on Oct. 1, 1920.

$400.00 Hartford, Conn., July 12, 1922.

Six months after date, we jointly and severally promise to pay to the order of Ralph Stone, Four Hundred Dollars with interest at 6%. Value received.

Myron E. Hayden
John A. Storrs
Harry Pritchard.

(16) The preceding note is a joint-and-several note, and each signer is responsible for the whole amount due at settlement. Find the amount due "six months after date."

\$250.00 Springfield, Mass., July 3, 1919.
Thirty days after date I promise to pay to the order of John A. Prouty
Two Hundred Fifty.....Dollars
At the First National Bank of Springfield.
Value Received
Carl A. Healey.

125. Discounting a note. On July 3, 1919, Carl A. Healey buys goods to the amount of \$250, giving the above note in payment. John A. Prouty, wishing to use the money which this note represents, immediately, asks a bank to buy it; this the bank does, being satisfied that both the maker and the holder are reliable persons. After Mr. Prouty endorses the note (writes his name on the back of the note) the bank accepts it, and pays to John A. Prouty the sum named in the note, less the interest for thirty days at the rate agreed upon, 6%. \$250—\$1.25 = \$248.75. This is discounting the note. The bank is now the owner of the note, and when the note becomes due (30 days after July 3, or, Aug. 2), Carl Healey must pay to the bank the sum named in the note, instead of to Mr. Prouty.

126. Bank discount. Bank discount is the sum deducted from the face of the note by the bank, for its services; thus, \$1.25 is the bank discount on the above; bank discount is simple interest, in advance, on the value of a note when it becomes due, or matures.

1. The value of a note at maturity is its face value, if the note does not bear interest; if it bears interest the value of

a note at maturity is its face value plus the interest for the time stated.

2. The value at maturity of a note for $500 due in 90 days without interest is $500; if it bears interest at 6%, the value at maturity is $500+$7.50, or $507.50.

3. The amount paid by the bank for the note is the proceeds; this is the value of the note at maturity, less the bank discount. Thus, $248.75 is the proceeds of the note shown on p. 351.

4. The date of maturity is the date when a note becomes due, or matures. The date of maturity is found as follows:

(1) When a note is given for a stated number of months, the number of months is added to the date of the note.

Thus, a note for three months dated Mar. 5, becomes due in the corresponding day of the third month after, or June 5. If dated March 31 and due in three months, the date of maturity is June 30. Since there are not enough days in June to correspond with the thirty-first day of March, the note becomes due on the last day of June.

(2) The exact number of days is added to the date of the note, when a note is given for a stated number of days.

Thus, a note for 90 days dated Apr. 6 is due exactly 90 days after Apr. 6, or on July 5.

5. The term of discount is the time from the date when the note is discounted until it becomes due, and is found by counting the exact number of days from the date of discount to the date of maturity.

ILLUSTRATIVE EXAMPLE (1): Find the date of maturity and the term of discount of a three months' note, dated Apr. 6, and discounted May 2.

Solution: April 6+3 mo. = July 6, = date of maturity.
From May 2 to July 6 = 65 days = term of discount.

Explanation: Find the date of maturity by counting forward 3 mo. from April 6, which is July 6. Find the term of discount by

counting all the days from the date of discount, May 2, to the date of maturity, July 6.

Thus,—May.....29 days
June.....30 "
July.....6 "
Sum.....65 "

ILLUSTRATIVE EXAMPLE (2): Find the date of maturity and the term of discount of a 90 days' sight draft dated July 5, accepted July 8, and discounted Aug. 20.

July 8+90 da. = Oct. 6, date of maturity.

From Aug. 20 to Oct. 6 = 47 da., term of discount. A draft drawn at 90 days' sight does not mature until 90 da. after it is accepted, therefore the date of maturity in the above draft is 90 da. after July 8, the exact number of days being counted. The term of discount is the exact number of days from Aug. 20 to Oct. 6 (47 days).

Problems in Bank Discount

(1) A note dated Feb. 12, 1914, payable in 6 months with interest at 6%, was discounted at a New York bank March 1, 1914. Find the discount and the proceeds, if the face of the note was $350.

Explanation:

6 months after Feb. 12, 1914, or Aug. 12, 1914, was the date of maturity. March 1 was the date of discount.

From March 1 to Aug. 12 is 165 days, the term of discount.

$0.0275 = the bank discount (or interest) on $1 for 165 days.

$350 × $0.0275 or $9.625 = the required bank discount.

$350 + $10.50 (interest 6 mo. at 6%) = $360.50 = amount of note at maturity.

$360.50 − $9.625 = $350.87½ = proceeds.

(2) Find the proceeds of a note of $500 dated Aug. 1, 1916, payable in 6 mo. with interest at 5%, if this note was discounted Nov. 30, 1916 at 6%.

Explanation:

$500 —the face of the note
12.50—the interest for 6 mo. at 5%
$512.50—the amount of the note at maturity
5.12—the bank discount for 60 da. at 6%
$517.62—the proceeds.

(3) Find the proceeds of the following notes:

	Face	Time	Date	Interest	Date of Discount	Rate of Discount
1	$2800	3 mo.	July 1	5%	Aug. 15	5%
2	$4320	5 mo.	Apr. 12	6%	July 1	6%
3	$340.80	60 days	Sept. 15	6%	Oct. 1	6%
4	$950.50	4 mo.	Oct. 1	5%	Dec. 15	5%
5	$500.00	30 days	Dec. 31		Jan. 3	7%
6	$325.75	2 mo.	Apr. 30		May 15	$4\frac{1}{2}$%
7	$4200	3 mo.	Mar. 10		Mar. 25	6%

(4) Find date of maturity, term of discount, bank discount, and proceeds of following notes and drafts, each bearing interest at 5%.

	Face	Time	Date of Paper	Date of Acceptance	Date of Discount	Rate of Discount
1	$515.50	2 mo.	July 1		Aug. 12	6%
2	$350.75	60 days	July 15		" 5	6%
3	$2600	90 "	Mar. 10		Apr. 11	5%
4	$1000	1 mo.	Aug. 5		Aug. 18	5%
5	$2250	30 days after date	May 10	May 15	May 24	6%
6	$1850	3 mo. after date	June 1	Aug. 10	Sept. 1	6%
7	$500.00	30 days sight	Sept. 30	Oct. 15	Oct. 25	5%

(5) Many short-sighted merchants fail to take advantage of discounts by paying bills promptly. Most wholesale dealers give a liberal discount for receiving their money within 10 days of the date of the bill. It is sometimes a saving to borrow money at the bank to discount the bills.

Example:
Amount of bill $100, if paid in 30 days
3% discount if paid within 10 days ($3.00)
This means $3.00 for 20 days; the interest on $100 for 20 days at 6% is $.33$\frac{1}{2}$. This shows a saving of $2.67. The saving on $1000 would be $26.70.

(6) A bill of $250 is due in 60 days. A discount of 5% is allowed if paid within 10 days. How much is saved if the bill is paid on the tenth day?

(7) Why is it economy to borrow money and discount bills?

$1425.00 Providence, R. I., Dec. 6, 1921.

At three (3) days' sight pay to the order of

PHOENIX NATIONAL BANK, PROVIDENCE, R. I.

Fourteen hundred twenty-five Dollars,
and charge the same to account of
Value received.

To S. E. Duran
Newport, R. I.

DAVOL RUBBER COMPANY
James A. Hall,
Acting Treas.

(8) Find the proceeds of the following sight drafts:

	Face	Collection and exchange.
a.	$ 560.00	$\frac{1}{2}\%$
b.	1280.00	$\frac{1}{8}\%$
c.	2460.00	$\frac{1}{4}\%$
d.	4500.00	$\frac{1}{4}\%$
e.	5672.00	$\frac{1}{10}\%$

CHAPTER XXIII

TAXES AND INSURANCE

1. Name some of the imported goods in a large department store. How does the price of these goods compare with the price of domestic goods? What is the cause of the difference in price?

2. When a tax is imposed on goods imported into the country from foreign countries, what is the tax called? How are these duties authorized?

3. For what is the revenue received from duties and customs used? Tell what you can of the purpose of duties and customs.

4. Does the United States Government receive revenue from any goods manufactured in this country? What is the tax from this source called?

5. Name some articles manufactured in this country from which Internal Revenue is obtained.

6. What imports are taxed heavily for the sake of revenue only? Is this justifiable?

7. Are these articles of luxury or of necessity?

8. From which five countries does the United States buy most of its goods? From what countries do most of the important goods in your store come?

9. How will knowledge of this subject benefit you as a store employee? As a customer buying goods? As a citizen of the United States?

127. National taxation. The National Government obtains the money with which it pays its bills from two sources:

(1) Duties (customs or tariff), which are taxes levied upon goods imported from foreign countries.

(2) Internal revenue (excises), or taxes levied upon certain articles which are grown or manufactured in this country (cigars, tobacco, etc.). Congress fixes a tariff or list of duties in the United States. These duties are collected at

the Custom House, established by the government for the transaction of business relating to duties, at the various Ports of Entry.

1. The Collector of the Port is the officer in charge of the Custom House; his duties are to supervise all entries and papers relating to them, estimate all moneys and employ assistants. (Pupils should visit the Custom House if possible.) Import duties are of two kinds:

(1) An ad valorem duty, which is a certain per cent of the value of the imported goods.

(2) A specific duty, which is a tax levied upon the number, weight, or measure of the goods (ton, bale, pound, barrel, etc.) without regard to their value.

2. When goods are purchased in a foreign country, an invoice or manifest (statement showing the names of the various articles being imported, with their actual cost or value, made out in the currency of the country from which they are imported) is filed with the United States Consul, in the place where the purchase is made. He sends a copy to the Collector of the Port where the goods are to be sent; when the goods arrive the cases are opened and the quantity and value are compared with those stated in the invoice. If they are found to be undervalued, a tax is imposed.

3. Travelers' baggage is also inspected for dutiable goods. Importing goods without paying the required duty or underestimating the value, is smuggling. Cite any instances of smuggling that you may know. How are smugglers treated by the government?

4. Duties are figured by United States officers in even dollars only; fifty cents or more is counted as another dollar; any amount smaller is not considered. Thus, if the duty value of goods was $125.25, the goods are valued at $125; but if the duty value is $125.65, it is counted as $126. This makes computation easier.

Problems

(1) What is the ad valorem duty on an invoice of serges at 15%, which cost $1240 in England?

(2) What is the duty on a bale of Irish linen which cost $4280, at 33⅓%?

(3) Skein silk, costing 5508 francs in Paris and weighing 625 lb. net, was taxed $.30 on the pound and 12% ad valorem. What was the amount of duty to be paid?

(4) What is the duty on 250 yd. of lace invoiced at $.25 per yard and 25% ad valorem?

(5) What is the duty on 2400 lb. of wool invoiced at $.16 a pound at 22% ad valorem?

(6) Goods are invoiced at 12,600 francs. At 36%, what is the duty? (1 franc = $.193 at the normal rate of exchange. What is its value now?)

(7) What is the duty at 60% upon a consignment of 425 dozen kid gloves, invoiced at 80 fr. a dozen?

(8) What is the specific duty on 800 quarts of oil at $1.20 a gallon? What is the duty at 5½¢ a pound on 42 boxes of dates weighing 158 lbs. each, tare (weight of covering), 18 lbs. on each box.

(9) What is the duty at 36¢ a square yard and 10% ad valorem on 1000 sq. yd. of carpet, invoiced at $1.75 a sq. yd.?

(10) *a*. A shipment of straw-matting, 50 pieces averaging 40 yd. each. *b*. 30 pieces of linen, 30 yd. each. *c*. 15 pieces tricotine, 50 yd. each.

(11) Estimate the duty on the above goods, at the present day quotations.

(12) Explain how the family living in a rented house helps pay the city bills.

(13) How does your mother, who buys the meat, help the butcher to pay his rent?

(14) If men overcharge the city for work that they do, how does this affect taxes?

(15) Explain how everyone has to help pay for schools.

(16) What kinds of buildings or other property in your town are not taxed?

(17) Explain how your father helps to pay the State tax.

(18) What is the difference between a tax and a license?

(19) What is a war tax? Who pays such a tax? Name five different ways in which you are helping to pay for the Great War.

(20) Why is the subject of taxes an important one to you? To the community? To the State? To the nation?

(21) Name some purposes for which cities and states levy taxes. Why should the people pay for public improvements, schools, care of the blind, insane, sick, etc., and salaries of public officials?

(22) What is a tax? A poll tax? Name some of the different kinds of taxes levied in your State.

(23) What is your city tax rate? State some causes for the increase or decrease in the tax levy. How are tax rates fixed?

(24) What is meant by an inheritance tax? An income tax? A property tax? A personal tax?

(25) What items are contained in a tax bill? (Tax bills should be brought to the class and real problems made from them.)

(26) What is a license? What is its purpose? How is a license obtained? What people are required to have a license?

(27) What is done with the revenues received from the sale of licenses?

128. Taxes. The State Legislature determines the amount of money required for taxes. The amount of taxable property in the state is determined by the assessors (local officers appointed to estimate the taxable value of property). The total amount of money to be collected is divided by the number of dollars of taxable property; this fixes the amount to be paid on one dollar, so a certain per cent of the property assessed is a tax. Thus, if the total amount of revenue needed by a state is \$1,500,000 and the assessed valuation of all property in the State is \$60,000,000, the tax rate is $\$1{,}500{,}000 \div \$60{,}000{,}000 = \frac{1{,}500{,}000}{60{,}000{,}000} = \frac{1}{40}$, which is $2\frac{1}{2}\%$ or 25 mills on the dollar.

If a man's assessment is \$2800 and he has to pay 25 mills on every dollar, his taxes $= 2\frac{1}{2}\%$ or $\frac{1}{40}$ of \$2800.

$\frac{1}{40}$ of \$2800 = \$70, the amount of tax.

Problems

(1) If your property is valued at \$4200 and the tax rate is $2\frac{1}{2}\%$ (or 25 mills on the dollar), what is your tax?

(2) A small town must raise \$8540 on property assessed at

$600,000. What must a person pay whose property is assessed at $4200?

(3) The taxable property of a town is valued at $1,250,000 and a tax of $8650 is voted for school purposes. What is the rate of taxation?

(4) When an owner's tax is $35 at the rate of $1\frac{3}{4}\%$, on what valuation is he taxed?

(5) How do the subjects of taxes and percentage differ? How are they similar?

(6) The assessed valuation of property in a town is $45,672,800. What is the amount of the tax, if the tax rate is $1.75 per $100?

(7) How does a merchant protect his merchandise against loss by fire or by accident?

(8) What is the purpose of insurance?

(9) Why are corporations, cities, towns, and all business people interested in insurance?

(10) What is meant by property insurance? By life insurance? What are the advantages of each?

(11) What is meant by a fireproof building? How does the rate of insurance on this kind of building compare with that on other buildings? State other advantages in making a building fireproof.

(12) How is the cost of fire insurance on a building affected by its nearness to other buildings? Give other causes which increase or decrease the cost of fire insurance.

(13) State different ways in which losses to property and life occur.

(14) Is life insurance a good investment? Why?

(15) Do you know how to ring in a fire alarm? Give reasons why knowledge of this subject adds to your efficiency in any employment.

(16) What are the fire insurance rates in your town on different kinds of buildings? Why do the rates differ? When property is insured against loss, it is a property insurance; when a person is insured against a loss due to accident, sickness, or death, it is a personal insurance.

(17) What is a contract? Who are the contracting parties in an insurance policy? Can you give examples of any other contracts?

129. Insurance. An insurance is a contract (policy) guaranteeing to make good a loss from a certain cause. This agreement is known as an insurance policy. Property insurance includes indemnity against damage or loss by fire,

called fire insurance; against damage or loss to a ship or its cargo, by accident at sea, called marine insurance; against loss or damage to horses, cattle, etc. (from lightning or other casualty), called live-stock insurance. The premium is the amount paid for the insurance for a certain period of time. Business buildings are usually insured against fire for a single year; dwellings and personal property for three years.

Problems

(1) Merchandise valued at \$3600 was insured at an annual premium of \$126. What was the rate?

(2) A cargo worth \$86.40 was insured for $\frac{9}{10}$ of its value at $4\frac{1}{2}\%$. What would be the actual loss to the owner in case of shipwreck?

(3) Goods worth \$6000 are insured for $\frac{2}{3}$ value. If the annual premium is \$30, what is the rate?

(4) I insure \$4800 worth of merchandise for $\frac{3}{4}$ of its value at 80¢ per \$100. What premium must I pay?

(5) If it costs \$98.04 a year to insure $\frac{2}{3}$ of the value of a store, at $1\frac{1}{2}\%$, what is the whole value of the store?

(6) If your life is insured for \$3500 and you pay an annual premium of \$25.00 on every \$1000 worth of insurance, how much will your insurance cost you every year?

(7) Mr. James had his life insured for \$1000 for the benefit of his wife. The annual premium was \$32.81. At the age of 40, Mr. James died after paying four annual premiums. How much more did his widow receive than Mr. James paid in premiums? How much did the company lose by the contract?

(8) Find the annual premiums on:

a. \$5000 worth of goods insured for $\frac{4}{5}$ of their value at 1%.

b. \$6000 worth of goods insured for $\frac{2}{3}$ of their value at $1\frac{1}{2}\%$.

c. \$7000 worth of goods insured for $\frac{1}{2}$ of their value at $1\frac{1}{4}\%$.

d. \$10,000 building insured for $\frac{1}{2}$ of its value at $1\frac{1}{2}\%$.

e. \$2000 building insured for $\frac{3}{4}$ of its value at $1\frac{1}{2}\%$.

(9) What is the premium on a house valued at \$6500 insured for $\frac{3}{4}$ of its value, at $1\frac{1}{4}\%$?

(10) What is the cost of insuring a row of 8 houses for \$5600 each, at $1\frac{3}{4}\%$?

(11) What will be the premium on an \$8000 policy where the rate of insurance is \$7.50 on a \$1000?

CHAPTER XXIV

BONDS AND STOCKS

1. Is a $1 bill a "promise to pay"? Is a $5 or a $10 bill? The sum written on the face of the bond is called the par value or face value.

Bonds issued by the United States Government are Government Bonds; those issued by State Governments are State Bonds; those issued by Cities, Counties, etc., are Municipal Bonds; and those issued by Business Corporations, are Industrial Bonds.

2. When a person borrows money, he usually gives a promissory note. When the government or a corporation borrows large sums of money, it usually issues bonds. The United States Government needed billions of dollars to carry on the war; it borrowed the money from the people and gave them bonds called Liberty Bonds.

130. Bonds. A bond is a written or printed promise to pay a sum of money at a certain time, with interest at regular intervals, at a fixed rate per cent per year.

1. A Liberty Bond is the direct and unconditional promise of the United States Government to pay, upon a certain date, a specified sum of money in gold, with interest semi-annually upon such sum until the bond is paid.

2. A bond states the time at which the principal is payable; it bears interest payable annually, semi-annually, or quarterly, as specified.

3. When a person owns a registered bond, his name is registered or recorded on the books of the corporation, and when the interest is due, a check is sent to the bondholder in payment.

4. When a person owns a coupon bond (one to which in-

terest certificates, called coupons, are attached) he clips the coupon from the bond on the date when interest payment is due and presents it at a bank for payment.

5. Bonds pay a fixed income and are usually safe investments. When a bond is sold the buyer usually pays not only the market value of the bond but also the interest which has accrued since the last interest day. Bonds are usually issued in denominations of \$50, \$100, \$500, or \$1000, so that they may be sold to small investors.

ILLUSTRATIVE EXAMPLE: A owned a $4\frac{1}{2}\%$ Government Bond which he wished to sell. The par value of the bond was \$1000.00 and it was quoted at 98. B bought the bond on April 1 at its market value, plus accrued interest. Interest is paid semi-annually, June 1 and December 1. How much did A receive?

Solution: Since the bond was sold at 98, A received 98% of its face value.

98% of \$1000.00 = \$980.00

Interest at $4\frac{1}{2}\%$ has accrued on the bond from Dec. 1 to April 1, four months. Interest on \$1000 for 4 months at $4\frac{1}{2}\%$—

\$15.00 accrued interest

Market value of bond—\$980.00

Accrued Int. 15.00

\$995.00—Selling Price.

6. The customary brokerage for large transfers is $\frac{1}{8}\%$ of the par value of the bonds.

Problems

(1) Find the cost of five \$100 U. S. bonds at $101\frac{1}{2}\%$, brokerage $\frac{1}{8}\%$.

(2) Find the proceeds from the sale of \$8000 Atlantic Gulf R. R. bonds at $102\frac{1}{2}\%$, brokerage $\frac{1}{8}\%$.

(3) How many \$1000 U. S. bonds at $104\frac{1}{2}\%$ can be bought for \$16,740, brokerage $\frac{1}{8}\%$?

(4) Find the annual income from six \$100 Liberty Bonds, U. S. 4s, 1932; from 10 \$100 U. S. Liberty Bonds, $4\frac{1}{4}$s, 1928; from 25 \$100 U. S. Liberty Bonds, $4\frac{1}{4}$s, 1933.

(5) Find the annual income from ten \$1000 Reading R. R. Co. $4\frac{1}{2}\%$ bonds.

(6) A bond house quotes Atlantic Gulf $4\frac{1}{2}\%$ bonds of \$1000.00 at par and accrued interest. Interest due March 15 and September 15. What is the cost of a bond purchased May 15? October 15?

131. Stocks. A corporation is a body doing business as a single person under a general law or a special charter.

Find out the advantages of a corporation over a partnership.

The amount of capital of a corporation is the capital stock, which is divided into a number of equal parts called shares. The face or original value of a share is its par value.

1. A certificate of stock is a document specifying the number of shares to which the holder is entitled and the par value of each.

2. Dividends are the profits of the corporation which are divided among the stockholders; dividends are usually a certain per cent of the par value of the stock. Thus, if a 5% dividend is declared on stock, par value \$100, the stock owners are paid 5% of \$100 or \$5 on each share owned.

3. If the corporation fails to produce any profits, not only is there no dividend, but the stock becomes worthless. In this case it cannot be sold and the investor's money is lost.

Which is the safer investment, stocks or bonds? Why?

4. Stock is usually bought and sold in the stock exchange, which is an auction room where brokers buy and sell stocks.

$\frac{1}{8}\%$ of the par value is the usual charge of the broker selling the share and of the broker buying the share.

Problems

(1) Find the cost of 10 shares of American Locomotive Stock (\$50) at $\$60\frac{1}{2}$ a share, brokerage $\frac{1}{8}\%$.

Explanation:

$\$50.00$ = the par value of 1 share
$\frac{1}{8}\%$ of \$50 or $\$0.0625$ = the brokerage on 1 share
$\$60.50$ = the market value on 1 share
$\$60.5625$ = the entire cost of 1 share
$10 \times \$60.5625 = \$605.625$ = the entire cost of 10 shares

(2) Find the annual income on 25 shares of Union Motor Company (\$50), which pays a quarterly dividend of 3%.

Explanation:

\$50 — the par value of 1 share.
3% of \$50 = \$1.50 = the quarterly dividend on 1 share.
6.00 = the annual income on 1 share.
$25 \times 6 = \$150$ = the annual income on 25 shares.

(3) Find the cost of 25 shares Lehigh Valley R. R. Stock at \$50 at $\$54\frac{7}{8}$ a share, brokerage $\frac{1}{8}\%$.

(4) Find the cost of 150 shares of United States Steel Stock at $\$91\frac{1}{4}$, brokerage $\frac{1}{8}\%$.

(5) What is the annual income from 10 shares of American Sugar (\$100) if it pays a quarterly dividend of $1\frac{3}{4}\%$?

(6) What is the annual income from 15 shares of United Fruit Co. (\$100) if it pays a quarterly dividend of 2%?

(7) Mr. Gray bought 12 shares of American Locomotive at \$82 and sold them at $\$101\frac{3}{4}$ per share. How much did he make?

(8) What yearly income will a person receive from \$12,000 invested in U. S. 4s?

(9) Which is the better investment on \$6000 and how much: to buy 4s at 120 or 6s at 150?

(10) A man who had \$10,000 loaned at 5% interest took his money and bought $4\frac{1}{2}\%$ bonds at 80. Was his income increased or decreased, and how much?

(11) If you sell 140 shares of $3\frac{1}{2}\%$ stock at \$75, and with the proceeds buy 5% stock at par (\$100), how much is your income increased?

(12) Bought 150 shares of railroad stock at $103\frac{3}{4}$, commission $\frac{1}{8}\%$. What was the total cost?

(13) How many shares of stock at $\$134\frac{1}{8}$ can be bought for \$20,137.50; brokerage $\frac{1}{8}\%$?

(14) A sold 240 acres of land at \$37.50 per acre, and with the

proceeds bought stock at \$90 a share. How many shares did he receive?

(15) A company with \$3,000,000 capital declares a 5% dividend. What does a holder of 100 shares (\$100 par value) receive?

ILLUSTRATIVE EXAMPLE: What must be the market value of 6% stock in order that it may yield 5% on the investment, brokerage $\frac{1}{8}$%?

Explanation:

\$100 par value of 1 share.
.06 × 100, \$6 annual dividend on 1 share.
5% of the investment = \$6
100% of the investment, \$120.
\$0.125, the brokerage on 1 share.
\$120—\$0.125, = \$119.875 the market value of 1 share.

(16) What must the market value of 7% stock be in order that it may yield 6% on the investment, including brokerage $\frac{1}{8}$%?

(17) What must be the market value of 5% stock to yield an investor 4% interest on his investment, brokerage $\frac{1}{8}$%?

(18) How many shares of stock (\$100 par value) paying 6% dividends must be purchased to produce an annual income of \$300?

(19) What is the amount paid in dividends on the following:

Number of Shares \$100 par value		Rate of Dividend on	
Preferred	*Common*	*Preferred*	*Common*
(1) 50	100	6%	$4\frac{1}{2}$%
(2) 125	250	7%	6%
(3) 500	1200	$5\frac{1}{2}$%	7%
(4) 625	1500	8%	5%
(5) 350	575	$6\frac{1}{2}$%	$7\frac{1}{2}$%

CHAPTER XXV

EXCHANGE AND MONEY ORDERS

How can money be sent to a person at a distance? What are the different ways of sending money? What is the easiest way of sending money? Have you ever sent a registered letter?

132. Exchange. The process of settling accounts at distant points without actually sending the money is called *exchange*. Money may be safely sent by mail in a letter that is registered at a post office. The cost of sending a registered letter is 10¢; a receipt showing the delivery of the letter is returned to the person sending it by the post office.

1. Exchange is of two kinds: domestic and foreign. Domestic exchange treats of the payment of debts between people in the same country without the sending of money. For example, if a Boston dealer owes a New York merchant for a bill of merchandise, he can pay his debt without sending money, in any one of the following ways:

(1) By his personal check (§ 101).
(2) By a bank draft (illustrated § 126).
(3) By a commercial draft on one of his debtors (§ 127).
(4) By a postal money order (§ 107).
(5) By an express money order (§133).
(6) By a telegraphic money order (§133).

133. Money orders. Postal money orders are most often used and may be purchased at any post office. These are orders directing the postmaster at the place where the other person lives, to pay him the money. The person who presents the order must be known by the postmaster or iden-

tified. A small fee is charged varying with the amount of the order. Money forwarded in this way to any part of the United States, certain parts of South America and the West Indies, is sent by Domestic Order. If money is to be sent to a foreign country, a Foreign or International Money Order must be used, which differs very little from the Domestic Money Order.

Express Check

1. Express money orders are issued by express companies in about the same form and at the same rate as post office money orders. A receipt for the money forwarded is given the sender, and if the order is lost, a refund may be obtained by guaranteeing the company against loss, should the lost order be cashed. Express companies and banks also pay by telegraph orders, instructing their agents at a certain place to deliver to the payee the amount given.

(1) Which of these ways would be used in an emergency? Why? Which is the quickest way? Why is this way more expensive than the others? Is money ever sent through the mails without registering, or by post office order? Is this a safe way? Why?

2. A telegraph money order is an order telegraphed by one agent of the company to another directing him to pay

a stipulated sum of money to a specified person either identified or unidentified.

(1) The cost of sending a telegraphic money order is added to the regular cost of the telegram and is as follows:

a. For $25 or less	$.25
b. Over $25 and not over $50	.35
c. Over $50 and not over $75	.60
d. Over $75 and not over $100	.85
e. For every $100 or over, to $3000, add	.25

(2) A code is used, which is understood only by the company and its agents for the telegraphic order between the depositing and paying agents.

Problems

(1) Compare the costs of sending $125 by postal money order, and by telegraphic money order, if a fifteen-word telegram costs $1.25.

(2) A business man pays the following bills: $79.50 by express money order; $156.00 by postal money order; $784.00 by telegraphic money order (telegram 75¢); and $3674.00 by draft purchased at $.9850. What is the total amount of settlement?

(3) A merchant wishes to settle a debt of $3780. Which is better, and how much, to send his check or to send a bank draft purchased at $.9985?

(4) To find the cost of a money order or draft.

At ½% premium, what is the cost of a draft on New York for $3000?

½% of $3000 = $15, premium
$3000 + $15 = $3015, cost.

Explanation: Exchange is selling at a premium of ½%; the premium amounts to ½% of $3000, or $15. The cost is the sum of $3000 and $15, or $3015.

(5) What is the cost of a draft for $4500, when exchange is selling at 50¢ discount?

Explanation: 50¢ discount means 50¢ discount on $1000.
4.5 × .50 = $2.25, discount on $4500.
$4500 — $2.25 = $4497.75, cost.

NOTE: The discount on \$1000 is 50¢, on \$4500 is $4\frac{1}{2}$ times \$.50 or \$2.25. \$4500 less \$2.25 = \$4497.75, the cost.

(6) To find the proceeds of a draft.

Collection and exchange being $\frac{1}{8}\%$, find the proceeds of a sight draft of \$1420.

NOTE: The bank charges $\frac{1}{8}\%$ for collection: $\frac{1}{8}\%$ of \$1420 = \$1.775. The proceeds is the difference between \$1420 and \$1.775, or \$1418.225.

Solution: $\frac{1}{8}\%$ of \$1420 = \$1.775, collection and exchange.

\$1420—\$1.775 = \$1418.225, proceeds.

(7) Find the proceeds of a 60-day commercial draft for \$1800, if sold on its date at $\frac{1}{2}\%$ discount, money being worth 6%.

Solution: \$18.00 = the interest for 60 days.

$\frac{1}{2}\%$ of \$1800 = \$9.00, the discount.

\$18.00 + \$9.00 = \$27.00, the total discount.

\$1800—\$27.00 = \$1773.00, the proceeds.

NOTE: The solution is identical with the work in bank discount.

CHAPTER XXVI

BANKS OR BUSINESS INSTITUTIONS

1. Care and economy in the spending of money is just as important as the earning of it. Economy does not mean spending a

Banking House

small amount, as much as it means getting the largest returns for the amount spent.

2. Does your cash account tell you that you are saving or wasting your money? Many people not having money on hand acquire the habit of wasting money by buying foolishly on credit, or on installments, realizing too late that much more has been paid for the

goods than they are worth. Other people acquire the habit of saving money by depositing it in banks where money earns money. Everyone ought to have a bank account. Banks pay a small sum to you for the use of every dollar you deposit. There are many different kinds of banks for the receiving and safe keeping of money and the making of loans.

3. Why do we save money? Why do people put money in the bank?

134. Banks. There are three different kinds of banks:

(1) Commercial banks or banks of deposit.

(2) Savings banks.

(3) Trust companies.

1. Banks of deposit or commercial banks are business men's banks; they receive money for safe keeping on deposit, and loan money to business men at interest. Notes and other negotiable paper are usually bought at such banks.

National banks, authorized by the United States and controlled by its laws, belong to this class. They have authority to issue paper money to the value of a certain amount of their stock.

2. Savings banks are established to care for people's savings. They receive money for safe keeping and pay interest to the depositor. Accounts may be opened at most savings banks with a small amount, the depositor being furnished with a bank book in which further deposits, interest, credits, and withdrawals are recorded.

Money deposited in savings banks may be withdrawn at any time, but it should not be taken out to spend except in cases of great need.

Coöperative banks are savings banks in which each depositor makes regular monthly payments of a stated amount; these banks receive deposits, lend money on first mortgages, and pay interest semi-annually on deposits.

The United States has established the Postal Savings Bank, the Post Office receiving the deposits for savings.

These banks are guaranteed to be absolutely safe, but the interest paid is less than in regular savings banks.

3. Trust companies are state institutions, carried on by corporations, which settle estates, care for money or property left in trust, and do a banking business.

Banks aid business in the following ways:

(1) They serve as safe places in which to deposit money. This is necessary, as money carried about in the pocket or kept in a cash box or drawer, is apt to be stolen. Business men put what they do not want to use, in a bank.

(2) They cause more money to be in circulation than would otherwise be possible. Many people who deposit money in banks do not have a sufficient amount over and above their needs to admit of loaning it; but when small amounts are gathered through these deposits, the bank can loan them in large sums. In this way the bank makes use of money that would otherwise be idle.

(3) Banks enable many payments to be made by check. The use of checks saves business men much trouble. How?

9. State other advantages of banks to the individual; to the community; to corporations. How can banks afford to receive money and cash checks for their depositors? How do banks get their revenue?

CHAPTER XXVII

RATIO, PROPORTION, AND PARTNERSHIP

What is the relation of 8 yd. to 4 yd.? Of 4 yd. to 8 yd.?

135. Ratio. Ratio is the relation of one number to another of the same kind expressed by the sign : written between the two numbers and equivalent to the sign ÷. Thus the ratio of 8 to 4 is written 8:4, and means 8÷4, or 2. The ratio of 4 to 8 is written 4:8, and equals $\frac{4}{8}$ or $\frac{1}{2}$. The numbers compared are the terms of the ratio.

1. The first term is the antecedent, and the second the consequent. Thus in the ratio 8:4, 8 is the antecedent and 4 the consequent.

$$\text{The ratio of 4 to 8} = \tfrac{4}{8} = \frac{\text{antecedent}}{\text{consequent}} = \frac{\text{dividend}}{\text{divisor}} = \frac{\text{numerator}}{\text{denominator}}$$

At sight, tell the ratios:

(1) 6:8	(4) 12 bu.:18 bu.	(7) 12 oz.:1 lb.
(2) 24:36	(5) 3 ft.:9 ft.	(8) 75¢:$1.50
(3) 1 pk.:1 bu.	(6) 8 qt.:3 gal.	(9) 1 lb. 2 oz.:2 lb.

2. A direct ratio is the quotient of the antecedent divided by the consequent.

3. An inverse ratio is the quotient of the consequent divided by the antecedent. Thus the inverse of 4:9 is 9:4.

4. All the principles of fractions apply to ratio since a ratio may be expressed in the form of a fraction whose numerator is the antecedent and whose denominator is the consequent. Ratio may be single or compound.

8:10 is a simple ratio. $\left.\begin{matrix} 4:7 \\ 9:10 \end{matrix}\right\}$ is a compound ratio.

Example $\frac{3}{4}$ to $\frac{5}{6}$.

Solution: $\frac{3}{4}=\frac{9}{12}$.

$\frac{5}{6}=\frac{10}{12}$.

$\frac{9}{12}:\frac{10}{12}=9:10=\frac{9}{10}$; or $\frac{3}{4}\div\frac{5}{6}=\frac{3}{\cancel{4}_{2}}\times\frac{\cancel{6}^{3}}{5}=\frac{9}{10}$.

5. Multiplying or dividing both terms of a ratio by the same number does not change the value of the ratio. Since ratio shows comparative size, it may exist only between quantities like in kind. Fractions having a common denominator are in the ratio of their numerators: fractions not having a common denominator should be reduced to equivalent fractions having a common denominator.

EXERCISE

State at sight the ratio of the first price to the second in the following increase in food stuffs in a period of five years:

(1) Quaker oats per package from 8¢ to 12¢.

(2) Beans per can from 12¢ to 19¢.

(3) Butter from 32¢ per lb. to 79¢.

1. Name two numbers that have the same ratio as 5 to 10; as 8 to 4; as 12 to 3.

136. Proportion. Proportion is an equality of ratios; thus, 5:8 = 10:16, is a proportion and may be read, the ratio of 5 to 8 equals the ratio of 10 to 16. The sign of proportion is the double colon (::); so 5:8::10:16 is read, 5 is to 8 as 10 is to 16.

1. The first and fourth terms are the extremes; the second and third terms are the means; the first and third, the antecedents, and the second and fourth, the consequents.

2. In every proportion the product of the means equals the product of the extremes.

Thus: in the proportion 6:8 = 3:4, the product of the means (8×3), 24 = the product of the extremes (6×4) 24.

3. Since the product of the means equals the product of the extremes, a missing extreme is found by dividing the product of the means by the given extreme.

Thus: in 6:8=3:?

$$\frac{8\times 3}{6}=4$$

EXERCISE

In these examples x stands for the missing term; find the value of x:

(1) $x:12=10:18$

(2) $24:16::x:8$

(3) $37\frac{1}{2}$yd.:$62\frac{1}{2}$ yd.::$\$12\frac{1}{2}$:$\$x$.

(4) If 8 yd. of cloth cost 96¢, what will 16 yd. cost?

Quantity	Cost
8	96¢
16	x¢

$8:16::96:x$

$$\frac{16\times 96}{8}=1.92=x$$

Ans. $1.92

Explanation: The answer is to be dollars, so we make 96¢ the third term. The same relation must exist between the cost of 8 yd. and the cost of 16 yd. as exists between 8 yd. and 16 yd. Since 16 yd. cost more than 8 yd., the first ratio must read 8:16, because if the fourth term is to be greater than the third term, the second term must be greater than the first.

ILLUSTRATIVE EXAMPLE (1):

If 24 rugs cost $384, what will 16 rugs cost?

Solution:

Quantity	Cost
24	$384
16	?

$24:16::\$384:?$

$$\frac{16\times 384}{24}=256$$

Ans. $256

Explanation: Since the answer is to be in dollars, we make $384 the third term. 16 rugs will cost less than 24 rugs. Since the fourth term is to be less than the third, the second term must be less than the first.

NOTE: In example No. (4) the cost is directly proportional to the quantity; in Example No. (1) the quantity is inversely proportional to the cost.

4. To solve problems in proportion:

(1) Make that number the third term which is of the same denomination as the required answer.

(2) If the answer is to be greater than the third term, write the larger of the two remaining terms for the second term; if the answer is to be less than the third term, write the smaller for the second term.

(3) Find the missing terms, cancelling when possible.

Problems

(1) At 65¢ a dozen, how much will 18 oranges cost?

(2) For how long a time should \$200 be loaned to balance a loan of \$500 for 9 months?

(3) An electric light burning $3\frac{1}{2}$ hr. each night will last about 220 nights: how many nights will it last if it were burned $2\frac{1}{2}$ hours each night?

(4) If $12\frac{3}{4}$ tons of coal cost $\$121\frac{1}{8}$, what will $8\frac{2}{3}$ tons cost?

(5) If $5\frac{3}{8}$ yd. of cloth cost $\$4.03\frac{1}{8}$, how many yards can be bought for $\$6\frac{3}{8}$?

137. Partnership.

1. An association of individuals called partners associated for the purpose of transacting business is called a partnership. A partnership is sometimes called a firm.

2. The amount which the partners invest in the business is the Capital Stock.

3. The assets of a firm are all the property belonging to the firm; the liabilities are all the debts of a firm.

4. Gains and losses are distributed among the partners in proportion to their respective shares, *when the shares of the partners are invested for equal times.*

5. Gains and losses are distributed among the partners in proportion to their respective shares and times, *when the shares of the*

partners are invested for unequal times, except when by special agreement some other plan is adopted.

Illustrative Examples (1): A, B, and C form a partnership. A invests \$2000, B \$1500, and C \$1000. What is each one's share of the gain of \$2500?

Explanation:

\$2000 + \$1500 + \$1000 = \$4500, the capital stock

$\frac{4}{9}$ of the capital stock = A's investment.

$\frac{1}{3}$ of the capital stock = B's investment.

$\frac{2}{9}$ of the capital stock = C's investment.

\$2500 = the gain to be divided.

The gain is distributed in proportion to the investments.

$\frac{4}{9}$ of \$2500, or $\$1111\frac{1}{9}$ = A's gain.

$\frac{1}{3}$ of \$2500, or $\$ 833\frac{1}{3}$ = B's gain.

$\frac{2}{9}$ of \$2500, or $\$ 555\frac{5}{9}$ = C's gain.

(2) Three girls invested their savings in a lunch-room. Mary invested \$250 Jan. 1, Frances \$230 Mar. 1, and Katherine \$220 Aug. 1. At the end of the year they had a profit of \$1260 to divide among them: What was each one's share?

Explanation:

Mary's capital was invested 12 mo., Frances' 10 mo., and Katherine's 5 mo.

Mary's gain = the gain on \$250 for 12 mo. or the gain on \$3000 for 1 mo.

Frances' gain = the gain on \$230 for 10 mo., or the gain on \$2300 for 1 mo.

Katherine's gain = the gain on \$220 for 5 mo., or the gain on \$1100 for 1 mo.

The total gain = the gain on (\$3000 + \$2300 + 1100), \$6400 for 1 mo. The total gain = \$1260.

$\frac{1}{3}\frac{5}{2}$ of \$1260, or $\$590.62\frac{1}{2}$ = Mary's gain.

$\frac{2}{6}\frac{3}{4}$ of \$1260, or $\$452.81\frac{1}{4}$ = Frances' gain.

$\frac{1}{6}\frac{1}{4}$ of \$1260, or $\$216.56\frac{1}{4}$ = Katherine's gain.

Problems in Partnership

(1) A merchant failing in business has property to the value of \$6400. He owes A \$9500, B \$5000, and C \$4000; how much should each receive?

(2) Three men invested in a $6000 hardware business. At the end of the first year their gains were $3000. The first invested $1600 the second $2000, and the third $2400. Find each one's share of the profit.

(3) A and B are partners; A invests $10,000 for 8 mo. and B $15,000 for 6 mo. Their gains amount to $6800. What is each man's share?

(4) The rent of a field is $72. A puts in 24 sheep for 30 weeks and B 40 sheep for 36 weeks. How much should each pay?

(5) A, B, and C are partners. A put into the business $6000, but withdrew $\frac{1}{2}$ of it at the end of 6 mo.; B put in $4000, and added $100 at the end of 4 mo.; C put in $5000 for the whole year. The gain during the year was $10,200. Find each one's share.

(6) A bankrupt's liabilities are $35,000, and his assets $31,250. If the assignee's expenses are $1500, what will a creditor whose claim is $3400 receive?

APPENDIX

Foreign money [1]—English.

Show English coins. Is foreign money ever received in stores? Customers from what foreign lands visit our stores?

1. The unit of United States money is the dollar; do you know the unit of English money? French money? German money? The interchange of commerce between the United States and foreign countries necessitates the exchange of foreign money into United States money.

2. An Englishman visiting this country has £50, 8s. 6d. What must he do with this money in order to pay his bills? Where can this exchange of money be made? Will he lose by this transaction? Why do banks charge for this exchange of foreign money into United States money?

3. English money is the currency of Great Britain, the unit being the pound (£).

TABLE OF ENGLISH MONEY WITH UNITED STATES EQUIVALENTS

4 farthings (far.)	=1 penny (d.)	=$0.02
12 pence	=1 shilling (s.)	=$0.243+
20 shillings	=1 pound sterling (£)	=$4.8665

(1) From the above table find the equivalent in United States money for £50, 8s. 6d.

(2) Of £25; of £7, 3s. 2d.

[1] Unless otherwise indicated the normal rate of exchange is used throughout. The present rate of exchange, the result of war conditions, is subject to rapid change, and it is uncertain when the normal rate will be reëstablished.

(3) What is the value of £300 in United States money?

(4) Change £15 to dollars; £35 to dollars.

(5) I sold 10 pr. shoes at 6 shillings a pr. How many pounds sterling did I receive?

(6) At 10d. per hour, how much English money did an Englishman receive in 6 days, if he worked 8 hours a day?

(7) How much United States money would he receive in exchange for £2, 10s?

(8) Solve the above problems on the basis of the current rate of exchange.

Foreign money [1]—French.

Show French coins. In France, the currency is decimal.

1. What advantage has a decimal currency? The unit of French money is the franc, which equals $0.193 or approximately 20¢ of United States money.

2. How many francs equal $1 of United States money?

3. How much United States money is equal to a ten-franc piece?

4. A twenty-franc piece?

5. A forty-franc piece?

6. A hundred-franc piece?

7. A two-franc piece?

8. Change 15 francs to United States money.

9. Change 23 francs to United States money.

10. Find the cost of 6 books at 8 francs apiece in United States money.

11. Change $30 to francs.

12. Change $45 to francs.

13. A Frenchman changed 500 francs for United States money. $1.50 was charged for the exchange. How much United States money did he receive?

[1] See note on page 381.

14. Solve the above problems on the basis of the current rate of exchange.

Foreign money[1]—German.

Show coins. The unit of German money is the mark. Its value is $.2385 in United States money, or 24¢.

1. What is the value of a 5-mark piece?
2. Of a 10-mark piece?
3. Of a 20-mark piece?
4. Change 19 m. to United States money.
5. Change $55 to German money.
6. Change 26 m. to United States money.
7. Change $100 to German money.
8. A clerk earns 4 m. per day. What is the equivalent in United States money?
9. How many marks should be received in exchange for $14.40?
10. If a German should exchange 900 marks for United States money, how much would he receive?
11. Solve the above problems on the basis of the current rate of exchange.

Equivalents. Approximate equivalents of foreign money in the United States:

One pound = $5.00	One mark (m.) = $.25
One shilling = $.25	One franc (fr.) = $.20
One lira = $.193	One rouble = $.514

Give the value of each of the above amounts to-day.

Problems

(1) An American sees the following prices in a London shop window:

Gloves, 4/8 (4s. 8d.); stockings, 1/10½; gown, 55/0; coat 2–5–6,

[1] See note on page 381.

(£2, 5s. 6d.); card case, 2/4½. What is the approximate equivalent in United States money?

(2) What is the total amount of the bill in the above example, if exchange is quoted at $4.87?

(3) An agent bought merchandise in Berlin for 200 m. What is this amount in United States money at the present rate of exchange?

(4) A Frenchman paid 16.75 francs for some silk. How many dollars and cents is this equal to at present-day quotations?

(5) 52 fr. was paid for merchandise. About how many dollars is this at the present rate of exchange?

(6) When 19s., English money, are worth $4.75, what are 25s. 1d. worth?

(7) On August 15, 1915, the British pound sterling was quoted at $4.64; in France the franc was quoted at 16.61 cents (6.02 francs to the dollar); in Italy the lira at 15.4 cents (6.48 lira to the dollar). What was the loss on the pound if the normal figure is $4.87¼? On the franc if the normal figure is 19.7 (5.06 fr. to a dollar)? On the lira if the normal figure is 19.3?

(8) Comparison of the results obtained by solving the following problems at the normal rate of exchange and at existing rate of exchange shows the importance of a stable rate in ordinary business transactions:

a. What is the value of 500 marks in United States money?

b. What will be the cost in United States money of 6 pr. of gloves at 6 shillings a pair?

c. What is the value of £120 in United States money?

d. How many pounds sterling are equal to $199.67?

e. Find the cost in dollars and cents of 150 yd. of silk at 10 francs per yard.

f. If $86.85 is exchanged for French money, how much will be received?

g. When olive oil is 3 lira a quart, what is the value of 60 quarts in United States money?

h. If a person exchanges 1000 roubles for United States money, pays a bank $7.50 for the exchange, how much money will he receive?

i. When marks are quoted at 150 for a dollar, how many will be received in exchange for $462?

The metric system. The metric system of weights and measures originated in France, and is based on a decimal scale, the only multiplier or divisor used being 10, or a power

of 10. All reductions are made by simply changing the position of the decimal point.

1. This system is used in nearly all the civilized nations in the world, except in England and in the United States. The United States Government however requires the use of the metric system in all medical work of the Navy and the other War Departments, and in the Public Health and Marine Hospital Service. Its use is obligatory in Porto Rico, and in the Philippine Islands.

2. The primary units are as follows:

For length, meter = 39.37 in.

For capacity, liter (lēter) = .908 dry quarts: 1.0567 liquid qts. (equivalent to a cube having an edge one-tenth of a meter long).

For weight, gram = 15.432 grains = weight of a cube of distilled water having an edge of $\frac{1}{100}$ of a meter.

3. The words "meter," "gram," and "liter" are combined with the seven prefixes, shown below to form the metric table.

Latin prefixes express fractional parts
- mili = .001 = (millimeter = $\frac{1}{1000}$ meter)
- centi = .01 = (centimeter = $\frac{1}{100}$ meter)
- deci = .1 = (decimeter = $\frac{1}{10}$ meter)

Greek prefixes express multiplier
- Deka or Deca = 10 = (decagram = 10 grams)
- Hekto or Hecto = 100 = (hectogram = 100 grams)
- Kilo = 1000 = (kilograms = 1000 grams)
- Myria = 10000 = (myriagrams = 10000 grams)

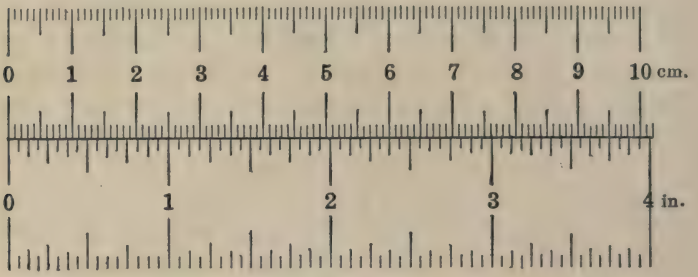
prefixed to

Units
- Meter, for length
- Gram for weight
- Liter for capacity

Measures of Length

4. The meter is the unit of length: it equals 39.37 inches. The following scales show the comparison between 10 centimeters and 4 inches. 10 centimeters = 1 decimeter.

The meter is used for the same purposes as are the foot and the yard in the English system. The smallest subdivisions are millimeters, which are used in fine measurements in machine construction.



Scale Showing Comparison between 10 cm. and 4 in.

Metric Linear Table

10 millimeters (mm.) = 1 centimeter (cm.) = .01 meter
10 centimeters (cm.) = 1 decimeter = .1 meter
10 decimeters (dm.) = 1 meter = 39.37 in. = 1. meter
10 meters (m.) = 1 decameter (Dm.) = 10. meters
10 decameters = 1 hectometer (Hm.) = 100 meters
10 hectometers = 1 kilometer (Km.) = 200 rd. = 1000 meters
10 kilometers = 1 myriameter (Mm.) = 10,000 meters

Note that the abbreviations for the Latin forms begin with small letters; those for the Greek forms, with capital letters.

Measures of Volume

The units of cubic measure are obtained by cubing the units of linear measure. The cubic meter is the unit of volume.

Table

1000 cubic millimeters (cu. mm.) = 1 cubic centimeter (cu. cm.) = .000001 cu. m.
1000 cubic centimeters = 1 cubic decimeter (cu. dm.) = .001 cu. m.
1000 cubic decimeters = 1 cubic meter (cu. m.) or stere (s) 35.315 cu. ft. = 1. cu. m.

EXERCISE

Complete the following table of comparisons.

a. 1 cu. meter = ? cu. in.
b. 1 cu. meter = ? cu. ft.
c. 1 cu. foot = ?cubic meters.
d. 1 cu. yd. = ? cubic meters.

Measures of Capacity

The unit of capacity measure is the liter, which is equal in volume to a cubic decimeter.

A liter is about a quart; it is equal to 1.057 qt. (liquid measure) and .908 qt. (dry measure). Small quantities are purchased by the liter. Large quantities (similar to those measured by the bu. in our system) are measured by the hektoliter. (Nearly 2.84 bu.)

Table

10 millimeters (ml.) = 1 centiliter (cl.) = .01 liter
10 centiliters = 1 deciliter (dl.) = .1 liter
10 deciliters = 1 liter (l.) = 1. liter
10 liters = 1 dekaliter (Dl.) = 10. liters
10 dekaliters = 1 hektoliter (Hl.) = 100. liters
10 hektoliters = 1 kiloliter (Kl.) = 1000 liters
1 centiliter = .01 of a liter: 1 hektoliter = 100 liters

EXERCISE

(1) Reduce 658.1 to Dl; Hl; cl.
(2) Reduce .03576 K. to l.; dl; Hl.
(3) One peck equals how many liters?
(4) One bushel equals how many liters?
(5) One hektoliter equals how many bushels?

Measures of Weight

The unit of weight is the gram. A liter of water weighs a kilogram or 1000 grams. A 5-cent piece weighs about

5 grams. A kilogram weighs 2.2046 pounds avoirdupois or 2.2 pounds.

TABLE

10 milligrams (mg.)	= 1 centigram (cg.)	= .01 gram
10 centigrams	= 1 decigram (dg.)	= .1 gram
10 decigrams	= 1 gram (g.)	= 1. gram
10 grams	= 1 dekagram (Dg.)	= 10. grams
10 dekagrams	= 1 hectogram (Hg)	= 100. grams
10 hectograms	= 1 kilogram (Kg.) (2.2046 lb.)	= 1000. grams
1000 kilograms	= 1 metric ton	= 1,000,000 grams

The gram ($\frac{1}{28}$ oz. avoirdupois) is used in weighing precious metals, medicines, etc.; the kilogram is used in weighing groceries; the metric ton (2205 lb.) is used in weighing coal, iron, ore, etc.

An amount expressed in different metric units may be expressed and read, as we express a sum of money made up of \$3000, \$15, 6d., 8¢, 2 m. in one denomination, \$3015.682, which is sometimes read 3015.682 dollars, and sometimes, three thousand fifteen dollars, sixty-eight cents, two mills.

Thus: 2 Km. 3 Dm. 2 M. 5 Cm. 4 Mm. is expressed as 2032.054 meters and is read two thousand thirty-two and fifty-four thousandths meters, or 2 kilometers, 32 meters, 54 millimeters.

Reductions from one denomination to another are made by moving the decimal point to the right or to the left.

Thus, 2938 mm. = 293.8 cm = 29.38 dm = 2.938 m.

0.567 m. = 5.67 dm. = 56.7 cm. = 567 mm.

EXERCISE

Read:

(1) 3 Km. in hectometers; in decameters; in meters.

(2) 824 mm. in centimeters; in decimeters; in meters.

Write:

(3) 568 mm. in meters.

(4) 2368.52 m. in kilometers, meters, and millimeters.

(5) Reduce 5.68 Km. to dm.
(6) Reduce .0496 Km. to M.

Square Measure

The unit of square measure is the square meter for small areas and the sq. decameter or 100 sq. m. for land areas. The units of square measure are obtained by squaring the units of linear measure.

Table of Square Measure

100 square millimeters (sq. mm.)	=1 square centimeter (sq. cm.) =.155+ sq. in.
100 square centimeters	=1 square decimeter (sq. dm.) +15.5+ sq. in.
100 square decimeters	=1 square meter (sq. m.) =1.196 + sq. yd.
100 square meters	=1 square dekameter (sq. Dm.) =119.6034 sq. yd.
100 square dekameters	=1 square hectometer (sq. Hm.) =2.47114A.
100 square hectometers	=1 square Kilometer (sq. Km.) =247.114A =.3861 sq. mi.

EXERCISE

(1) Reduce 362 sq. m. to sq. Dm.; sq. Hm.
(2) Reduce 8 sq. m. to sq. cm.; sq. mm.
(3) One square meter equals how many square inches?
(4) One square meter equals how many square feet?
(5) One square foot equals how many sq. centimeters?
(6) One square yard equals how many square meters?
(7) Reduce 3672 g. to Kg.; cg.; mg.
(8) Reduce .08 cg. to mg.; g.; Dg.
(9) One pound avoirdupois equals how many grams?
(10) One kilogram equals how many ounces avoirdupois?
(11) One kilogram equals how many pounds Troy?

1 centigram = .01 of a gram: 1 kilogram called a kilo (K.), = 1000 grams. The interrelation of the different measures is shown by the following:
1 milliliter of water = 1 cu. centimeter in volume; 1 gram in weight.
1 liter of water = 1 cu. decimeter in volume; 1 kilogram in weight.

Approximate equivalents in United States measures:

1 meter = $1\frac{1}{10}$ yd.	1 hectare = $2\frac{1}{2}$ A.
1 kilometer = $\frac{5}{8}$ mi.	1 stere = $\frac{1}{4}$ cd.
1 liter = $\frac{9}{10}$ qt. (dry)	1 gram = $\frac{1}{30}$ oz. (avoir.)
1 liter = 1 qt. (liquid)	1 kilogram = $2\frac{1}{5}$ lb. (avoir.)

EXERCISE

(1) Measure the edges of the cover of your largest book, using the meter stick in making the measurements. State the measurements; measure again, using the foot rule. Compare the results.

(2) How many square centimeters in the surface of the cover? How many square inches?

(3) Make a cube from cardboard which will contain 1 liter.

Make a cube from cardboard which will contain 1 cu. inch.

Make a cube from cardboard which will contain 1 cu. cm.

Compare results.

(4) I buy 16 meters of cloth at $1.35 per meter. What do I pay for it? How many yards would this piece of cloth measure? At the same rate what would be the cost per yard?

(5) The distance from Boston to New York is about 200 miles. How many kilometers?

Reference tables of weights and measures.

LINEAR MEASURE

Linear measure is used for measuring distances.

Table

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.) 36 in.
$5\frac{1}{2}$ yd.	= 1 rod (rd.) $16\frac{1}{2}$ ft.
320 rd.	= 1 mile (mi.) 1760 yd. = 5280 ft. = 63,360 in.

The unit of length is the yard.

Square Measure

Square measure is used to measure the area of surfaces.

Table

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yd.	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.) = 102,400 sq. rd. 27,848,400 sq. ft. = 4,014,489,600 sq. in.

A square is 100 sq. ft. (the unit used in roofing).

The acre is the unit in measuring land, other than city lots.

The square yard is the unit in measuring other surfaces.

Cubic Measure

Cubic measure is used to measure the volume of solids and the contents or capacity of hollow bodies.

Table

1728 cu. inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
$24\frac{3}{4}$ cubic feet	= 1 perch (p.)
128 cubic feet	= 1 cord (cd.)
1 cubic yard (of earth)	= 1 load

A perch of stone or masonry is $16\frac{1}{2}$ ft. (1 rd.) long, $1\frac{1}{2}$ ft. wide, and 1 ft. high.

A cord of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high.

A cubic foot of water weighs $62\frac{1}{2}$ lb. (avoir.)

Approximate Measure

A ton of hay is about 450 cu. ft.
A ton of stove coal is about $34\frac{1}{2}$ cu. ft.

Liquid Measure

Liquid measure is used for measuring liquids, and in estimating the capacity of cisterns, tanks, reservoirs, etc.

Apothecaries' Weight

Apothecaries' weight is used by druggists and physicians in compounding and prescribing medicines.

Table

20 grains (gr.) = 1 scruple (sc.)
3 scruples = 1 dram (dr.) = 60 gr.
8 drams = 1 ounce (oz.) = 24 sc. or 480 gr.
12 oz. = 1 pound (lb.) = 96 dr., 288 sc. = 5760 gr.

Avoirdupois Weight

Drugs and chemicals are bought and sold wholesale by avoirdupois weight. Avoirdupois weight is used in weighing all sorts of coarse, heavy articles.

Table

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweight = 1 ton (T.)
1 T. or 20 cwt. = 2000 lb. = 32,000 oz.

The long ton (2240 lb.) is used in the U. S. Custom House in determining the duty on merchandise taxed by the ton. Coal and iron are sold wholesale at the mines by the long ton.

Time Measure

60 seconds (sec.)	= 1 minute (min.)	52 weeks	= 1 year (yr.)
60 minutes	= 1 hour (hr.)	12 months	= 1 year
24 hours	= 1 day (da.)	365 days	= 1 common year
7 days	= 1 week (wk.)	366 days	= 1 leap year
30 days	= 1 month (mo.)	100 years	= 1 century.

Circular or Angular Measure

Circular or angular measure is used in measuring angles or areas of circles as applied to latitude, longitude, surveying, etc.

Table

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circle (cir.) = 21,600′ = 1,296,000″

Paper Measure

24 sheets = 1 quire (qr.)
20 quires = 1 ream (rm.)
2 reams = 1 bundle (bdl.)
5 bundles = 1 bale (bl.)

Counting Table

20 units = 1 score
12 units = 1 dozen (doz.)
12 dozen = 1 gross (gro.)
12 gross = 1 great gross (gr. gro.)

Table

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.) = 8 gi.
4 quarts = 1 gallon (gal.) = 8 pt. = 32 gi.
$21\frac{1}{2}$ gal. = 1 barrel (bbl.) = 126 qt.
2 bbls. = 1 hogshead (hhd.) = 63 gal. = 252 qt.

The unit of liquid measure is the wine gallon of 231 cu. in. A gallon of water weighs about $8\frac{1}{3}$ lb. (avoir.).

Apothecaries' Fluid Measure

Apothecaries' fluid measure is used by druggists in prescribing and compounding liquid medicine.

Table

60 minims (m.) = 1 fluid drachm (f. ʒ) (fl. dr.)
8 fluid drachms = 1 fluid ounce (f. ℥) (fl. oz.) = 480 m.
10 fluid ounces = 1 pint (o) = 128 fl. dr. = 7680 m.
8 pints = 1 gallon (cong.) = 128 fl. oz. = 1024 fl. dr.

United States Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.) = 16 pt.
4 pk. = 1 bushel (bu.) = 32 qt. = 64 pt.

Liquid and Dry Measure Compared

	Gallon	*Quart*	*Pint*
Liquid	= 231 cu. in.	$57\frac{3}{4}$ cu. in.	$28\frac{7}{8}$ cu. in.
Dry	= $268\frac{4}{5}$ cu. in. ($\frac{1}{2}$ peck)	$67\frac{1}{5}$ cu. in.	$33\frac{3}{5}$ cu. in.

Troy Weight

Troy weight is used in weighing gold, silver, diamonds, and other precious minerals. It is used by the government in weighing coins at the mint, by the jewelry trade and manufacturers, and by importers and exporters of gold and silver.

The troy pound is the standard of weight in the United States.

Table

44 grains (gr.) = 1 pennyweight (dwt.)
20 pennyweights = 1 ounce (oz.) = 480 gr.
12 ounces = 1 pound (lb.) = 240 dwt. = 5760 gr.

Carat

The term "carat" is a denomination of weight used in weighing diamonds, and is equal to 3.168 gr.

Carat is used to denote the fineness of gold, and means $\frac{1}{24}$ part. For example, gold marked 18 K. (18 carats) is $\frac{18}{24}$, by weight, pure gold, and $\frac{6}{24}$ alloy. A mark of 24 carats on an object means that it is pure gold.

Comparative Table of Weights and Measures

1 inch	= 25.4001 millimeters
1 foot	= .304801 meter
1 yard	= .914402 meter
1 mile	= 1.60935 kilometers
1 square inch	= 945.16 square millimeters
1 square foot	= .09290 square meter
1 square yard	= .8361 square meter
1 square mile	= 2.59 square kilometers
1 cubic foot	= .02832 cubic meter
1 cubic yard	= .7646 cubic meter
1 acre	= .4047 hectare
1 liquid quart	= .94636 liter
1 liquid gallon	= 3.78543 liters
1 dry quart	= 1.1012 liters
1 peck	= 8.80982 liters
1 bushel	= .35239 hectoliter
1 grain	= .06480 gram
1 ounce (Avoir.)	= 28.3495 grams
1 ounce (Troy)	= 31.10348 grams
1 pound (Avoir.)	= .45359 kilogram
1 pound (Troy)	= .37324 kilogram
1 millimeter	= .03937 inch

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